

Midterm #1 of ECE 301-004, (CRN: 13890)
8–9pm, Wednesday, February 2, 2022, FRNY G140.

1. Do not write answers on the back of pages!
2. After the exam ended, you will have 5 additional minutes to write down your name and Purdue ID on each of the pages.
3. If you need additional sheets to write down your answers, please let the instructor/TA know. We will hand out additional answer sheets then.
4. Enter your student ID number, and signature in the space provided on this page.
5. This is a closed book exam.
6. This exam contains multiple-choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and first working on those that you know how to solve.
7. The instructor/TA will hand out loose sheets of paper for the rough work.
8. Neither calculators nor help sheets are allowed.

Name:

Student ID:

As a Boiler Maker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

Signature:

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Question 1: [16%, Work-out question] Consider the following function:

$$X(\omega) = \begin{cases} e \cdot e^{-\omega} & \text{if } \omega > 1 \\ 0 & \text{if } -1 \leq \omega \leq 1 \\ e \cdot e^{\omega} & \text{if } \omega < -1 \end{cases} \quad (1)$$

We construct another function

$$y(t) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \quad (2)$$

1. [12%] What is the value of $y(\frac{\pi}{2})$?
2. [4%] What is the imaginary part of $y(\frac{\pi}{2})$?

For Q1.2 (not for Q1.1), you must simplify your answer to its simplest form. For example, if you see something like $\cos(\frac{\pi}{2})$, you need to write it as 0. If you see something like $3 \cos(\pi/3) - 2 \cos(\pi/3)$, you need to write it as $\cos(\pi/3) = 0.5$.

Hint: If you do not know the answer to Q1.1 and thus do not know the answer of Q1.2, you can answer the following alternative question. You will receive full mark if your answer of the alternative question is correct.

Consider a complex value $z = \frac{e^{j\frac{2\pi}{3}}}{1+\sqrt{3}j} + \frac{e^{-j\frac{2\pi}{3}}}{1-\sqrt{3}j}$. Find the amplitude and angle of z .

Answer:

$$\begin{aligned} 1. \quad y\left(\frac{\pi}{2}\right) &= \frac{1}{2\pi} \int_{\omega=-\infty}^{-1} e e^{\omega} e^{j\omega t} d\omega \\ &\quad + \frac{1}{2\pi} \int_{\omega=1}^{\infty} e e^{-\omega} e^{j\omega t} d\omega \\ &= \frac{e}{2\pi} \int_{\omega=-\infty}^{-1} e^{(1+\frac{\pi}{2}j)\omega} d\omega + \frac{e}{2\pi} \int_{\omega=1}^{\infty} e^{(-1+\frac{\pi}{2}j)\omega} d\omega \\ &= \frac{e}{2\pi} \cdot \frac{e^{(1+\frac{\pi}{2}j)\omega}}{1+\frac{\pi}{2}j} \Big|_{\omega=-\infty}^{-1} + \frac{e}{2\pi} \cdot \frac{e^{(-1+\frac{\pi}{2}j)\omega}}{-1+\frac{\pi}{2}j} \Big|_{\omega=1}^{\infty} \\ &= \frac{e}{2\pi} \left(\frac{e^{-(1+\frac{\pi}{2}j)}}{1+\frac{\pi}{2}j} + \frac{e^{-1+\frac{\pi}{2}j}}{-1+\frac{\pi}{2}j} \right) \end{aligned}$$

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This sheet is for Question 1.

$$= \frac{e}{2\pi} \frac{e^{-(1+\frac{\pi}{2}j)} + e^{-1+\frac{\pi}{2}j} - \frac{\pi}{2}j e^{-(1+\frac{\pi}{2}j)} + \frac{\pi}{2}j e^{-1+\frac{\pi}{2}j}}{(1+\frac{\pi}{2}j)(1-\frac{\pi}{2}j)}$$

$$= \frac{e}{2\pi (1+(\frac{\pi}{2})^2)} \left(e^{-(1+\frac{\pi}{2}j)} + e^{-1+\frac{\pi}{2}j} - \frac{\pi}{2} e^{-1-\frac{\pi}{2}j+\frac{\pi}{2}j} - \frac{\pi}{2} e^{-1+\frac{\pi}{2}j-\frac{\pi}{2}j} \right)$$

$$= \frac{e}{2\pi (1+(\frac{\pi}{2})^2)} \left(e^{-1-\frac{\pi}{2}j} + e^{-1+\frac{\pi}{2}j} - \frac{\pi}{2} e^{-1} - \frac{\pi}{2} e^{-1} \right)$$

$$= \frac{e}{2\pi (1+(\frac{\pi}{2})^2)} (-\pi e^{-1})$$

$$= -\frac{1}{2 + \frac{\pi^2}{2}}$$

$$= -\frac{2}{4 + \pi^2}$$

2. $|y(\frac{\pi}{2})| = 0$

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Question 2: [15%, Work-out question]

