

**Midterm #3 of ECE 301-003 and OL1, (CRN: 11474 and 26295)  
8–9pm, Wednesday, April 7, 2021, Online Exam.**

1. Enter your student ID number, and signature in the space provided on this page **now!**
2. This is a closed book exam.
3. This exam contains exclusively work-out questions. You have one hour to complete it. The students are suggested not spending too much time on a single question, and first working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

As a Boiler Maker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

Signature:

Date:

*Question 1:* [18%, Work-out question] Consider two discrete-time periodic signals

$$x[n] = \sin\left(\frac{6\pi n}{5}\right) \quad (1)$$

$$y[n] = \cos\left(\frac{2\pi n}{5}\right) \quad (2)$$

$$(3)$$

Denote the DTFS coefficients of  $x[n]$  by  $a_k$  and denote the DTFS coefficients of  $y[n]$  by  $b_k$ . Answer the following questions.

1. [2%] Find the period of  $x[n]$ .
2. [6%] Plot the values of  $a_k$  for the range of  $k = -5$  to  $5$ .
3. [10%] Define  $z[n] = x[n] \cdot y[n]$  and denote the corresponding DTFS coefficients as  $c_k$ . Plot the values of  $c_k$  for the range of  $k = -5$  to  $5$ .

Hint: If you do not know the answer to this question, please write down the expression of  $c_3$  in terms of  $a_k$  and  $b_k$  formulas. Namely, you can just assume  $a_k$  and  $b_k$  are known numbers and use them to find your  $c_3$  value. You will receive 7 points if your answer is correct.



*Question 2:* [22%, Work-out question] Consider a CT-LTI system with the impulse response as follows.

$$h(t) = \begin{cases} \pi & \text{if } -\pi \leq t \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

We use  $y(t)$  to denote the output of the system when the input is  $x(t) = 2 + \cos(4t)$ . Answer the following questions:

1. [6%] Let  $X(j\omega)$  denote the CTFT of the input signal  $x(t)$ . Plot  $X(j\omega)$  for the range of  $-7 \leq \omega \leq 7$ .
2. [2%] What is the definition of the *frequency response* of an LTI system?
3. [10%] Let  $H(j\omega)$  denote the CTFT of  $h(t)$ . Plot  $H(j\omega)$  for the range of  $-7 \leq \omega \leq 7$ . Please carefully mark all the special points of your figure.
4. [4%] Find the expression of  $y(t)$ .



*Question 3:* [16%, Work-out question] We know that if

$$x(t) = e^{-|t|} \quad (5)$$

the corresponding CTFT is

$$X(j\omega) = \frac{2}{1 + \omega^2} \quad (6)$$

Answer the following questions:

1. [8%] Find the value of  $\int_{-\infty}^{\infty} \left(\frac{2}{1+\omega^2}\right)^2 d\omega$ .
2. [8%] Find the value of  $\int_{-\infty}^{\infty} \left(\frac{2}{1+\omega^2}\right) \cdot e^{j3\omega} d\omega$ .



Question 4: [14%, Work-out question] Consider the following periodic CT signal  $x(t)$

$$x(t) = \begin{cases} 1 & -1.5 \leq t \leq 1.5 \\ 0 & 1.5 < t < 6.5 \\ \text{periodic with period 8} & \end{cases} \quad (7)$$

Plot  $X(j\omega)$  for the range of  $-\frac{\pi}{3} \leq \omega \leq \frac{\pi}{3}$ .

Hint 1: This type of computation is termed the *generalized CTFT* in the lecture.

Hint 2: If you do not know how to find the CTFT of  $x(t)$ , you should find the CTFS of  $x(t)$  instead. You will receive 9 points if your answer is correct.

Hint 3: The CTFS of  $x(t)$  actually helps you find the CTFT of  $x(t)$ .





*Question 5:* [12%, Work-out question] Consider a discrete time signal  $x[n]$  and we know that the DTFT  $X(e^{j\omega})$  satisfies

$$X(e^{j\omega}) = \cos(\omega) + j \sin(3\omega) \quad (8)$$

Plot the signal  $x[n]$  for the range of  $-5 \leq n \leq 5$ .

