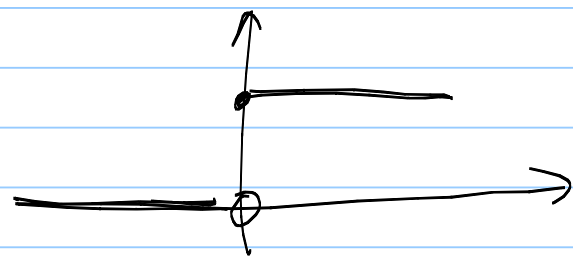
* Watch Online Video 1.4.1

* $\boxed{CT}$ Unit step is a signal

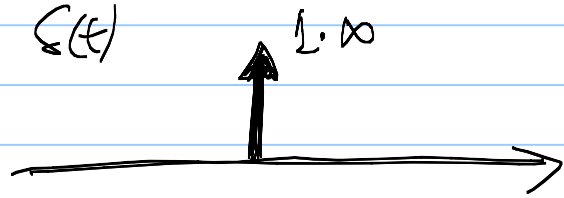
$$u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$



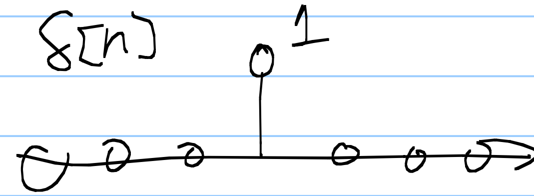
Unit impulse is a signal

$$\delta(t) = \begin{cases} 0 & \text{if } t \neq 0 \\ \infty & \text{if } t = 0 \end{cases} \quad \text{s.t.}$$

the area underneath the "infinite spike" is 1.



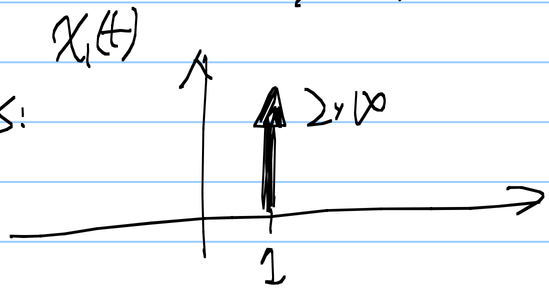
Comparison

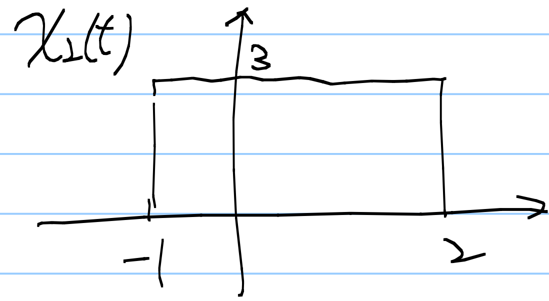


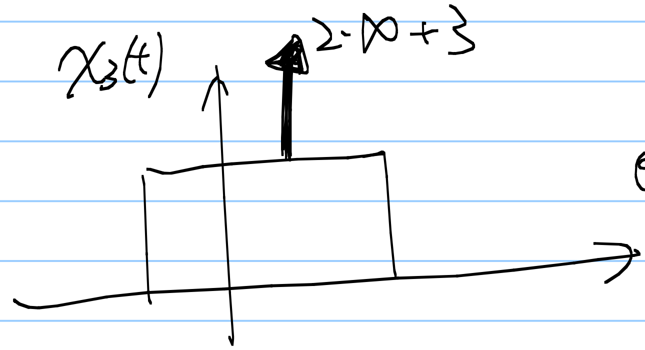
Example: Q1: Plot $x_1(t) = 2\delta(t-1)$, $x_2(t) = 3u(t+1) - 3u(t-2)$

$$x_3(t) = x_1(t) + x_2(t).$$

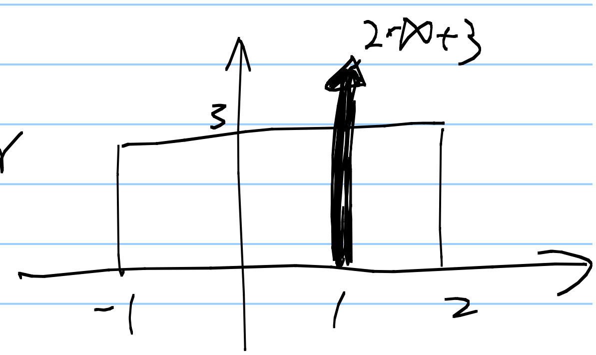
Ans:







or



Q2: Plot $y(t) = \int_{-\infty}^{t} x_3(s)\,ds$

Ans:

Case 1: $t \leq -1$

$$y(t) = 0$$

Case 2: $-1 < t < 1$

$$y(t) = \int_{-1}^{t} 3\,ds$$

$$= 3(t-(-1)) = 3t+3$$

Case 3: $1 < t \leq 2$

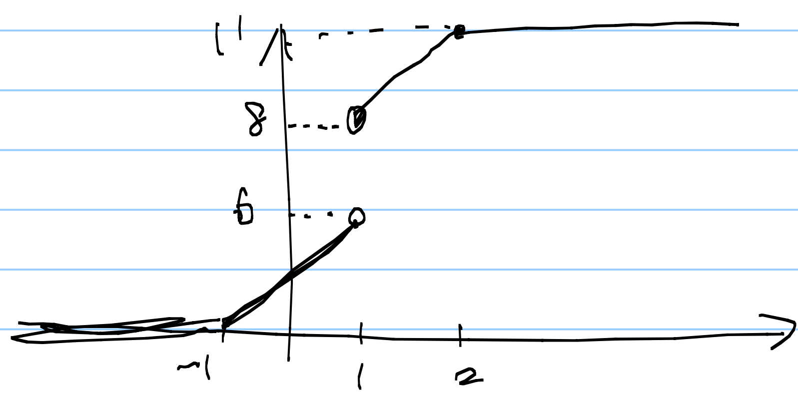
$y(t) = \int_{-1}^{t} 3\,ds$ + the area of the impulse

$$= (3t+3)+2 = 3t+5$$

Case 4: $2 < t$

$y(t) = \int_{-1}^{2} 3\,ds$ + the area of the impulse

$$= 9+2 = 11$$



* Properties of CT unit step / impulse

- Conversion between $u(t)$ & $\delta(t)$

① $\delta(t) = \frac{d}{dt} u(t)$

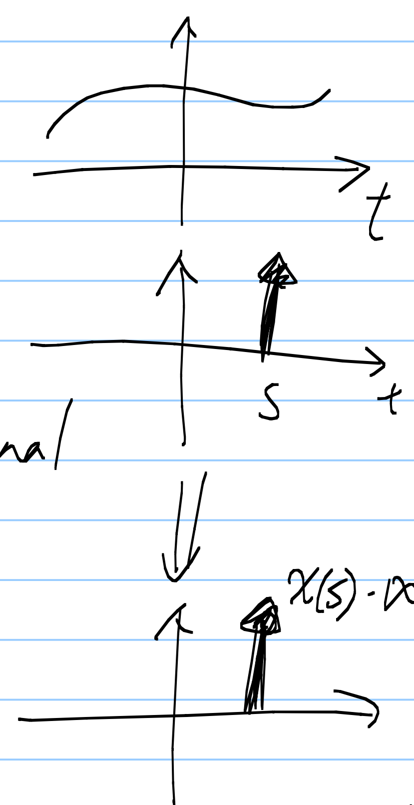
② $u(t) = \int_{-\infty}^{t} \delta(s)\, ds$

③ $u(t) = \int_{0}^{\infty} \delta(t-s)\, ds$

- Integration of $\delta(t)$

$$\int_{-\infty}^{\infty} \delta(s)\, ds = 1 = \int_{-\infty}^{\infty} \delta(t-s)\, ds$$

