Note Title 9/12/2014

Q: What is the instantaneous power of $x(t) = |c| e^{\sigma t} e^{j(\omega t + \phi)}$

Ans: $|x(t)|^2 = |c|^2 |e^{\sigma t}|^2 |e^{j(\omega t + \phi)}|^2$

$$= |c|^2 e^{2\sigma t} \times 1$$

* CT harmonically related complex exponentials (HRCEs)

— A family of signals:

$$x_k(t) = e^{jk\omega t}, \quad k = 0, \pm 1, \pm 2, \ldots$$

any integer

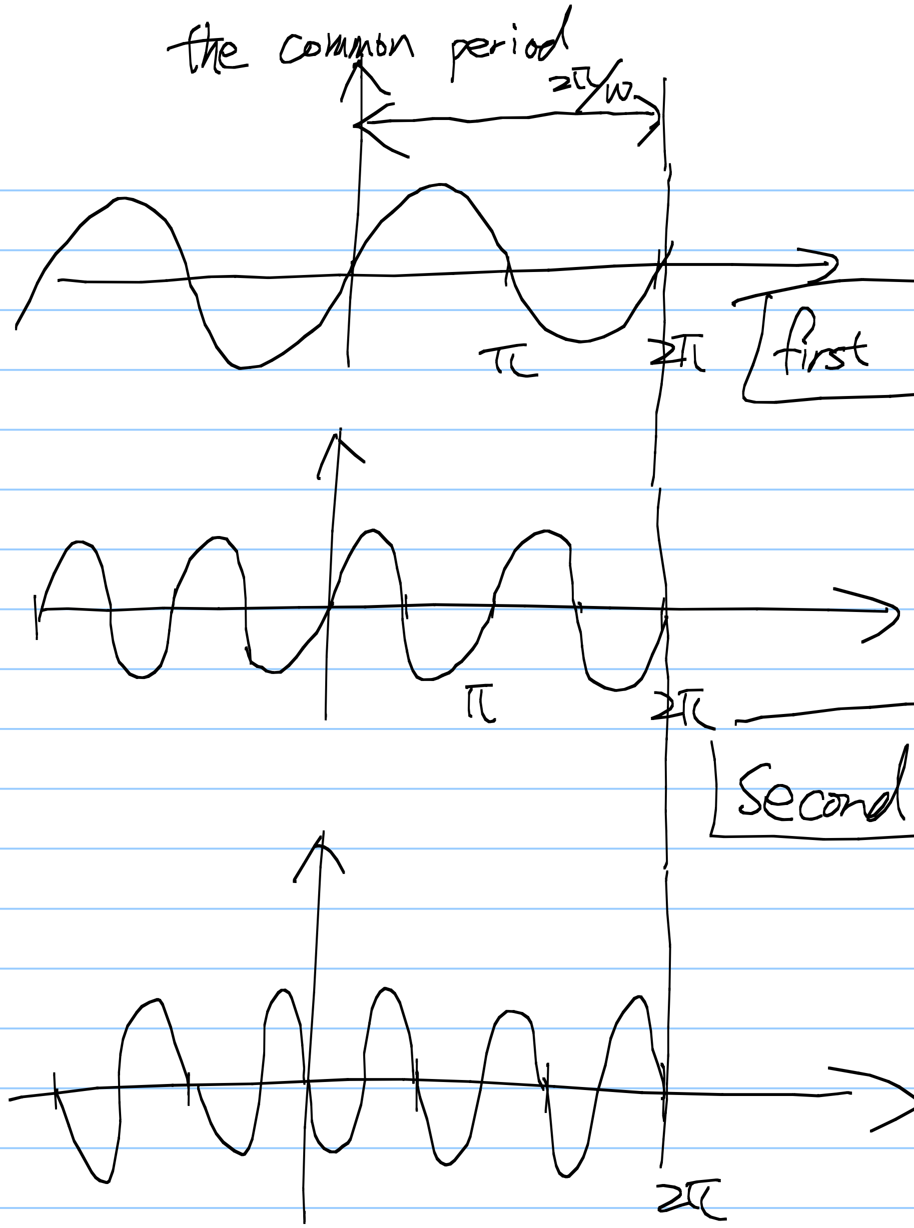
— For any ω, how many CT HRCEs do we have

Ans: ∞

— All these signals are periodic

— Their fundamental freq are $|k|\omega$

fundamental periods are $\frac{2\pi}{|k|\omega}$



$\sin(\omega t)$ $\omega = 1$

$k = 1$

$\boxed{\text{first harmonic}}$

$\sin(2\omega t)$ $\omega = 1$

$k = 2$

$\boxed{\text{Second harmonic}}$

$\sin(-3\omega t)$

$\omega = 1$

$k = -3$

$\boxed{\text{third harmonic}}$

— The name "harmonic" follows from music.

