Note Title 8/28/2014

* HW1Q6 $f(t)=|1-t|$ $g(t)=\int_{t-1}^{t+2} f(s)\,ds$

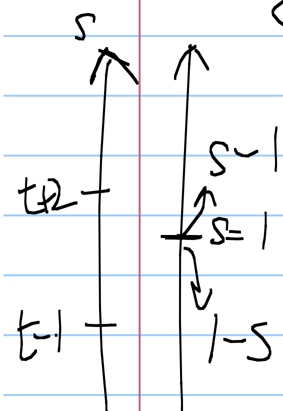
Q: $g(3)$, $\int_1^4 g(s)\,ds$, $\int_{-1}^1 f(s)g(1-s)\,ds$

A: $g(3)=\int_{3-1}^{3+2} |1-s|\,ds$

$$= \int_2^5 (s-1)\,ds = \frac{s^2}{2}-s\Big|_2^5 = \frac{15}{2}$$

Let us plot $g(t)$.

$$g(t)=\int_{t-1}^{t+2} |1-s|\,ds$$



Case 1: if $t+2<1 \iff t<-1$

$$g(t)=\int_{t-1}^{t+2} (1-s)\,ds$$

$$= s-\frac{s^2}{2}\Big|_{t-1}^{t+2} = \left((t+2)-\frac{(t+2)^2}{2}\right)$$

$$-\left((t-1)-\frac{(t-1)^2}{2}\right)$$

$$= -3t+\frac{3}{2}.$$

Case 2: $t-1<1<t+2$

$$\iff -1<t<2$$

$$g(t)=\int_{t-1}^{1} (1-s)\,ds + \int_1^{t+2} (s-1)\,ds$$

$$= \left(s-\frac{s^2}{2}\right)\Big|_{t-1}^{1} + \left(\frac{s^2}{2}-s\right)\Big|_1^{t+2}$$

$$= t^2-t+5/2$$

Case 3: $1 < t-1 \iff 2 < t$

$$\int_{t-1}^{t+2} s-1\,ds = \left(\frac{s^2}{2} - s\right)\Bigg|_{t-1}^{t+2} = 3t - \frac{3}{2}$$

$$\int_1^4 g(s)\,ds = \int_1^2 s^2 - s + \frac{5}{2}\,ds + \int_2^4 3s - \frac{3}{2}\,ds$$

$$= 55/3$$

$\int_{-1}^{1} f(s)\,g(1-s)\,ds$ Change of variable

$s' = 1-s$ $\quad ds' = -ds$

$$= \int_2^0 (-1) f(1-s')\,g(s')\,ds' = \int_0^2 f(1-s')\,g(s')\,ds'$$

$$\boxed{f(1-s') = |1-(1-s')| = |s'|} = \int_0^2 s'(s'^2 - s' + 5/2)\,ds'$$

$$= 19/3$$

Computation speed is important and will be tested in the exam.