

**Final Exam of ECE 301-003 and OL1, (CRN: 11474 and 26295)  
10:30am–12:30pm, Thursday, May 6, 2021, Online Exam.**

1. Enter your student ID number, and signature in the space provided on this page now!
2. This is a closed book exam.
3. This exam contains multiple-choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have two hours to complete it. The students are suggested not spending too much time on a single question, and first working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

As a Boiler Maker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

Signature:

Date:

Question 1: [23%, Work-out question]

1. [1%] What does the acronym AM-DSB stand for? *double side-band amplitude modulation*
2. [1%] What does the acronym FDM stand for? *frequency division multiplexing*

Prof. Wang wanted to transmit an AM-SSB signal. To that end, he wrote the following MATLAB code.

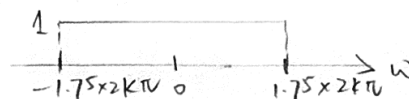
```
% Initialization
duration=8;
f_sample=44100;
t=((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5)/f_sample;

% Read two different .wav files
[x1, f_sample, N]=audioread('x1.wav');
x1=x1';
[x2, f_sample, N]=audioread('x2.wav');
x2=x2';

% Step 0: Initialize several parameters
W_1=pi*3500;
W_2=pi*2000;
W_3=????;
W_4=????;
W_5=????;
W_6=????;
W_7=????;
```

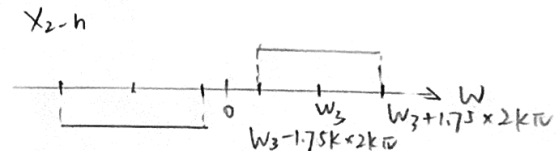
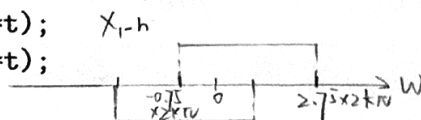
% Step 1: Make the signals band-limited.

```
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);
```



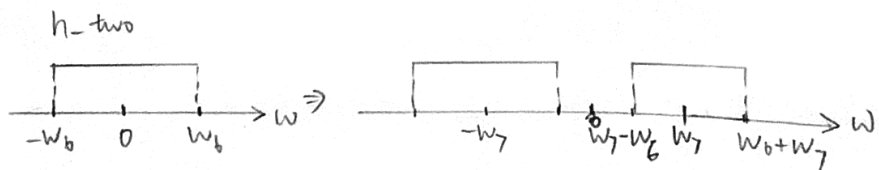
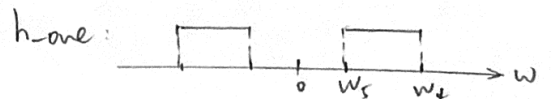
% Step 2: Multiply x1\_new and x2\_new with a sinusoidal wave.

```
x1_h=x1_new.*sin(W_2*t);
x2_h=x2_new.*sin(W_3*t);
```



% Step 3: Keep one of the two side bands

```
h_one=1/(pi*t).*(sin(W_4*t)-sin(W_5*t));
h_two=1/(pi*t).*(sin(W_6*t).*(2*cos(W_7*t)));
x1_sb=ece301conv(x1_h, h_one);
x2_sb=ece301conv(x2_h, h_two);
```



```
% Step 4: Create the transmitted signal
y=x1_sb+x2_sb;
audiowrite('y.wav', y, f_sample);
```

3. [1%] What is the carrier frequency (Hz) of the signal x1\_new?
4. [3%] Our goal is to transmit the "upper-side bands" for both x1 and x2 signals. What should be the values of W\_4 and W\_5 in the MATLAB code?
5. [3%] Suppose I choose the value of W\_3 to be  $7000\pi$ , what should be the values of W\_6 and W\_7 in the MATLAB code?
6. [2%] What would be the smallest possible value of W\_3 in the MATLAB code without negatively impacting the sound quality of either x1 and x2 signals? (Here we assume that we still use the upper side band and also the W\_6 and W\_7 values are adjusted accordingly when we choose the smallest possible W\_3 value.)
7. [1.5%] If I choose the value of W\_3 to be  $5000\pi$ , please describe in words what would be the impact to the sound quality?

