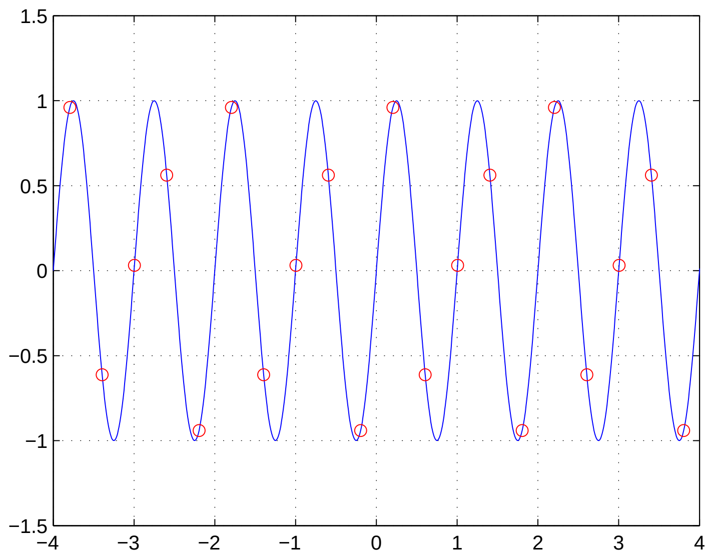
ECE 301, A Digital Differentiator Demonstration

Input: Given an band-limited input $x(t)$ with bandwidth $W_M = 2.5\pi$. Sample it with sampling period $T = 2/5$ ($\omega_s = 5\pi$). Let $x[n]$ denote the sampled discrete-time (digital) array.

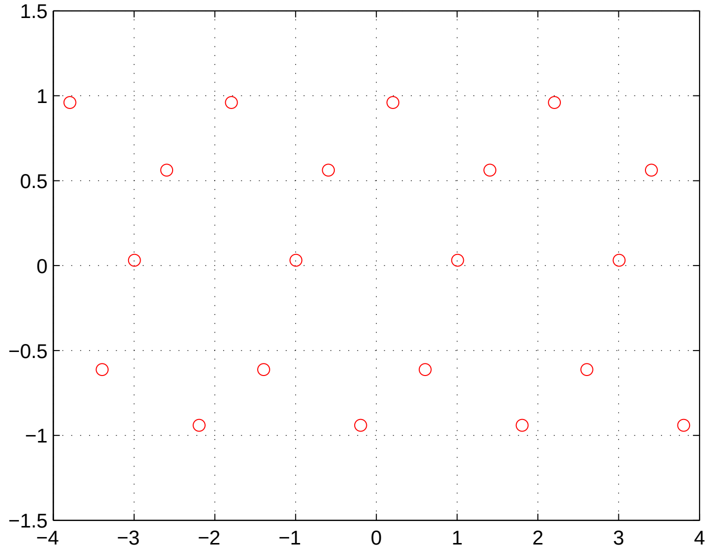
Goal: Design a discrete-time processing $h[n]$ satisfying the following. Let $y[n]$ denote the output of the discrete-time system: $y[n] = x[n] * h[n]$. Use perfect band-limited reconstruction to generate a continuous signal $y(t)$. We desire that $y(t)$ being the first-order derivative of $x(t)$. (Note that all we can handle/manipuate is the samples $x[n]$, not the original signal $x(t)$.)