* DT Complex Exponential

$$x[n] = \underline{C}\, e^{\underline{\alpha} n}$$

→ complex numbers

To study this signal, rewrite

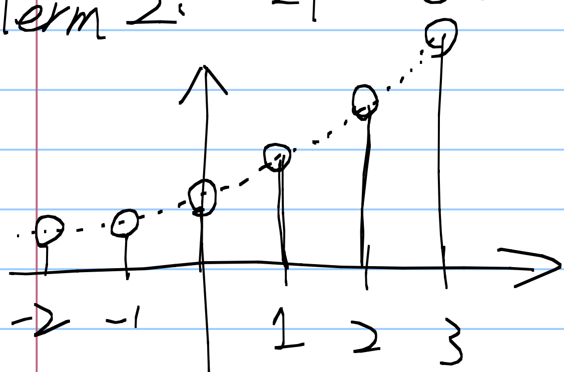
$$C = |C|\, e^{j\phi} \quad \text{— polar form}$$

$$\alpha = \sigma + j\omega \quad \text{— rectangular}$$

$$\Rightarrow x[n] = \underbrace{|C|}_{\text{Term 1}} \cdot \underbrace{e^{\sigma n}}_{\text{Term 2}} \cdot \underbrace{e^{j(\omega n + \phi)}}_{\text{Term 3}}$$

Term 1: Amplitude Scaling

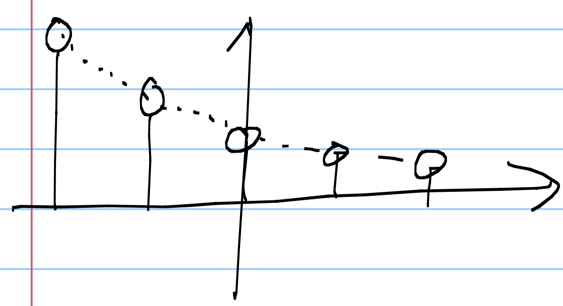
Term 2: If $\sigma > 0$



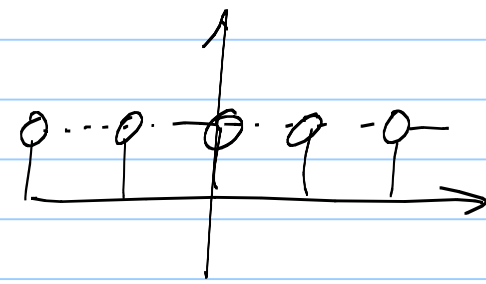
Use a dotted continuous curve for auxiliary purposes. The conti. curve is NOT a discrete-time signal

$e^{\sigma n}$ vs. $e^{\sigma t}$

If $r<0$

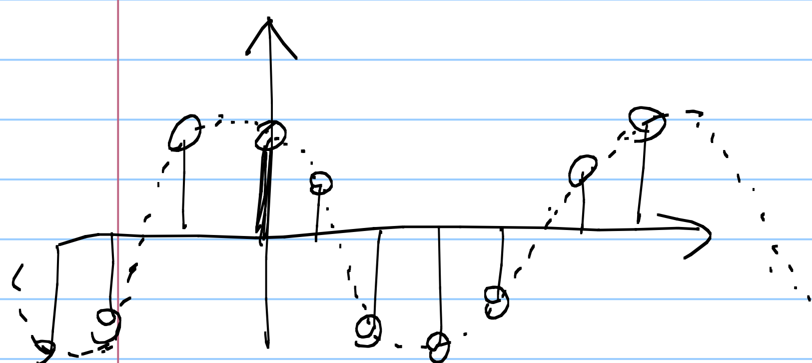


If $r=0$



Term 3: $e^{j(\omega n+\phi)}$

$\mathrm{Real}(e^{j(\omega n+\phi)})$



Again we rely on continuous signals to help us plot the DT signals.

...... $\cos(\omega t+\phi)$

When we combine them together

$\mathrm{Real}(e^{j\omega n})$