3.  $2000\pi$  rad/s

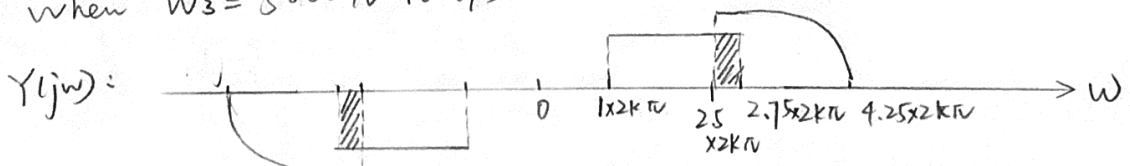
4.  $W_4 = 5500\pi$  rad/s       $W_5 = 2000\pi$  rad/s

5. 
$$\begin{cases} W_7 - W_6 = W_3 = 7000\pi \\ W_7 + W_6 = W_3 + 3500\pi = 10500\pi \end{cases}$$

$\Rightarrow$  
$$\begin{cases} W_6 = 1750\pi$$
 rad/s  
 $W_7 = 8750\pi$  rad/s

6.  $W_3 > 5500\pi$  rad/s

7. when  $W_3 = 5000\pi$  rad/s



The higher frequency components of  $x_1$  and the lower frequency components of  $x_2$  are overlapped, then distorted

Prof. Wang decided to use the upper-side-band transmission with  $W_3$  set to  $7000\pi$  and used the code in the previous page to generate the “y.wav” file. A student tried to demodulate the output waveform “y.wav” by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5)/f_sample;

% Read the .wav files
[y, f_sample, N]=audioread('y.wav');
y=y';

% Initialize several parameters
W_8=????;
W_9=????;
W_10=????;

% Create the low-pass filter.
h_M=1/(pi*t).*(sin(W_8*t));

% demodulate signal 1
y1=y.*sin(W_9*t);
x1_hat=ece301conv(y1,h_M);

sound(x1_hat,f_sample)

% demodulate signal 2
y2=y.*sin(W_10*t);
x2_hat=ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```

8. [4.5%] Continue from the previous questions. What should the values of  $W_8$  to  $W_{10}$  be in the MATLAB code?
9. [6%] It turns out that the above MATLAB code is not written correctly and the end results do not sound right. Neither signal  $x1\_new$  nor signal  $x2\_new$  can be correctly/perfectly demodulated. Please use 2 to 3 sentences to answer (i) what kind of problem does  $x1\_new$  have, i.e., how does the problem impact the sound quality of “sound(x1\_hat,f\_sample)”? (ii) how can the MATLAB code be corrected so that the playback/demodulation can be performed successfully?

"sound(x1\_hat, f\_sample)"? (ii) how can the MATLAB code be corrected so that the playback/demodulation can be performed successfully?

10. Please use 2 to 3 sentences to <sup>answer</sup> (iii) what kind of problem does x2\_new have, i.e., how does the problem impact the sound quality of "sound(x2\_hat, f\_sample)"? (ii) how can the MATLAB code be corrected so that the playback/demodulation can be performed successfully? Please explain your changes very carefully for this question.

Sub-questions 7 and 8 would be graded together. The two sub-questions combined will have 6 points.

Hint: If you do not know the answers of Q1.3 to Q1.9, please simply draw the AMSSB modulation (using lower side band) and demodulation diagrams and mark carefully all the parameter values. You will receive 12 points for Q1.3 to Q1.9 if your system diagrams are correct and all parameter values are marked correctly.

