

Plot $y(t) = \frac{1}{3} x(4 - 0.5t)$.

Ans: Good pts:

$$t = 10$$

$$4 - 0.5t = -1$$

$$y(10) = \frac{1}{3} x(-1)$$

$$t = 4$$

2

$$y(4) = \frac{1}{3} x(2)$$

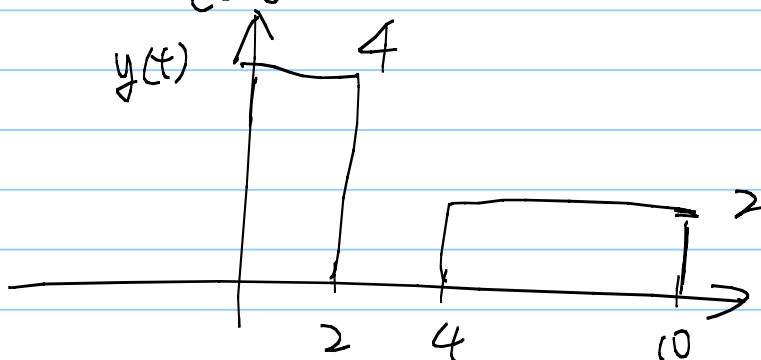
$$t = 2$$

3

$$x(2)$$

$$t = 0$$

4



$$y(2) = \frac{1}{3} x(3)$$

$$y(0) = \frac{1}{3} x(4)$$

Prof. Balakrishnan's handout.

* Classification #3: By the period.

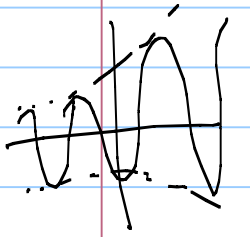
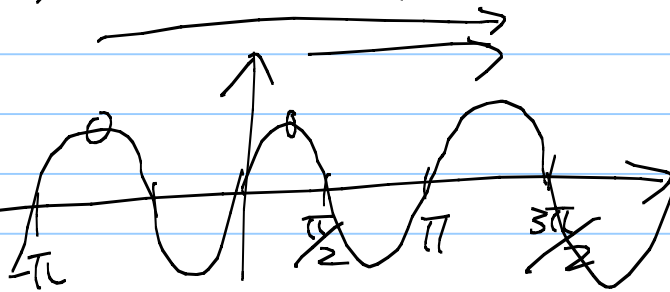
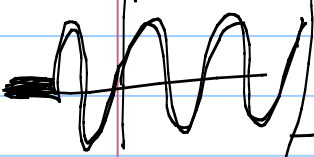
* We say $x(t)$ is a periodic signal with period T if we let $y(t) = x(t-T)$ be the shifted version of $x(t)$, then the new signal "looks" exactly like the old signal: sometimes we just write

$$x(t) = x(t-T)$$

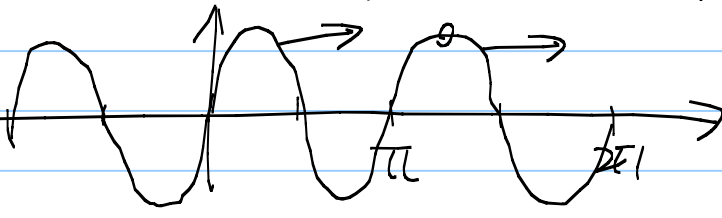
For DT: if $x[n] = x[n-N]$, then $x[n]$ is periodic with period N

Ex: $x(t) = \sin(2t)$ Plot $x(t)$ vs. t .

Not periodic



$y(t) = \sin(2(t-\pi))$

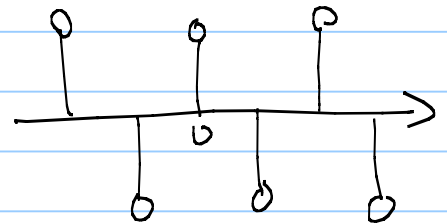


$\Rightarrow x(t)$ is periodic with period π

Ex: $x[n] = (-1)^n$

Q: Is $x[n]$ periodic?

A: $x[n]$
 1 -1 1 -1 1 -1 1 -1



\Rightarrow periodic with period 2.

* If $x(t)$ is periodic with period T then it is periodic with period mT for any $m \geq 1$ integers

Ex: $\pi, 2\pi, 3\pi, \dots$ are all periods

for $x(t) = \sin(2t)$

* Def: The fundamental period is the smallest period of a periodic signal $x(t)$ or $x[n]$.

* If $x(t) = x_{\text{Re}}(t) + jx_{\text{Im}}(t)$ is a periodic complex signal, then both $x_{\text{Re}}(t)$ & $x_{\text{Im}}(t)$ are periodic real signals.

Proof: $x(t)$ is periodic with period T

$$\Leftrightarrow x(t) = x(t-T)$$

$$\Leftrightarrow x_{\text{Re}}(t) + jx_{\text{Im}}(t) = x_{\text{Re}}(t-T) + jx_{\text{Im}}(t-T)$$

$$\Rightarrow \begin{cases} x_{\text{Re}}(t) = x_{\text{Re}}(t-T) \\ x_{\text{Im}}(t) = x_{\text{Im}}(t-T) \end{cases}$$

both $x_{\text{Re}}(t)$ & $x_{\text{Im}}(t)$ are periodic

Question for the teams

If both $x_1(t)$ and $x_2(t)$ are periodic, must $x_1(t) + x_2(t)$ be periodic?

How to decide the period of a signal?

Ans: ① Inspection

② If $x_1(t)$ has fund. period T_1
 $x_2(t)$ has fund. period T_2

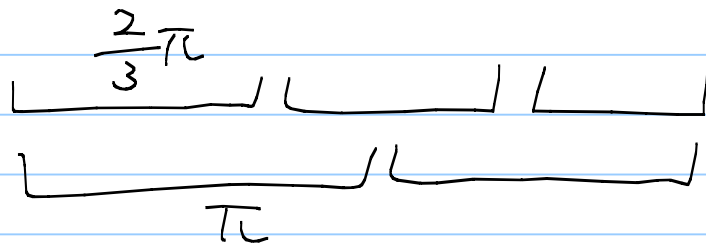
then $x_1(t) + x_2(t)$ is of period

Least Common Multiple

$L.C.M.(T_1, T_2)$

ex: $\cos(3t) + \sin(2t)$

$$T_1 = \frac{2\pi}{3} \quad T_2 = \frac{2\pi}{2} = \pi$$



$$LCM\left(\frac{2\pi}{3}, \pi\right) = 2\pi \text{ new period}$$