Hint: The DTFT *analysis formula*  $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$  may be useful for solving this question.



*Question 6:* [18%, Work-out question] Consider the following sequential operations. The input signal is  $x(t) = \frac{\sin(20\pi t)}{\pi t}$ . Step 1: We multiply  $x(t)$  by  $\cos(10\pi t)$ . That is,

$$y(t) = x(t) \cdot \cos(10\pi t).$$

Step 2: We pass  $y(t)$  through a low pass filter with cutoff frequency 10Hz. Denote the output by  $z(t)$ .

1. [5%] Plot the CTFT  $X(j\omega)$  for the range of  $\omega = -60\pi$  to  $60\pi$ .
2. [5%] Plot the CTFT  $Y(j\omega)$  for the range of  $\omega = -60\pi$  to  $60\pi$ .
3. [4%] Plot the CTFT  $Z(j\omega)$  for the range of  $\omega = -60\pi$  to  $60\pi$ .
4. [4%] Find out the expression of  $z(t)$ .



Discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \quad (1)$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n} \quad (2)$$

Continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \quad (3)$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt \quad (4)$$

Continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \quad (5)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad (6)$$

Discrete-time Fourier transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega \quad (7)$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \quad (8)$$

Laplace transform

$$x(t) = \frac{1}{2\pi} e^{\sigma t} \int_{-\infty}^{\infty} X(\sigma + j\omega) e^{j\omega t} d\omega \quad (9)$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \quad (10)$$

Z transform

$$x[n] = r^n \mathcal{F}^{-1}(X(re^{j\omega})) \quad (11)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \quad (12)$$

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$x(t)$ } Periodic with period $T$ and $y(t)$ } fundamental frequency $\omega_0 = 2\pi/T$	$a_k$ $b_k$
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} x(t) = e^{jM(2\pi/T)t} x(t)$	$a_{k-M}$
Conjugation	3.5.6	$x^*(t)$	$a_{-k}^*$
Time Reversal	3.5.3	$x(-t)$	$a_{-k}$
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period $T/\alpha$ )	$a_k$
Periodic Convolution		$\int_T x(\tau)y(t - \tau)d\tau$	$T a_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk \frac{2\pi}{T} a_k$
Integration		$\int_{-\infty}^t x(t) dt$ (finite valued and periodic only if $a_0 = 0$ )	$\left(\frac{1}{jk\omega_0}\right) a_k = \left(\frac{1}{jk(2\pi/T)}\right) a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a_{-k}^* \\ \Re\{a_k\} = \Re\{a_{-k}\} \\ \Im\{a_k\} = -\Im\{a_{-k}\} \\  a_k  =  a_{-k}  \\ \angle a_k = -\angle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	$a_k$ real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	$a_k$ purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\begin{cases} \Re\{a_k\} \\ \Im\{a_k\} \end{cases}$
Parseval's Relation for Periodic Signals			
$\frac{1}{T} \int_T  x(t) ^2 dt = \sum_{k=-\infty}^{+\infty}  a_k ^2$			

three examples, we illustrate this. The last example in this section then demonstrates how properties of a signal can be used to characterize the signal in great detail.

### Example 3.6

Consider the signal  $g(t)$  with a fundamental period of 4, shown in Figure 3.10. We could determine the Fourier series representation of  $g(t)$  directly from the analysis equation (3.39). Instead, we will use the relationship of  $g(t)$  to the symmetric periodic square wave  $x(t)$  in Example 3.5. Referring to that example, we see that, with  $T = 4$  and  $T_1 = 1$ ,

$$g(t) = x(t - 1) - 1/2. \quad (3.40)$$

Thus, in general, *none* of the finite partial sums in eq. (3.52) yield the exact values of  $x(t)$ , and convergence issues, such as those considered in Section 3.4, arise as we consider the problem of evaluating the limit as the number of terms approaches infinity.

### 3.7 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

There are strong similarities between the properties of discrete-time and continuous-time Fourier series. This can be readily seen by comparing the discrete-time Fourier series properties summarized in Table 3.2 with their continuous-time counterparts in Table 3.1.