- Sampling Property



$$\underbrace{x(t)}_{\text{signal}} \cdot \underbrace{\delta(t)}_{\text{signal}} = \underbrace{x(0)}_{\text{const}} \underbrace{\delta(t)}_{\text{signal}}$$

$$x(t) \cdot \delta(t-s) = x(s)\, \delta(t-s)$$

- Decomposing $x(t)$ as a weighted integral

$$x(t) = x(t) \cdot 1$$

$$= x(t) \cdot \int_{-\infty}^{\infty} \delta(t-s)\, ds$$

$$= \int_{-\infty}^{\infty} x(t)\, \delta(t-s)\, ds$$

Comparison

DT : $x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k]$

$$\boxed{x(t) = \int_{-\infty}^{\infty} \underbrace{x(s)}_{\text{coeff}}\, \underbrace{\delta(t-s)}_{\text{(test) signal}}\, ds}$$

The test signals are

$x_s(t) = \delta(t-s)$ indexed by s.

$$\therefore\ x(t) = \int_{-\infty}^{\infty} \alpha_s\, x_s(t)\, ds$$

$$= \int_{-\infty}^{\infty} x(s)\, \delta(t-s)\, ds$$

For Linear Sys.

$$\therefore\ y(t) = \int_{-\infty}^{\infty} \alpha_s\, y_s(t)\, ds$$

$$= \int_{-\infty}^{\infty} \underbrace{x(s)}_{\text{coeff}}\, \underbrace{y_s(t)}_{\text{signal}}\, ds$$

For this semester, our test signals are either HRCE or shifted unit impulses

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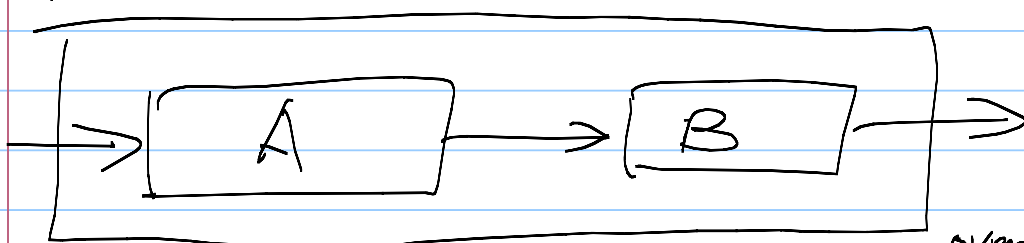
Enough of signals. Let us focus on the systems.

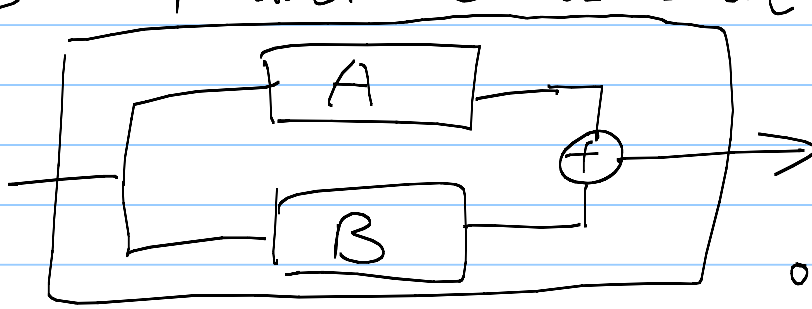
* Systems can be interconnected

Sys 1: Serial concatenation



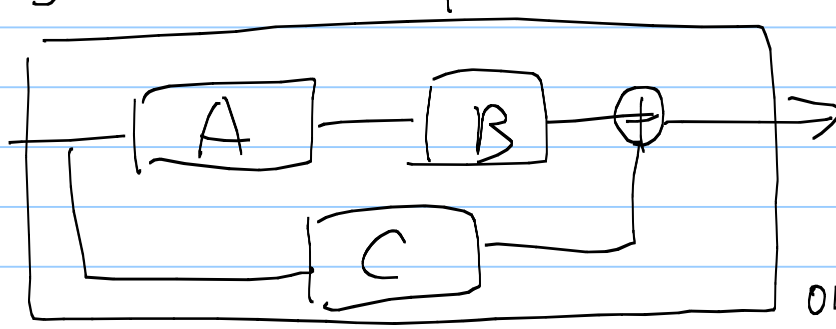
overall system

Sys 2 Parallel concatenation



overall sys.

Sys 3: Serial / parallel concatenation



overall sys