8.  $\omega_s = 3500\pi \text{ rad/s}$     $\omega_g = 2000\pi \text{ rad/s}$     $\omega_{10} = 7000\pi \text{ rad/s}$

9. i) The loudness of  $x_{1\_new}$  is degraded.

ii) Fixed signal 1 demodulation code:

$$y_1 = y \cdot 4 \sin(\omega_g * t);$$

$$x_{1\_hat} = \text{ece301conv}(y_1, h_m);$$

10. i) The frequency band  $[1500\pi \text{ rad/s}, 3500\pi \text{ rad/s}]$  are distorted.  
And the loudness of signal 2 is degraded.

ii) Fixed signal 2 demodulation code:

$$h_b = \frac{1}{\pi t} \cdot (\sin(10500\pi t) - \sin(7000\pi t))$$

$$y_{2b} = \text{ece301conv}(y, h_b);$$

$$y_2 = y_{2b} \cdot 4 \sin(\omega_{10} * t);$$

$$x_{2\_hat} = \text{ece301conv}(y_2, h_m);$$

Question 2: [14%, Work-out question]

1. [3%] Consider a continuous time signal  $x(t)$

$$x(t) = \frac{\sin(3\pi t)}{\pi t}. \quad (1)$$

Plot  $x(t)$  for the range of  $-1 \leq t \leq 1$ . Please carefully mark all the important points in the figure.

2. [3%] We sample  $x(t)$  by an *impulse train sampler* with sampling frequency 4Hz. That is,

$$x_p(t) = x(t) \cdot \left( \sum_{k=-\infty}^{\infty} \delta(t - 0.25k) \right) \quad (2)$$

Plot  $x_p(t)$  for the range of  $-0.4 \leq t \leq 0.4$ . Please carefully mark all the important points in the figure.

Hint:  $\sin(0.25\pi) = \sin(0.75\pi) = \frac{\sqrt{2}}{2}$ .

3. [4%] We know that the frequency domain  $X(j\omega)$  of the signal  $x(t)$  is

$$X(j\omega) = \begin{cases} 1 & \text{if } -3\pi \leq \omega \leq 3\pi \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Plot the frequency domain  $X_p(j\omega)$  of  $x_p(t)$  for the range of  $-12\pi \leq \omega \leq 12\pi$ .

4. [4%] We then pass  $x_p(t)$  over a *Band-Pass Filter* with the following frequency response

$$h_{\text{BPF}}(t) = \frac{1}{8} \cdot \frac{\sin((8000\pi + 4\pi) \cdot t) - \sin((8000\pi - 4\pi) \cdot t)}{\pi t}. \quad (4)$$

That is,  $y(t) = x_p(t) * h_{\text{BPF}}(t)$ . Plot  $y(t)$  for the range of  $-1 \leq t \leq 1$ .

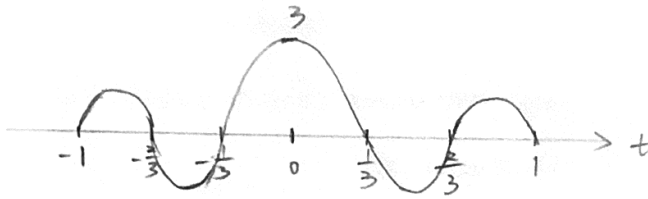
Hint 1: There is a connection between this question and the concept of AM-DSB. Such a connection will be critical when solving this question.

Hint 2: If you do not know how to solve Q2.4, you can write down the description of AM *asynchronous demodulation*. You will receive 1.5 points if your answer is correct.

Hint 3: This sub-question is considered to be slightly more advanced. You may consider working on other questions first and then come back to this one later.

Q2 Answer:

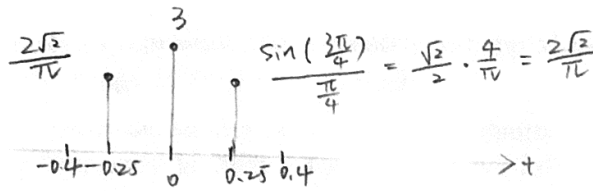
1.  $X(t) = \frac{\sin(3\pi t)}{\pi t}$



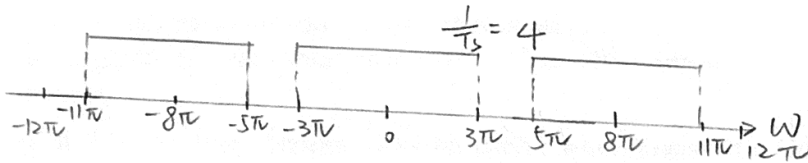
zero points:  
 $3\pi t = k\pi \Rightarrow t = \frac{k}{3}$

main lobe:  
 $\lim_{t \rightarrow 0} \frac{\sin(3\pi t)}{3\pi t} = 3 \cdot 1 = 3$

2.