**TABLE 3.2** PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$x[n]$ } Periodic with period $N$ and $y[n]$ } fundamental frequency $\omega_0 = 2\pi/N$	$a_k$ } Periodic with $b_k$ } period $N$
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n} x[n]$	$a_{k-M}$
Conjugation	$x^*[n]$	$a_{-k}^*$
Time Reversal	$x[-n]$	$a_{-k}$
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period $mN$ )	$\frac{1}{m} a_k$ (viewed as periodic) (with period $mN$ )
Periodic Convolution	$\sum_{r=(N)} x[r]y[n-r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=(N)} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^n x[k]$ (finite valued and periodic only) (if $a_0 = 0$ )	$\left(\frac{1}{1 - e^{-jk(2\pi/N)}}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a_{-k}^* \\ \Re\{a_k\} = \Re\{a_{-k}\} \\ \Im\{a_k\} = -\Im\{a_{-k}\} \\  a_k  =  a_{-k}  \\ \angle a_k = -\angle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	$a_k$ real and even
Real and Odd Signals	$x[n]$ real and odd	$a_k$ purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{O}\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\begin{cases} \Re\{a_k\} \\ j\Im\{a_k\} \end{cases}$
Parseval's Relation for Periodic Signals		
$\frac{1}{N} \sum_{n=(N)}  x[n] ^2 = \sum_{k=(N)}  a_k ^2$		



## 4.6 TABLES OF FOURIER PROPERTIES AND OF BASIC FOURIER TRANSFORM PAIRS

In the preceding sections and in the problems at the end of the chapter, we have considered some of the important properties of the Fourier transform. These are summarized in Table 4.1, in which we have also indicated the section of this chapter in which each property has been discussed.

In Table 4.2, we have assembled a list of many of the basic and important Fourier transform pairs. We will encounter many of these repeatedly as we apply the tools of

**TABLE 4.1** PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$
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4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t} x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta)Y(j(\omega - \theta))d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^t x(t)dt$	$\frac{1}{j\omega} X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j \frac{d}{d\omega} X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \Re\{X(j\omega)\} = \Re\{X(-j\omega)\} \\ \Im\{X(j\omega)\} = -\Im\{X(-j\omega)\} \\  X(j\omega)  =  X(-j\omega)  \\ \angle X(j\omega) = -\angle X(-j\omega) \end{cases}$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}\{x(t)\}$ [ $x(t)$ real] $x_o(t) = \mathcal{O}\{x(t)\}$ [ $x(t)$ real]	$\Re\{X(j\omega)\}$ $j\Im\{X(j\omega)\}$
-----			
4.3.7	Parseval's Relation for Aperiodic Signals		
		$\int_{-\infty}^{+\infty}  x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty}  X(j\omega) ^2 d\omega$	

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	$a_k$
$e^{j\omega_0 t}$	$2\pi \delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$ , otherwise
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$ , otherwise
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$ , otherwise
$x(t) = 1$	$2\pi \delta(\omega)$	$a_0 = 1$ , $a_k = 0$ , $k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$ )
Periodic square wave		
$x(t) = \begin{cases} 1, &  t  < T_1 \\ 0, & T_1 <  t  \leq \frac{T}{2} \end{cases}$	$\sum_{k=-\infty}^{+\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
and $x(t + T) = x(t)$		
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all $k$
$x(t) \begin{cases} 1, &  t  < T_1 \\ 0, &  t  > T_1 \end{cases}$	$\frac{2 \sin \omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, &  \omega  < W \\ 0, &  \omega  > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi \delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at} u(t), \operatorname{Re}\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$t e^{-at} u(t), \operatorname{Re}\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!} e^{-at} u(t), \operatorname{Re}\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—

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$-\operatorname{Im}\{X(-j\omega)\}$

( $j\omega$ )