Define two DT signals:

$$x[n] = \begin{cases} 3^{-\pi n} & \text{if } n \geq 4 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

and

$$h[n] = \begin{cases} e^{jn} & \text{if } n \leq 4 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Compute the expression of the following summation

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \quad (5)$$

Hint: If $|r| < 1$, then we have the following formulas for computing the infinite sums of a geometric sequence.

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$$
$$\sum_{k=1}^{\infty} kar^{k-1} = \frac{a}{(1-r)^2}.$$

If $r \neq 1$, then we have the following formula for computing the finite sum of a geometric sequence.

$$\sum_{k=1}^K ar^{k-1} = \frac{a(1-r^K)}{1-r}.$$

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This sheet is for Question 2.

Answer:

$$h[n-k] = \begin{cases} e^{j(n-k)} & \text{if } n-k \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$y[n] = \sum_{k=4}^{\infty} 3^{-\pi k} h[n-k]$$

Case 1: $n-4 \leq 4$

$$\begin{aligned} y[n] &= \sum_{k=4}^{\infty} 3^{-\pi k} e^{j(n-k)} \\ &= \sum_{k=4}^{\infty} e^{jn} \cdot (3^{-\pi} e^{-j})^k \\ &= e^{jn} \frac{(3^{-\pi} e^{-j})^4}{1 - 3^{-\pi} e^{-j}} \end{aligned}$$

Case 2: $n-4 > 4$

$$\begin{aligned} y[n] &= \sum_{k=n-4}^{\infty} 3^{-\pi k} e^{j(n-k)} \\ &= e^{jn} \frac{(3^{-\pi} e^{-j})^{n-4}}{1 - 3^{-\pi} e^{-j}} \\ &= \frac{3^{-\pi(n-4)} \cdot e^{j4}}{1 - 3^{-\pi} e^{-j}} \end{aligned}$$

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Question 3: [17%, Work-out question] Consider two DT signals

$$x[n] = e^{j\omega n} \tag{6}$$

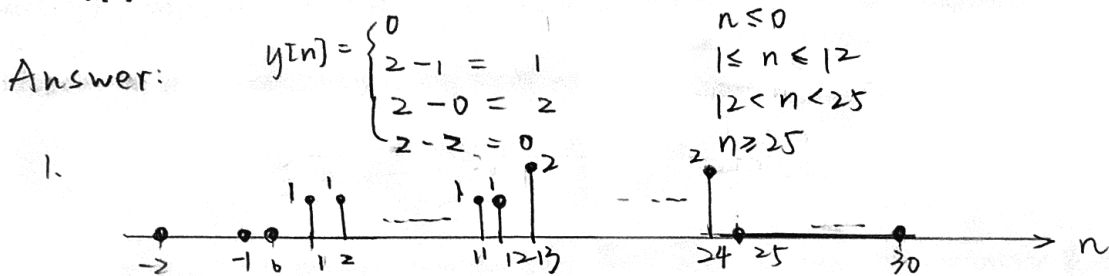
$$y[n] = 2\hat{U}[n-1] - U[-n+12] - 2U[n-25] \tag{7}$$

- [6%] Plot $y[n]$ for the range of $-2 \leq n \leq 30$.
- [11%] Find out the expression of $z[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k]$.

Hint 1: It may be easier to apply some change of variables.

Hint 2: You can leave (part of) your answer to be of the form like $\frac{e^{3jt} - 2 - 12t}{3 + 10j}$. There is no need to further simplify the expression.