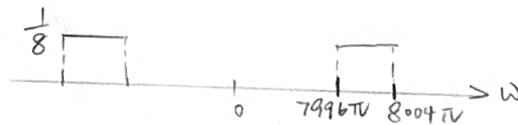


3.

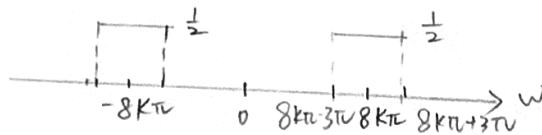


$\omega_s = \frac{2\pi}{T_s} = 8\pi$

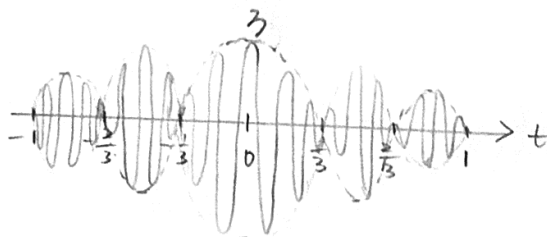
4.  $H_{BPF} =$



$Y(j\omega) = X_P(j\omega) \cdot H_{BPF}(j\omega)$



$y(t) =$



Question 3: [12%, Work-out question]

1. [3%] Consider the following continuous time signal

$$x(t) = \cos(5\pi t) \quad (5)$$

We sample  $x(t)$  with the sampling frequency 20Hz and denote the sampled values by  $x[n]$ . Plot  $x[n]$  for the range of  $-3 \leq n \leq 3$ .

2. [3%] We use  $x_{\text{ZOH}}(t)$  to represent the reconstructed signal using “zero-order hold”. Plot  $x_{\text{ZOH}}(t)$  for the range of  $-0.15 \leq t \leq 0.15$ .

Hint: if you do not know the answer of  $x[n]$ , you can assume that  $x[n] = 2(-1)^{n+1} - 2$  and the sampling frequency is 20Hz. You will receive full points if your answer is correct.

3. [3%] We use  $x_{\text{opt}}(t)$  to represent the optimal band-limited reconstruction of  $x(t)$ . Plot  $x_{\text{opt}}(t)$  for the range of  $-0.4 \leq t \leq 0.4$ .

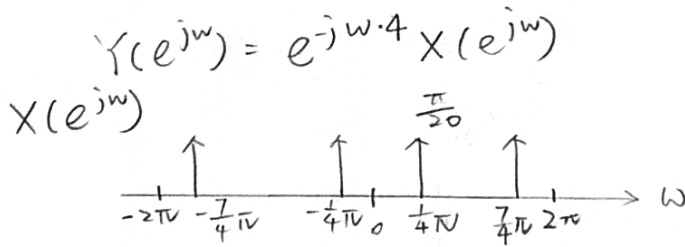
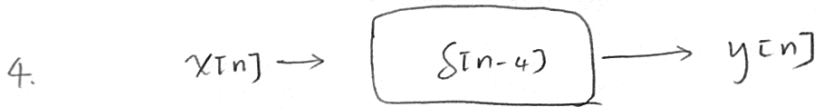
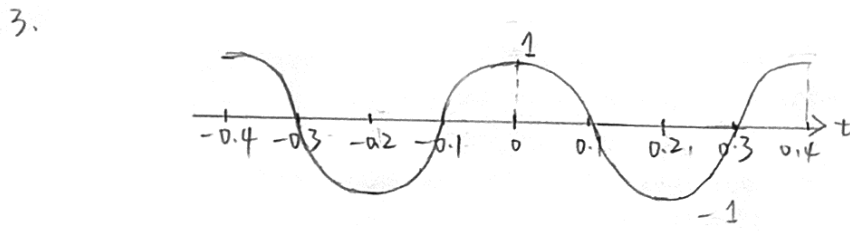
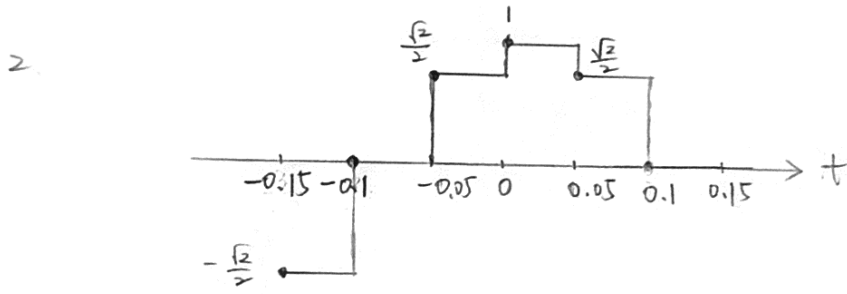
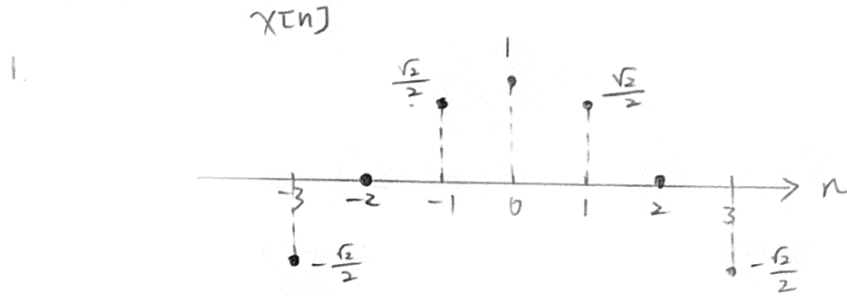
Hint 1: if you do not know the answer of this sub-question, please write down the description of “sampling theorem”. You will receive 1.5 points if your answer is correct.

