* A special class of signals

CT, complex exponential signals.

$$x(t) = C \cdot e^{\alpha t}$$

complex numbers

To study this signal, write

$C = |C| e^{j\phi}$ (polar form)

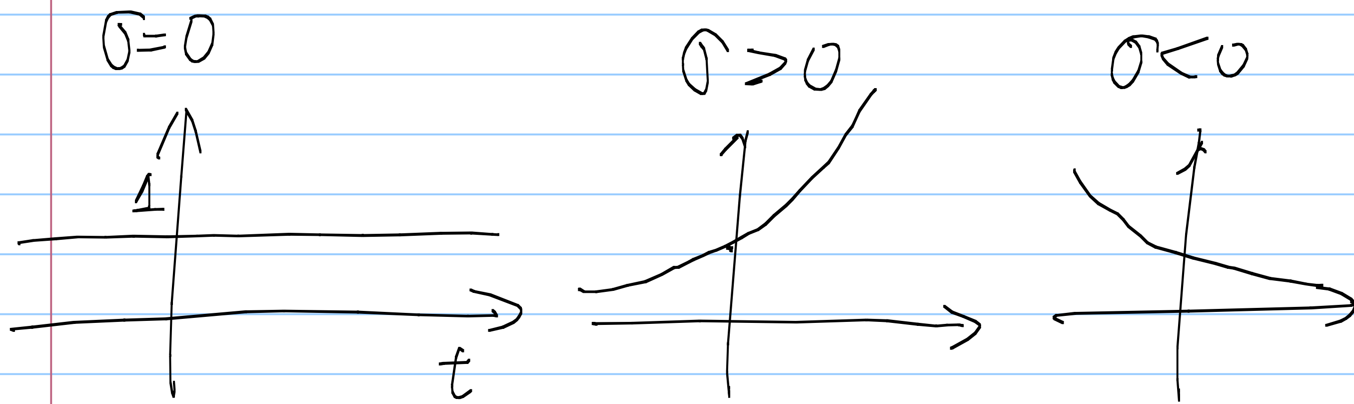
$\alpha = \sigma + j\omega$ (rectangular / cartesian form)

Then
$$x(t) = |C| e^{j\phi} \cdot e^{(\sigma + j\omega)t}$$
$$= \underbrace{|C|}_{\text{Term 1}} \underbrace{e^{\sigma t}}_{\text{Term 2}} \cdot \underbrace{e^{j(\omega t + \phi)}}_{\text{Term 3}}$$

Let us study the terms separately & then put them together.

Term 1: $|C|$ simply scales the signal

Term 2: $e^{\sigma t}$ (real exponential)



Term 3: $e^{j(\omega t+\phi)}$

$$= \cos(\omega t+\phi) + j\sin(\omega t+\phi)$$

It is periodic with fundamental period $\frac{2\pi}{\omega}$

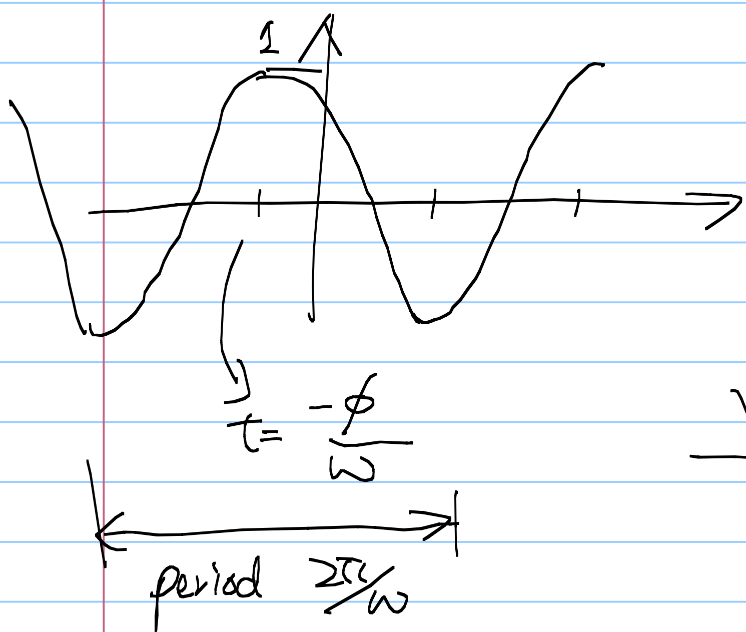
$$\therefore e^{j\omega(t-\frac{2\pi}{\omega})+\phi} = e^{j(\omega t - 2\pi + \phi)}$$