Hint 3: If you do not know the answer of Q3.1, you can assume $y[n] = 1$ if $0 \leq n \leq 50$ and $y[n] = 0$ otherwise. You will receive 9 points if your answer is correct.



$$\begin{aligned} 2. \quad y[n] &= \sum_{k=-\infty}^{\infty} x[k]y[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]y[k] \\ &= \sum_{k=1}^{12} e^{j\omega(n-k)} + \sum_{k=13}^{24} 2e^{j\omega(n-k)} \\ &= \frac{e^{j\omega n} \cdot e^{-j\omega} (1 - e^{-j\omega 12})}{1 - e^{j\omega}} + \frac{2e^{j\omega n} \cdot e^{-j\omega 13} (1 - e^{-j\omega 12})}{1 - e^{j\omega}} \\ &= \frac{e^{j\omega n} (e^{-j\omega} - e^{-j\omega 13})}{1 - e^{j\omega}} + \frac{2e^{j\omega n} (e^{-j\omega 13} - e^{-25j\omega})}{1 - e^{j\omega}} \end{aligned}$$

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Question 4: [16%, Work-out question] Consider the following CT linear system. We know for a fact that if the input signal is an *even* signal $x(t)$, the output signal $y(t)$ of this linear system is

$$y(t) = x(t - 4). \quad (8)$$

For example, we know that if the input signal is $x(t) = |t| \cos(t)$, then the output signal is $y(t) = |t - 4| \cos(t - 4)$. If the input signal is $x(t) = \sin^2(t)$, then the output signal is $y(t) = \sin^2(t - 4)$. And so on so forth.

We also know that if the input signal is an *odd* signal $x(t)$, the output signal $y(t)$ of this linear system is

$$y(t) = |x(t)|. \quad (9)$$

For example, we know that if the input signal is $x(t) = t^3$, then the output signal is $y(t) = |t^3|$.

Suppose we are encountering a new input signal

$$x_{\text{new}}(t) = \begin{cases} t + 2 & \text{if } -2 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

Since this new $x_{\text{new}}(t)$ is neither even nor odd, we cannot directly shift-by-minus-4 or taking-the absolute-value to find the output $y_{\text{new}}(t)$. Answer the following questions:

1. [2%] Plot $x_{\text{new}}(t)$ for the range of $-8 \leq t \leq 8$.
2. [14%] What is the output $y_{\text{new}}(t)$. In your answers, please plot $y_{\text{new}}(t)$ for the range of $-8 \leq t \leq 8$.

Hint: If you do not know the answer to this question, please plot the odd part of the signal $x_{\text{new}}(t)$ for the range of $-8 \leq t \leq 8$. You will receive 8 points if your answer is correct.

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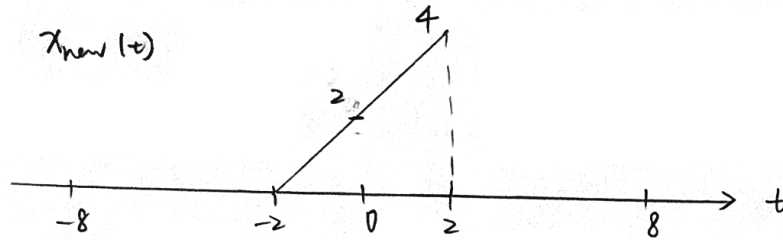
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This sheet is for Question 4.

Answer:

$x_{new}(t)$

1)



2)

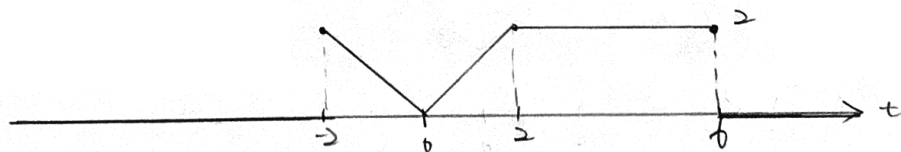
$$x_{new}(t) = \frac{x_{new}(t) + x_{new}(-t)}{2} + \frac{x_{new}(t) - x_{new}(-t)}{2}$$

$$= \underset{\substack{\uparrow \\ \text{even part}}}{2} + \underset{\substack{\uparrow \\ \text{odd part}}}{t}$$

$$x_{\text{even}}(t) = 2 \rightarrow y_{\text{even}}(t) = x_{\text{even}}(t-4) = 2 \quad 2 \leq t \leq 6$$

$$x_{\text{odd}}(t) = t \rightarrow y_{\text{odd}}(t) = |x_{\text{odd}}(t)| = \begin{cases} t & 0 \leq t < 2 \\ -t & -2 \leq t < 0 \end{cases}$$

$$y_{\text{new}}(t) = \begin{cases} -t & -2 \leq t < 0 \\ t & 0 \leq t < 2 \\ 2 & 2 \leq t \leq 6 \end{cases}$$



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Question 5: [16%, Work-out question]

Consider two CT HRCEs.

$$x(t) = e^{j\pi t} \quad (11)$$

$$y(t) = \sin(\pi t) \quad (12)$$

1. [6%] What is the average power of $x(t)$ during the interval $1 < t < 3$?
2. [10%] Consider a new signal $z(t) = x(t) + y(t)$. What is the average power of $z(t)$ during the interval $1 < t < 3$?