Hint 2: Sampling theorem may be useful when answering this question.

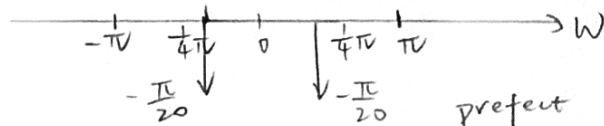
4. [3%] Suppose we pass the sampled array  $x[n]$  through a DT-LTI system with impulse response  $h[n] = \delta[n - 4]$  and denote the output array to be  $y[n]$ . We use  $y(t)$  to represent the optimal band-limited reconstruction based on the output array  $y[n]$  and the fact that the sampling frequency is 20Hz. Plot  $y(t)$  for the range of  $-0.4 \leq t \leq 0.4$ .



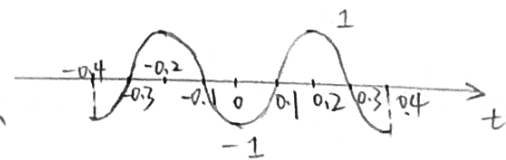
Q3 Answer



$Y(e^{j\omega}) = e^{-j\omega \cdot 4} X(e^{j\omega})$



perfect  
reconstruction



Question 4: [9%, Work-out question]

1. [2.5%] Consider a discrete time  $x[n]$  and we know its DTFT  $X(e^{j\omega})$  inside the range of  $-\pi \leq \omega \leq \pi$  being

$$X(e^{j\omega}) = \pi\delta(\omega - 0.1\pi) + \pi\delta(\omega + 0.1\pi) \quad \text{if } -\pi \leq \omega \leq \pi \quad (6)$$

Plot  $X(e^{j\omega})$  for the range of  $-3\pi \leq \omega \leq 3\pi$ .

2. [2.5%] Find the expression of  $x[n]$ .
3. [4%] Define  $Y(e^{j\omega})$  as

$$Y(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) *_{\text{cyc.}} \left( \frac{2\pi}{4} \sum_{k=-\infty}^{\infty} \delta\left(\omega - k \cdot \frac{2\pi}{4}\right) \right) \quad (7)$$

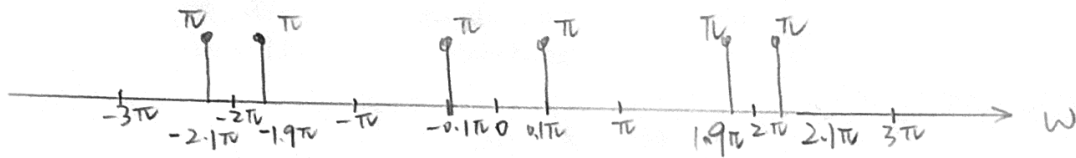
where “\*<sub>cyc.</sub>” represents the *cyclic convolution*. Find (i) the expression of  $y[n]$ ; (ii) the value of  $y[2]$ ; and (iii) the value of  $y[4]$ . The three answers will be graded together.