$$= e^{-j2\pi} \cdot e^{j(\omega t+\phi)}$$

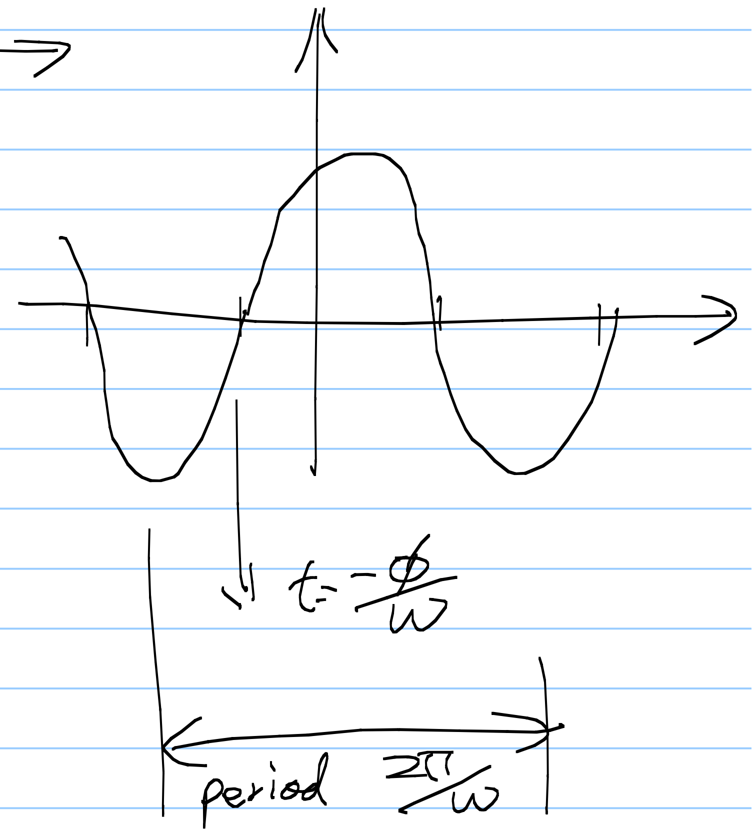
$$= 1 \cdot e^{j(\omega t+\phi)}$$

Q: How to plot $e^{j(\omega t+\phi)}$?

Real Part: (shifted & scaled) cosine



Imaginary part:



ω: omega is called the fundamental freq.

smaller $\omega \Rightarrow$ slower oscillation

larger $\omega \Rightarrow$ faster oscillation

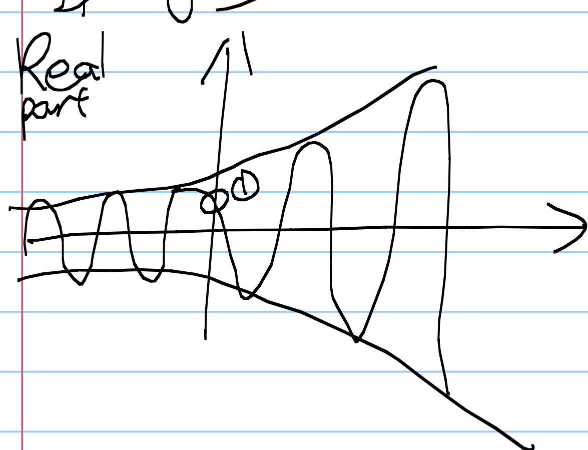
$$\boxed{T=\frac{2\pi}{\omega} \iff \overset{\text{fund. freq.}}{\omega}=\frac{2\pi}{\underbrace{T}_{\text{fund. period}}}}$$

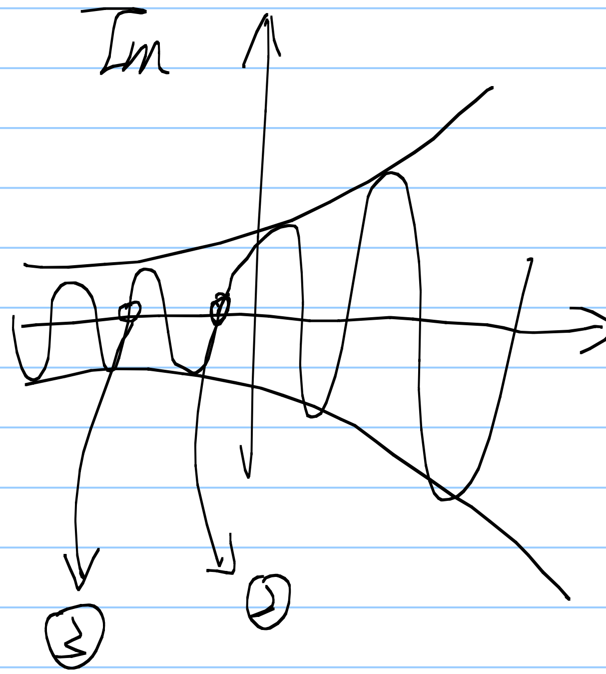
ϕ: is called the phase since it changes the angle of each sinusoidal

Combine all terms together

$$x(t) = |c|\, e^{\sigma t} e^{j(\omega t + \phi)}$$

If $\sigma > 0$





Q: Find the (x, y) coordinates of points ①, ②, ③ ?

Ans: $① = (0, |c| \cos(\phi))$

$② = \left(-\frac{\phi}{\omega}, 0\right)$

$③ = \left(-\frac{\phi}{\omega} - \frac{2\pi}{\omega}, 0\right)$

Note Title 9/12/2014

Q: What is the instantaneous power of $x(t) = |c| e^{\sigma t} e^{j(\omega t + \phi)}$

Ans: $|x(t)|^2 = |c|^2 |e^{\sigma t}|^2 \left| e^{j(\omega t + \phi)} \right|^2$

$$= |c|^2 e^{2\sigma t} \times 1 \quad \#$$

* CT harmonically related complex exponentials (HRCEs)
  - A family of signals:

    $x_k(t) = e^{jk\omega t}$, $k = 0, \pm 1, \pm 2, \ldots$ any integer
  - For any ω, how many CT HRCEs do we have

    Ans: ∞
  - All these signals are periodic
  - Their fundamental freq are $|k|\omega$

    fundamental periods are $\frac{2\pi}{|k|\omega}$