Hint 1: Q5.2 is a work-out question. An answer without detailed/correct justification reasons would receive 0 point.

Hint 2: The following trigonometric formulas may be useful.

$$\cos(\alpha)^2 = 0.5(\cos(2\alpha) + 1) \quad (13)$$

$$\sin(\alpha)^2 = 0.5(1 - \cos(2\alpha)) \quad (14)$$

$$\cos(\alpha)\sin(\alpha) = 0.5\sin(2\alpha) \quad (15)$$

Answer: 1.
$$P_1 = \frac{\int_{t=1}^3 |x(t)|^2 dt}{3-1} = 1$$

2.
$$z(t) = e^{j\pi t} + \sin(\pi t)$$

$$= (\cos(\pi t) + j\sin(\pi t) + j\sin(\pi t))$$

$$P_2 = \frac{1}{3-1} \int_{t=1}^3 |(\cos(\pi t) + \sin(\pi t) + j\sin(\pi t))|^2 dt$$

$$= \frac{1}{2} \int_{t=1}^3 |(\cos(\pi t) + \sin(\pi t))|^2 + (\sin(\pi t))^2 dt$$

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This sheet is for Question 5.

$$= \frac{1}{2} \int_{t=1}^3 1 + 2 \cos(\pi t) \sin(\pi t) + (\sin(\pi t))^2 dt$$

$$= 1 + \int_{t=1}^3 \cos(\pi t) \sin(\pi t) dt + \frac{1}{2} \int_{t=1}^3 (\sin(\pi t))^2 dt$$

$$= 1 + \frac{1}{2} \int_{t=1}^3 \sin(2\pi t) dt + \frac{1}{2} \cdot \frac{1}{2} \int_{t=1}^3 (1 - \cos(2\pi t)) dt$$

$$= 1 + 0 + \frac{1}{2} - \frac{1}{4} \cdot 0$$

$$= \frac{3}{2}$$

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Question 6: [20%, Multiple Choices]

The following questions are multiple-choice questions and there is no need to justify your answers. Consider two continuous-time signals:

$$x_1(t) = e^{\sin(t)} + e^{-\sin(t)} \quad (16)$$

$$x_2(t) = \int_{s=t-5}^{t+1} \sin(3s+6) ds = -\frac{\cos(3s+6)}{3} \Big|_{s=t-5}^{t+1} \quad (17)$$

and two discrete-time signals:

$$x_3[n] = \cos(0.2\pi n) \cdot \left(\sum_{k=-\infty}^{\infty} \delta[n-4k-1] \right) \quad (18)$$

$$x_4[n] = e^{j(0.5\pi n^2)} \quad (19)$$

- [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not, *respectively*. If it is periodic, write down the fundamental period. Please state explicitly which signal is periodic and which is not.
- [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them, *respectively*. Please state explicitly which signal is even, which is odd, and which is neither.

Answer:

1. $x_1(t)$: periodic

$$T = \pi$$

$x_2(t)$: periodic

$$T = \frac{2\pi}{3}$$

$x_3[n]$: periodic

$$T = \text{LCM}(T_1, T_2)$$

$$= \text{LCM}(10, 4) = 20$$

$x_4[n]$: periodic

(Find the smallest integer such that $e^{j(\frac{\pi}{2}(n+N)^2)} = e^{j\frac{\pi}{2}n^2}$)

$$N = 2$$

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This sheet is for Question 6.

2. $x_1(t) = \text{even}$

$$\begin{aligned}x_1(t) &= e^{\sin(1-t)} + e^{-\sin(1-t)} \\ &= e^{-\sin t} + e^{\sin t}\end{aligned}$$

$x_2(t) = \text{odd}$

$$x_2(t) = \frac{\cos(3t-9)}{3} - \frac{\cos(3t+9)}{3}$$

$$\begin{aligned}x_2(-t) &= \frac{\cos(3t+9)}{3} - \frac{\cos(-3t+9)}{3} \\ &= -x_2(t)\end{aligned}$$

$x_3[n] = \text{neither even or odd}$

$x_4[n] = \text{even}$