Hint 1: The DTFT table would be very useful when solving this question. Specifically, the table can help you find the relationship between  $y[n]$  and  $x[n]$ .

Hint 2: If you do not know the answer to this sub-question, please write down the definition of *cyclic convolution*. You will receive 2 points if your answer is correct.

Q4 answer:

1.



2.  $X[n] = \cos[0.1\pi n]$

3. By the multiplication property,

$$y[n] = x[n] \cdot h[n]$$

$$\text{where } h[n] = \sum_{k=-\infty}^{+\infty} \delta[n-4k]$$

$$y[n] = \sum_{k=-\infty}^{+\infty} \cos[0.1\pi n] \delta[n-4k]$$

$$y[2] = 0, \quad y[4] = \cos\left[\frac{2\pi}{5}\right]$$

Question 5: [8%, Work-out question]

Consider the following continuous time signals

$$x(t) = \sum_{k=1}^2 e^{j \cdot (kt - \sqrt{k})} \quad (8)$$

$$h(t) = e^{-|t-3|} \quad (9)$$

Find the expression of  $y(t) = x(t) * h(t)$ .

Hint: For a signal  $w(t) = e^{-|t|}$ , we know its Fourier transform being  $W(j\omega) = \frac{2}{1+\omega^2}$ .  
You may use this fact plus the time-shift property to simplify your computation.

Q5 Answer:

$$y(t) = x(t) * h(t)$$

By the convolution property, we first find

$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$X(j\omega) = \mathcal{F}\{e^{j(t-1)} + e^{j(2t-\sqrt{2})}\}$$

$$= e^{-j} \delta(\omega-1) + e^{-\sqrt{2}j} \delta(\omega-2)$$

$$H(j\omega) = \mathcal{F}\{e^{-|t-3|}\}$$

$$= e^{-j3\omega} \mathcal{F}\{e^{-|t|}\}$$

$$= e^{-j3\omega} \cdot \frac{2}{1+\omega^2}$$

$$Y(j\omega) = e^{-j} e^{-j3} \cdot \frac{2}{1+1} \delta(\omega-1) + e^{-\sqrt{2}j} e^{-j6} \cdot \frac{2}{1+4} \delta(\omega-2)$$

$$= e^{-4j} \delta(\omega-1) + \frac{2}{5} e^{-(6+\sqrt{2})j} \delta(\omega-2)$$

$$\Rightarrow y(t) = e^{j(t-4)} + \frac{2}{5} e^{j(2t-6-\sqrt{2})}$$

Question 6: [9%, Work-out question]

Consider the following discrete-time signal

$$x[n] = \begin{cases} \frac{n+2}{3} & \text{if } -2 \leq n \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

Denote the corresponding DTFT by  $X(e^{j\omega})$ .

1. [4%] Find the value of  $\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega} d\omega$ .
2. [5%] Find the value of  $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$ .

Answer: 1. Compare to DTFT synthesis formula:

$$\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega} d\omega = 2\pi X[1] = 2\pi.$$

2. By the Parseval's Relation

$$\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 2\pi \sum_{n=-\infty}^{+\infty} |x[n]|^2$$

$$= 2\pi \left( \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + 1 \right)$$

$$= \frac{14}{9} \times 2\pi$$

$$= \frac{28}{9} \pi.$$

Question 7: [10%, Work-out question]

Consider the following discrete time signal:

$$x[n] = \begin{cases} (3j)^{n+1} & \text{if } n \leq -1 \\ 0 & \text{if } n = 0 \\ 0.5^{n-1} & \text{if } 1 \leq n \end{cases} \quad (11)$$

- Derive the Z-transform expression  $X(z)$  of  $x[n]$ ;
- and plot the corresponding region of convergence.

