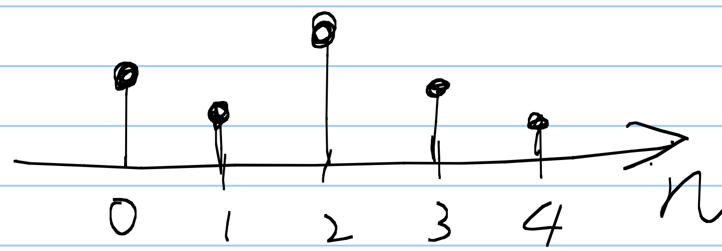
Visualization:



- Continuous-Time (CT)

Real

$$x(t) = \frac{t}{2}$$

$$x\left(\frac{1}{3}\right) = \frac{1}{6}$$

$$x(\pi) = \frac{\pi}{2}$$

Complex

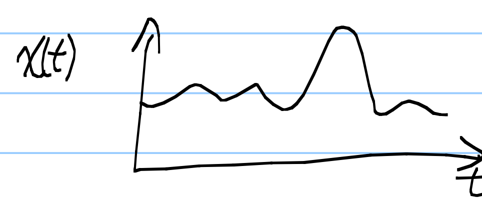
$$x(t) = \frac{t}{2} + (1-2t)j$$

$$= x_{Re}(t) + j x_{Im}(t)$$

$$x\left(\frac{1}{3}\right) = \frac{1}{6} + \frac{1}{3} j$$

$$x(\pi) = \frac{\pi}{2} + (1-2\pi) j$$

visualization



* Classification #2: By energy & by power

infinite (∞)

finite

energy

power

signals

(Four different types)

* Definition: Energy for CT signals

Energy between (t_1, t_2) interval is

$$\int_{t_1}^{t_2} |x(t)|^2 dt = \int_{t_1}^{t_2} \left(x_{Re}^2(t) + x_{Im}^2(t)\right) dt$$

$$\because |a+bj|^2 = a^2 + b^2$$

For DT signals:

Energy between $[n_1, n_2]$ interval is

$$\sum_{n=n_1}^{n_2} |x[n]|^2 = \sum_{n=n_1}^{n_2} \left(x_{Re}^2[n] + x_{Im}^2[n]\right)$$

- Total Energy (between $(-\infty, \infty)$)

CT:

$$\int_{-\infty}^{\infty} |x(t)|^2 dt$$

DT:

$$\sum_{n=-\infty}^{\infty} |x[n]|^2$$

- Avg power between (t_1, t_2) between $[n_1, n_2]$

CT:

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 dt$$

DT:

$$\frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$$

(Overall) Avg. Power.

CT:

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

DT:

$$\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

We can also define "instantenous power" by

$$|x(t)|^2 \quad \text{or} \quad |x[n]|^2$$

Q: If the total energy is 3, what the overall avg power?

A: 0 $\because \frac{3}{\infty} = 0$

Q: If the overall avg power is 3, what is the total energy?

A: ∞ $\because \frac{x}{\infty} = 3 \Rightarrow x = 3 \cdot \infty = \infty$.

* Implication: We do not have finite energy, ∞-power signals.

Q: $x(t) = \cos(t) + j\sin(t)$. What is the avg-power between $(-\frac{1}{2}, 1)$?

A: Avg power $= \frac{1}{1-(-\frac{1}{2})} \int_{-\frac{1}{2}}^{1} \underbrace{|x(t)|^2}_{=1} dt$

$$= \frac{1}{3/2} \int_{-\frac{1}{2}}^{1} \cos^2(t) + \sin^2(t)\, dt = \frac{1}{3/2} \int_{-\frac{1}{2}}^{1} 1\, dt$$

Note Title 8/27/2010

* Let us briefly digress to the "algebra of signals"

* Signals are just functions. So given two signals x_1, x_2 (can be $x_1(t)$ $x_2(t)$ or $x_1[n], x_2[n]$)
We can write,

New signals Old signals

- $y = x_1 + x_2$

  means $y(t) = x_1(t) + x_2(t)$ for all t.

- $y = \alpha x_1 \Rightarrow y[n] = \alpha x[n]$ for all n

- $y = x_1 \cdot x_2^2 \Rightarrow y(t) = (x_1(t)) \cdot (x_2(t))^2$

These operations are used/implemented quite often in a real system. Ex: amplifiers in a linear circuit. Ex: Graphic Processing Unit (GPU)

Ex: Voltage controlled amplifier.