ECE 301-001&003, Final Exam
1–3pm, Thursday, May 4, 2023, PHYS 203 and BHEE 170.

1. **Do not write answers on the back of pages!**
2. **After the exam ends, you will have 5 additional minutes to write down your name and Purdue ID on each of the pages.**
3. **If you need additional sheets of paper to write down your answers, please let one of the proctors know. We will hand out additional answer sheets as needed.**
4. Write your student ID number and signature in the space provided on this page.
5. This is a closed book exam. Neither calculators nor help sheets are allowed. A separate formula packet has been provided to you.
6. You have **120 minutes** to complete the exam. There are 8 multi-part questions.
7. You must **show all work** used to arrive at your answer. This is required to receive full credit, and also is helpful for you in getting partial credit.

Name:

Student ID:

As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

Signature: Date:

Last Name: First Name: Purdue ID:

Question 1: [12%]

Consider a continuous time system with input $x(t)$ and output $y(t)$ related as:

$$y(t) = \int_{s=-\infty}^{t} x(s-\pi)e^{(3+j)(t-s)}ds$$

(a) [4%] Is the above system linear? Please carefully justify your answer. A correct answer without any justification will receive only 1.5 points.

(b) [3%] Is the above system causal? Please carefully justify your answer. A correct answer without any justification will receive only 1.5 points.

(c) [5%] Find the expression of the impulse response of the above system.

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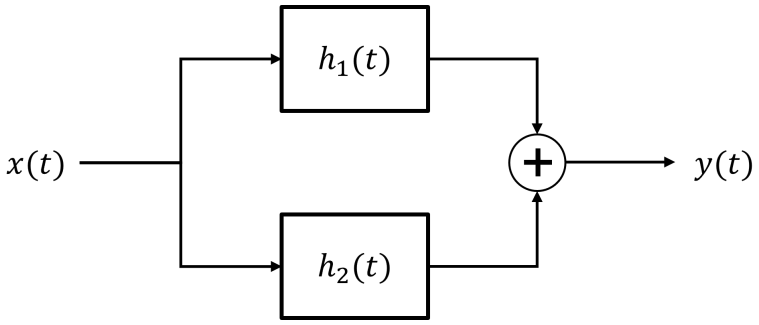
Question 2: [11.5%]

Consider the CT-LTI system shown in the figure below, which is comprised of two LTI sub-systems with impulse responses $h_1(t)$ and $h_2(t)$:



Let us define $h_1(t)$ and $h_2(t)$ as follows:

$$h_1(t) = \begin{cases} 2 & -1 \leq t < 0 \\ 0 & \text{else} \end{cases}$$

$$h_2(t) = \begin{cases} 2 & 0 \leq t \leq 1 \\ 0 & \text{else} \end{cases}$$

(a) [7%] Find the output of the system, $y(t)$, when the input $x(t)$ is defined as:

$$x(t) = \begin{cases} t & -1 \leq t \leq 1 \\ 0 & \text{else} \end{cases}$$

[Hint: You may find it useful to first specify the impulse response of the overall system.]

(b) [4.5%] With $x(t)$ from part (a), sketch a plot of the signal $z(t)$ defined as

$$z(t) = x(-2t + 1) + 1$$

for the range $-2 \leq t \leq 2$.

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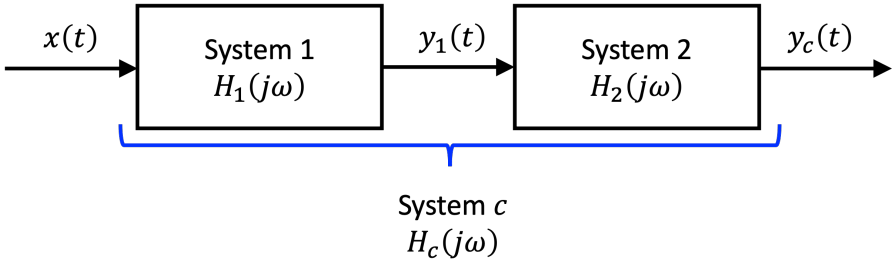
Question 3: [13%]

Consider the following cascade of two CT-LTI systems, with the composite system referred to as System c:



When we input

$$x(t) = te^{-4t}u(t)$$

to System 1, we observe an output

$$y_1(t) = t^2e^{-4t}u(t)$$

from System 1.

(a) [7%] Find the frequency response $H_1(j\omega)$.

(b) [3%] Suppose the frequency response of System 2 is

$$H_2(j\omega) = 10$$

Is System c invertible? Please carefully justify your answer.

(c) [3%] Repeat part (b) if

$$H_2(j\omega) = \sum_{k=-\infty}^{\infty} \delta(\omega - k\pi)$$

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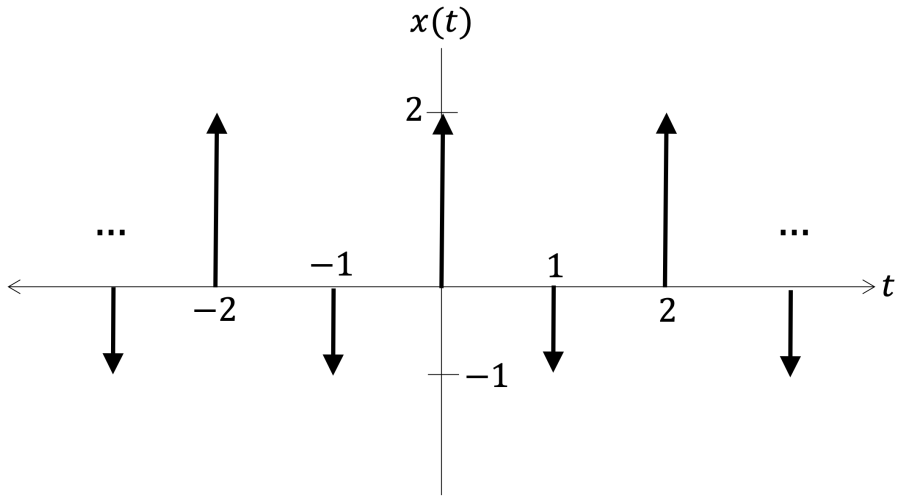
Question 4: [13%]

