ECE 301-001 and 301-003, Midterm #2
8–9:30pm, Wednesday, March 1, 2023, CL50 Rm224.

1. **Do not write answers on the back of pages!**
2. **After the exam ends, you will have 5 additional minutes to write down your name and Purdue ID on each of the pages.**
3. **If you need additional sheets of paper to write down your answers, please let one of the proctors know. We will hand out additional answer sheets as needed.**
4. Write your student ID number and signature in the space provided on this page.
5. This is a closed book exam. Neither calculators nor help sheets are allowed.
6. You have **90 minutes** to complete the exam. There are 6 multi-part questions.
7. You must **show all work** used to arrive at your answer. This is required to receive full credit, and also is helpful for you in getting partial credit.

Name:

Student ID:

As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

Signature: Date:

Last Name: First Name: Purdue ID:

Question 1: [18%] Consider an LTI system with impulse response

$$h(t) = \begin{cases} 0.5^{(t-4)} & \text{if } t \geq 1 \\ 0 & \text{if } t < 1 \end{cases}.$$

(a) [13%] If the input is

$$x_1(t) = u(-t+1),$$

where $u(t)$ is the unit step signal, denote the corresponding output by $y_1(t)$. Find the expression of $y_1(t)$.

[Hint: the following equality may be useful: $0.5 = e^{-\ln(2)}$.]

(b) [5%] Now, suppose we change the input to

$$x_2(t) = \begin{cases} 0 & \text{if } t < -1 \\ 2 & \text{if } -1 \leq t < 2 \\ 3 & \text{if } t \geq 2 \end{cases}.$$

Denote the corresponding output by $y_2(t)$. Write down the expression of $y_2(t)$.

[Hint: For part (b), rather than doing a direct computation, consider the relationship between $y_2(t)$ and $y_1(t)$. You will receive 4% just for correctly identifying this relationship even if you are unable to find the expression of $y_2(t)$.]

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Question 2: [16%] Consider an LTI system with impulse response

$$h(t) = \begin{cases} 0.5e^{-t} & \text{if } 0 \le t \le 1 \\ 0.5e^{t} & \text{if } -1 \le t < 0 \\ 0 & \text{otherwise} \end{cases}.$$

(a) [8%] Find the frequency response $H(j\omega) = \int_{-\infty}^{\infty} h(\tau)e^{-j\omega\tau}d\tau$ of the system.

(b) [8%] If we input the signal

$$x(t) = \frac{e^{j5t} + e^{-j5t}}{2} + 3e^{j2\pi t}$$

to the system, find the output $y(t)$.

[Hint: Your answers can be something like $\frac{e^{(1+j0.5)t}}{2-\pi j} - 5je^{-3t}$. There is no need to further simplify them.]

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Question 3: [22%] Consider a system with input-output relationship

$$y[n+2] = \alpha^{|n|} x[n+2]x[n]$$

for some constant α.

(a) [6%] Is the system time invariant? Prove or disprove it.

(b) [5%] Is the system causal? Is it memoryless?

(c) [5%] Is the system invertible? Prove it or provide a counter example.

(d) [6%] Is the system stable for $\alpha = -0.5$? How about for $\alpha = 2$? Explain.

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Question 4: [18%] Consider an LTI system with input-output relationship

$$y(t) = \int_t^{\infty} x(\tau) d\tau.$$

(a) [8%] What is the impulse response $h(t)$ of the system?

(b) [5%] Use $h(t)$ to prove whether the system is causal or not.

(c) [5%] Use $h(t)$ to prove whether the system is stable or not.

[Hint: If you are unsure how to use $h(t)$ for (b) and (c), you can answer by reasoning directly from the input-output relationship to obtain up to 3% on each part.]

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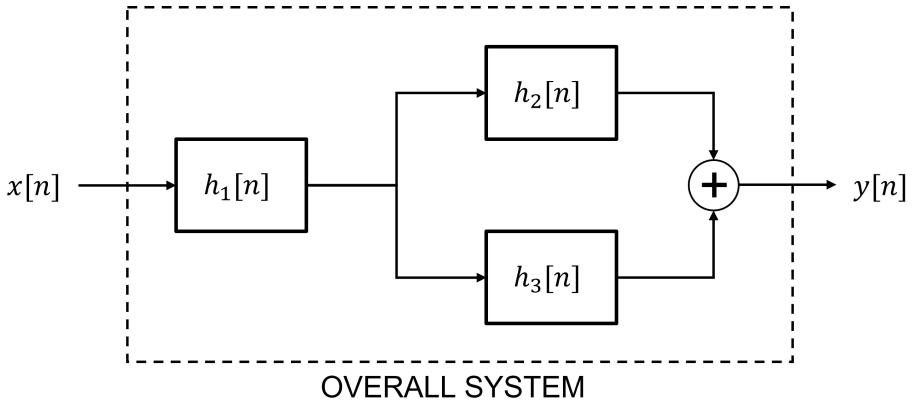
Question 5: [16%] Consider a DT system comprised of three LTI sub-systems with impulse responses $h_1[n]$, $h_2[n]$, and $h_3[n]$ connected in the configuration shown below. The three sub-systems have the following impulse responses:

$$h_1[n] = \delta[n-2] \qquad h_2[n] = u[n+2] \qquad h_3[n] = -u[n-2]$$



(a) [7%] Give an expression for the impulse response $h[n]$ of the overall system.

[Hint: Solve this question analytically, not graphically.]

(b) [3%] Sketch $h[n]$ from (a).

(c) [6%] Give an expression for the output $y[n]$ of the overall system when the input is:

$$x[n] = \delta[n-2] - \delta[n+4].$$

[Hint: If you are unsure how to answer (c), you can compute $x[n] * h_1[n]$ to receive up to 4%.]

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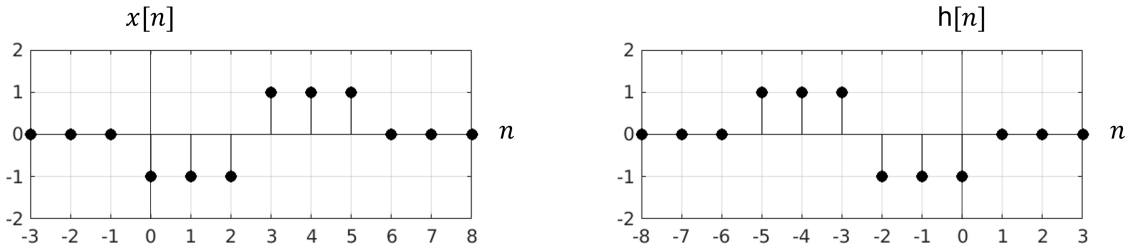
[Hint: This is a reasonably time consuming question that is worth only 10 points. The recommendation is to start working on this after you have finished the other questions to a reasonable degree.]

Question 6: [10%] Consider a discrete-time LTI system with an impulse response $h[n]$ shown on the right below. Suppose we input to this system the signal $x[n]$ shown on the left (note that $h[n] = x[-n]$).



(a) [7%] Determine the output $y[n]$ of the system.

(b) [3%] Sketch a plot of $y[n]$.

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