$X(-j\omega)$

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imaginary and odd

**TABLE 5.1** PROPERTIES OF THE DISCRETE-TIME FOURIER TRANSFORM

Section	Property	Aperiodic Signal	Fourier Transform
		$x[n]$	$X(e^{j\omega})$ } periodic with
		$y[n]$	$Y(e^{j\omega})$ } period $2\pi$
5.3.2	Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
5.3.3	Time Shifting	$x[n - n_0]$	$e^{-j\omega n_0} X(e^{j\omega})$
5.3.3	Frequency Shifting	$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$
5.3.4	Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
5.3.6	Time Reversal	$x[-n]$	$X(e^{-j\omega})$
5.3.7	Time Expansion	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
5.4	Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
5.5	Multiplication	$x[n]y[n]$	$\frac{1}{2\pi} \int_{2\pi} X(e^{j\theta})Y(e^{j(\omega - \theta)})d\theta$
5.3.5	Differencing in Time	$x[n] - x[n - 1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
5.3.5	Accumulation	$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1 - e^{-j\omega}} X(e^{j\omega})$
5.3.8	Differentiation in Frequency	$nx[n]$	$j \frac{dX(e^{j\omega})}{d\omega}$
5.3.4	Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \Re\{X(e^{j\omega})\} = \Re\{X(e^{-j\omega})\} \\ \Im\{X(e^{j\omega})\} = -\Im\{X(e^{-j\omega})\} \\  X(e^{j\omega})  =  X(e^{-j\omega})  \\ \angle X(e^{j\omega}) = -\angle X(e^{-j\omega}) \end{cases}$
5.3.4	Symmetry for Real, Even Signals	$x[n]$ real and even	$X(e^{j\omega})$ real and even
5.3.4	Symmetry for Real, Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
5.3.4	Even-odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}\{x[n]\}$ [ $x[n]$ real ] $x_o[n] = \mathcal{O}\{x[n]\}$ [ $x[n]$ real ]	$\Re\{X(e^{j\omega})\}$ $j\Im\{X(e^{j\omega})\}$
5.3.9	Parseval's Relation for Aperiodic Signals	$\sum_{n=-\infty}^{+\infty}  x[n] ^2 = \frac{1}{2\pi} \int_{2\pi}  X(e^{j\omega}) ^2 d\omega$	

a duality relationship between the discrete-time Fourier transform and the continuous-time Fourier series. This relation is discussed in Section 5.7.2.

### 5.7.1 Duality in the Discrete-Time Fourier Series

Since the Fourier series coefficients  $a_k$  of a periodic signal  $x[n]$  are themselves a periodic sequence, we can expand the sequence  $a_k$  in a Fourier series. The duality property for discrete-time Fourier series implies that the Fourier series coefficients for the periodic sequence  $a_k$  are the values of  $(1/N)x[-n]$  (i.e., are proportional to the values of the original

**TABLE 5.2 BASIC DISCRETE-TIME FOURIER TRANSFORM PAIRS**

Signal	Fourier Transform	Fourier Series Coefficients (if periodic)
$\sum_{k=(N)} a_k e^{jk(2n/N)n}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k$
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} 1, & k = m, m \pm N, m \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2}, & k = \pm m, \pm m \pm N, \pm m \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$ $a_k = \begin{cases} \frac{1}{2j}, & k = r, r \pm N, r \pm 2N, \dots \\ -\frac{1}{2j}, & k = -r, -r \pm N, -r \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, &  n  \leq N_1 \\ 0, & N_1 <  n  \leq N/2 \end{cases}$ and $x[n + N] = x[n]$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N \sin[2\pi k/2N]}, k \neq 0, \pm N, \pm 2N, \dots$ $a_k = \frac{2N_1 + 1}{N}, k = 0, \pm N, \pm 2N, \dots$
$\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N}$ for all $k$
$a^n u[n],  a  < 1$	$\frac{1}{1 - ae^{-j\omega}}$	—
$x[n] = \begin{cases} 1, &  n  \leq N_1 \\ 0, &  n  > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$	—
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} \text{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \leq  \omega  \leq W \\ 0, & W <  \omega  \leq \pi \end{cases}$ $X(\omega)$ periodic with period $2\pi$	—
$\delta[n]$	1	—
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi \delta(\omega - 2\pi k)$	—
$\delta[n - n_0]$	$e^{-j\omega n_0}$	—
$(n + 1)a^n u[n],  a  < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$	—
$\frac{(n + r - 1)!}{n!(r - 1)!} a^n u[n],  a  < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$	—