Example: $x(t) = \sin(2\pi t)$

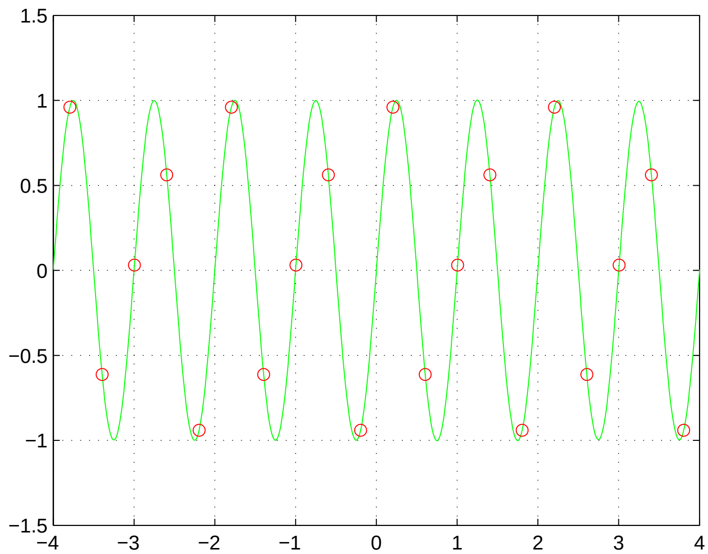
Original signal



What you really have is the sampled values:

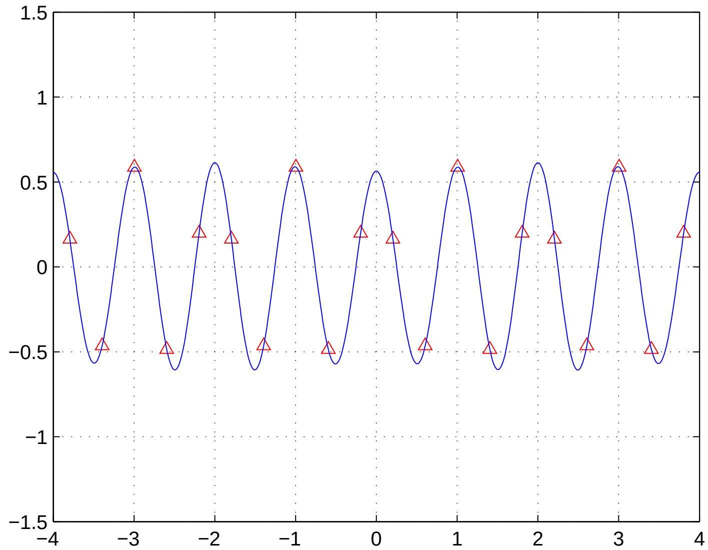


Perfect reconstruction without any processing:



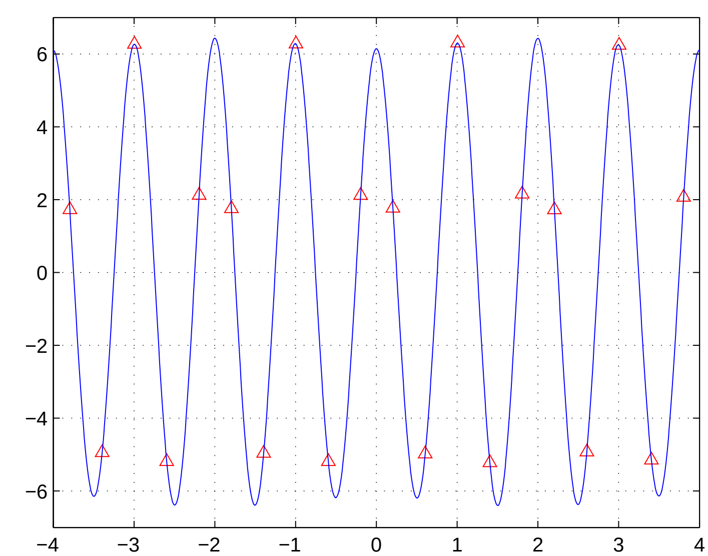
Naive Differentiator (in discrete time) + Perfect Reconstruction:

$$y[n] = \frac{1}{2}\left(\frac{x[n]-x[n-1]}{T} + \frac{x[n+1]-x[n]}{T}\right) \tag{1}$$



True Digital Differentiator + Perfect Reconstruction: (see lecture notes)

$$h[n] = \begin{cases} \frac{(-1)^n}{nT} & \text{if } n \neq 0 \\ 0 & \text{if } n = 0 \end{cases}. \tag{2}$$



Comparison: $x(t) = \sin(2\pi t)$ and $x'(t) = 2\pi\cos(2\pi t)$.

Reconstructed original $\hat{x}(t)$, a naive differentiator $y_1(t)$, and the true differentiator $y_2(t)$.