Consider the following periodic signal $x(t)$ consisting of impulses alternating between weights of 2 and -1:



(a) [2%] Is $x(t)$ even, odd, or neither? Please carefully justify your answer.

(b) [2%] Let $\{a_k\}$ denote the Fourier series coefficients of $x(t)$. Will the magnitude $|a_k|$ be even, odd, or neither? You must carefully justify your answer without performing any calculations.

(c) [2%] Again without performing any calculations, can we conclude that the phase $\angle a_k = 0$ for all k? Please carefully justify your answer.

(d) [7%] Compute $\{a_k\}$, and use them to write $x(t)$ as its Fourier series synthesis.

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Question 5: [12%]
Consider a discrete time signal

$$x[n] = \begin{cases} e^{j\frac{14\pi}{3}n} + \delta[n] & \text{if } 0 \le |n| \le 3 \\ \text{periodic with period 6} \end{cases}.$$

Let $\{a_k\}$ denote the DTFS coefficients of $x[n]$.

(a) [8%] Plot a_k for the range of $0 \le k \le 3$.

[Hint: Try breaking down $x[n] = x_1[n] + x_2[n]$ to solve this question.]

Consider a discrete-time ideal low-pass filter with cutoff frequency $W = 0.4\pi$ rad/sec. Namely, its frequency response satisfies

$$H(e^{j\omega}) = \begin{cases} 1 & \text{if } |\omega| \le 0.4\pi \\ 0 & \text{if } 0.4\pi < |\omega| \le \pi \end{cases}$$

Let $y[n]$ denote the output when feeding $x[n]$ through the above discrete-time low pass filter.

(b) [4%] Let $\{b_k\}$ denote the DTFS coefficients of the output $y[n]$. Answer the following yes/no questions,

i) Is $b_0 = 0$?

ii) Is $b_1 = 0$?

iii) Is $b_2 = 0$?

iv) Is $b_3 = 0$?

v) Is $b_4 = 0$?

vi) Is $b_5 = 0$?

and briefly explain your answers.

[Hint: If you do not know the answer to this subquestion, you can write down the relationship between b_k and a_k in terms of $H(e^{j\omega})$. You will receive 1.5 points for this subquestion if your answer is correct.]

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Question 6: [13%]

Consider a causal DT-LTI system characterized by the difference equation

$$y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 2x[n-1].$$

(a) [6%] Determine the frequency response $H(e^{j\omega})$ of the system.

(b) [4%] Determine the impulse response $h[n]$ of the system.

(c) [3%] Is the system stable? Justify your answer. A correct answer without any justification will receive only 1 point.

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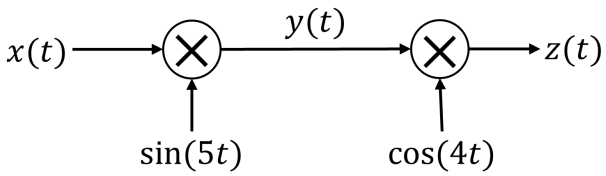
Question 7: [13.5%]



Suppose we feed the above system with the following continuous time signal $x(t)$:

$$x(t) = \sin(t)$$

(a) [2%] Plot $X(j\omega)$ for the range of $-12 < \omega < 12$. Please carefully mark both the horizontal and vertical axes of your figure. For simplicity, if the value $X(j\omega)$ is $3j$, then you can mark the vertical axis as $3j$.

[Hint 1: If you do not know how to plot $X(j\omega)$, you can simply find the expression of $X(j\omega)$. You will receive 1.5 points if your answer is correct.]

Define $y(t) = x(t) \cdot \sin(5t)$.

(b) [5%] Plot $Y(j\omega)$ for the range of $-12 < \omega < 12$. Please carefully mark both the horizontal and vertical axes of your figure, as in part (a).

[Hint 2: If you do not know how to solve this subquestion, please write down the relationship between $Y(j\omega)$ and $X(j\omega)$. You will receive 3 points if your answer is correct.]

Define $z(t) = y(t) \cdot \cos(4t)$.

(c) [6.5%] Plot $Z(j\omega)$ for the range of $-12 < \omega < 12$. Please carefully mark both the horizontal and vertical axes of your figure, as in part (a).

[Hint 3: If you do not know how to solve this subquestion, please solve the following question instead. Suppose $z_3(t) = (\sin(5t))^2$. Find out the corresponding Fourier transform $Z_3(j\omega)$. You will receive 5 points if your answer is correct.]

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Question 8: [12%]

Let us define a DT signal $x[n]$ as

$$x[n] = e^{j\pi n} \cdot (u[n+2] - u[n-3])$$

(a) [6%] Find the DTFT $X(e^{j\omega})$.

(b) [6%] Find the value of the expression

$$\int_{\pi}^{3\pi} |X(e^{j\omega})|^2 d\omega$$

[Hint: Consider how this expression relates to total energy.]

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