* DT signals are very different from CT signals:

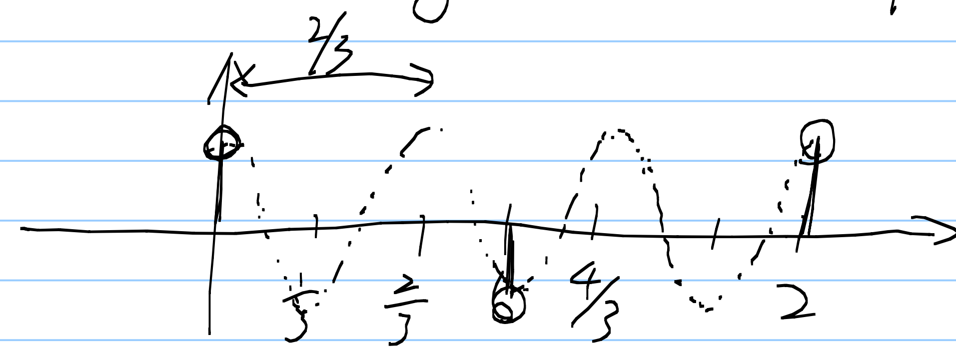
Ex: Compute the fund. period of

$x_1[n] = e^{j(3\pi n)}$ or $x_2[n] = \cos(3\pi n)$

Ans: If it is a CT signal

$\cos(3\pi n) \Rightarrow$ fund. period $= \frac{2\pi}{3\pi} = \frac{2}{3}$

$\Rightarrow$ The auxiliary curve has period $\frac{2}{3}$



But the true $x[n]$ only samples the curve at integer points

$\Rightarrow$ The period of $\cos(3\pi n)$

is $L.C.M(\frac{2}{3}, \underbrace{1}_{\text{only at integers}}) = 2$

which is different from $\frac{2}{3}$.

Ex: Q: Is e^{jn} periodic?

A: $e^{jt} = \cos(t) + j\sin(t)$ has period

$\frac{2\pi}{1} = 2\pi$.

e^{jn} period $= L.C.M(2\pi, 1)$ for 2π to get realigned with integers, which does not exist. $\Rightarrow e^{jn}$ is aperiodic

* The fundamental period of a DT signal is always an integer.

* The fund. freq. of DT signal is always $\frac{2\pi}{\text{fund. period}} = \frac{2\pi}{\text{integer}}$

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* DT. Harmonically Related Complex Exponentials

Consider a DT fund. freq $\omega = \frac{2\pi}{N}$

The DT HRCE is

$$x_k[n] = e^{jk\left(\frac{2\pi}{N}\right)n} \quad \text{for } k = 0, \cdots \boxed{N-1}$$

Q: How many distinct DT HRCEs can we have?

Ans: N ※

$$\because \; e^{jN\left(\frac{2\pi}{N}\right)n} = e^{j2\pi n} = e^{j0} = 1$$

$$e^{j(N+1)\left(\frac{2\pi}{N}\right)n} = e^{j2\pi n + \frac{2\pi}{N}n}$$

$$= e^{j2\pi n} \cdot e^{j\frac{2\pi}{N}n}$$

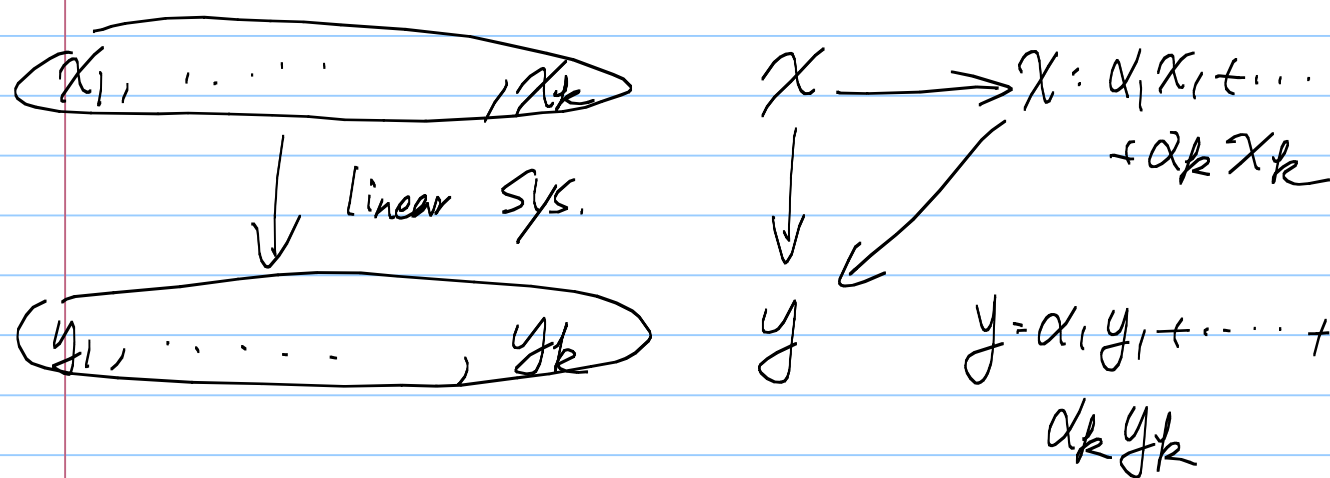
$$= e^{j\left(\frac{2\pi}{N}\right)n}$$

$$\underbrace{\begin{array}{cccc}
\Rightarrow \; e^{j(-N)\frac{2\pi}{N}n}, & \cdots & \cdots & e^{j(-1)\frac{2\pi}{N}n} \\
\| & \| & & \| \\
e^{j0\left(\frac{2\pi}{N}\right)n}, & e^{j1\left(\frac{2\pi}{N}\right)n}, & \cdots, & e^{j(N_d-1)\frac{2\pi}{N}n} \\
\| & \| & & \| \\
e^{jN_d\left(\frac{2\pi}{N}\right)n}, & e^{j(N_d+1)\frac{2\pi}{N}n} & & \\
\vdots & \vdots & &
\end{array}}_{\text{Only } N \text{ distinct DT HRCEs. for } k=0,\ldots,N-1}$$

This is a major difference between CT & DT signals.

Q: Why are we interested in HRCEs.

A: Recall that we are interested in linear systems.



* We use HRCEs as our test signals.

CT HRCE

Test sig. $x_k(t) = e^{jk\omega_c t}$ $\quad k = 0, \pm 1, \pm 2, \ldots$

New $x(t) = \sum_{k=-\infty}^{\infty} \alpha_k e^{jk\omega_c t}$

DT HRCE

Test sig. $x_k[n] = e^{jk\frac{2\pi}{N}t}$ $\quad k = 0, \ldots, N-1$

New $x[n] = \sum_{k=0}^{N-1} \alpha_k e^{jk\frac{2\pi}{N}n}$