These two questions will be graded together since it is testing your knowledge about how to derive the Z-transform. Please carefully write down your reasonings. If you use the table without explanation, then you will only receive 7 points even if your answer is correct.

Hint 1: You may need the following formula:

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r} \text{ if } |r| < 1 \quad (12)$$

Hint 2: If you do not know the answer to any of the above two question, please answer the following yes/no question: Suppose a signal  $x[n]$  has its Z-transform being  $\frac{z^7}{1-2jz^{-1}}$  with the ROC being  $|z| > 2$ . Does the DTFT of  $x[n]$  exist? Please carefully write down your reasons. If your reasons are correct, you will receive 4 points. A yes/no answer without justification will receive no point even if it may be correct.

Q7 Answer:

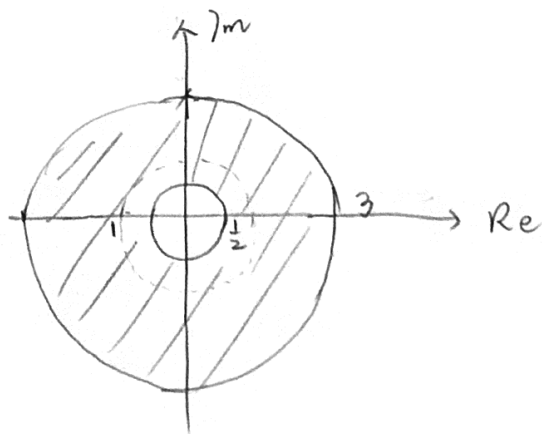
By z-transform analysis formula:

$$\begin{aligned}X(z) &= \sum_{n=-\infty}^{+\infty} x[n] z^{-n} \\&= \sum_{n=-\infty}^{-1} (3j)^{n+1} z^{-n} + \sum_{n=1}^{\infty} 0.5^{n-1} z^{-n} \\&= 3j \sum_{n=1}^{\infty} \left(\frac{z}{3j}\right)^n + 2 \sum_{n=1}^{\infty} \left(\frac{1}{2z}\right)^n \\&= 3j \frac{\frac{z}{3j}}{1 - \frac{z}{3j}} + 2 \cdot \frac{\frac{1}{2z}}{1 - \frac{1}{2z}} \\&= \frac{1}{\frac{1}{2} - \frac{1}{3j}} + \frac{1}{z - \frac{1}{2}}\end{aligned}$$

$$\left|\frac{z}{3j}\right| < 1 \quad \& \quad \left|\frac{1}{2z}\right| < 1$$

$$\Rightarrow \frac{1}{2} < |z| < 3$$

ROC:





Question 8: [15%, Multiple-choice question] Consider two signals

$$h_1(t) = \begin{cases} \left( \frac{e^{4jt} - e^{-4jt}}{2j\pi t} \right)^2 & \text{if } t \neq 0 \\ \frac{16}{\pi^2} & \text{if } t = 0 \end{cases} \quad (13)$$

and

$$h_2[n] = (\cos(n) + \cos(|n|)) \cdot (\sin(n) + \sin(|n|)) \quad (14)$$

*Handwritten note:  $2 \cos n \cdot 2 \sin(n) u(n) = 4 \sin(n) \cos(n) u(n)$*

1. [1.25%] Is  $h_1(t)$  periodic? *No*
2. [1.25%] Is  $h_2[n]$  periodic? *No*
3. [1.25%] Is  $h_1(t)$  even or odd or neither? *even*
4. [1.25%] Is  $h_2[n]$  even or odd or neither? *neither*
5. [1.25%] Is  $h_1(t)$  of finite power? *Yes*
6. [1.25%] Is  $h_2[n]$  of finite power? *Yes*

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless? *No*
2. [1.25%] Is System 2 memoryless? *No*
3. [1.25%] Is System 1 causal? *No*
4. [1.25%] Is System 2 causal? *Yes*
5. [1.25%] Is System 1 stable? *Yes*
6. [1.25%] Is System 2 stable? *No*