

Q25

Note Title

$$f(t) = \begin{cases} 1 & \text{if } -3 \leq t < 0 \\ 0 & \text{otherwise} \end{cases}$$

9/14/2014

$$g(t) = \begin{cases} e^{\pi t} & \text{if } t < 2 \\ 0 & \text{otherwise} \end{cases}$$

Q: Find $h(t) = \int_{s=-\infty}^{\infty} f(s)g(t-s)ds$

Ans: s

$$g(t-s) = \begin{cases} e^{\pi(t-s)} & \text{if } t-s < 2 \Leftrightarrow s > t-2 \\ 0 & \text{otherwise} \end{cases}$$

Case 1: $t-2 < -3 \Leftrightarrow t < -1$

$$h(t) = \int_{-3}^0 e^{\pi(t-s)} ds$$

$$= e^{\pi t} \left(\frac{e^{+3\pi} - 1}{\pi} \right)$$

Case 2: $-3 < t-2 < 0 \Leftrightarrow -1 < t < 2$

$$h(t) = \int_{t-2}^0 e^{\pi(t-s)} ds$$

$$= e^{\pi t} \left(\frac{e^{(2-t)\pi} - 1}{\pi} \right)$$

$$= \frac{e^{2\pi}}{\pi} - \frac{e^{\pi t}}{\pi}$$

Case 3: $h(t) = 0$