

①  $y[n] = x[n]x[n-1]$  invertible?

Ans:  $x_1[n] = 0$  for all  $n$ .

$$x_2[n] = \begin{cases} 0 & \text{for all even } n \\ 1 & \text{for all odd } n \end{cases}$$

$$x_1[n] \longrightarrow y_1[n] = 0 \quad \text{for all } n$$

$$x_2[n] \longrightarrow y_2[n] = 0 \quad \text{for all } n$$

$\Rightarrow$  Not invertible.

②  $y[n] = x[1-n]$  invertible?

$$\text{Ans: } x[k] = y[1-k] \quad k = 1-n, \quad n = 1-k$$

$\Rightarrow$  invertible.

③  $y(t) = \frac{d}{dt}x(t)$  invertible?

$$\text{Ans: } x_1(t) = 0, \quad x_2(t) = 1.$$

$$x_1(t) \longrightarrow y_1(t) = 0$$

$$x_2(t) \longrightarrow y_2(t) = 0$$

$\Rightarrow$  Not invertible.

④  $y[n] = \begin{cases} x[n/2] & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$  invertible?

$$\text{Ans: } x[k] = y[2k]$$

$\Rightarrow$  invertible.

\* Question for the teams

Consider a sliding window averaging system

$$y[n] = \begin{cases} \frac{1}{\min(5, n) + 1} \sum_{k=\max(0, n-5)}^n x[k] & \text{if } n > 0 \\ x[n] & n \leq 0. \end{cases}$$

Ans: The inverse system is

$$x[n] = \begin{cases} y[n] & \text{if } n \leq 0 \\ (n+1)y[n] - ny[n-1] & \text{if } 1 \leq n \leq 5 \\ 6y[n] - 6y[n-1] + x[n-6] & \text{if } 6 \leq n. \end{cases}$$