PURDUE

COURSENAME/SECTIONNUMBER
EXAM TITLE

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2. Write only on the front of the exam pages.
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4. Ensure that all pages are facing the same direction.
5. Answer all questions in the area designated for that answer. Do not run over into the next question space.

Midterm #2 of ECE 301-004, (CRN: 17102)
8–9pm, Tuesday, October 19, 2021, FRNY G140.

1. Enter your student ID number, and signature in the space provided on this page **now!**
2. This is a closed book exam.
3. This exam contains multiple-choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and first working on those that you know how to solve.
4. The instructor/TA will hand out loose sheets of paper for the rough work.
5. **If you need additional sheets to write down your answers, please let the instructor/TA know. We will hand out additional answer sheets then.**
6. Neither calculators nor help sheets are allowed.

Name:

Student ID:

As a Boiler Maker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

Signature: Date:

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Question 1: [20%, Work-out question]

Consider a DT-LTI system for which we know that if the input is

$$x_1[n] = \begin{cases} 2 & \text{if } n = 1 \\ 0 & \text{if } n \neq 1 \end{cases} \tag{1}$$

then the output is

$$y_1[n] = \begin{cases} 2 & \text{if } 1 \leq n \\ 2e^{n-1} & \text{if } n \leq 0 \end{cases} \tag{2}$$

1. [6%] Find the *impulse response* $h[n]$ of this system.

   Hint 1: If you do not know the answer to this question, please write down (i) What is the definition of *impulse response*; (ii) what does "LTI" stand for? If your answers are correct, you will receive 3 points; (iii) Plot $x_1[n]$ for the range of $-2 \leq n \leq 2$.

   Hint 2: Your answer to (i), (ii), and (iii) in the above hint plays an important role when solving Q1.1 as well.

2. [14%] If the input to this DT-LTI system is $x_2[n] = e^{-n}U[n]$, find the corresponding output $y_2[n]$.

   Hint 3: If you do not know the answer of the impulse response in Q1.1, you can assume $h[n] = y_1[n]$ (even though it is not true) and use $h[n] = y_1[n]$ when solving this question. You will receive full credit if your answer (under this false assumption) is correct.

   Hint 4: You may need the following formulas:

$$\text{if } |r| < 1, \text{ then } \sum_{k=1}^{\infty} a \cdot r^{k-1} = \frac{a}{1-r} \tag{3}$$

$$\text{if } r \neq 1, \text{ then } \sum_{k=1}^{K} a \cdot r^{k-1} = \frac{a \cdot (1 - r^K)}{1-r} \tag{4}$$

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Question 2: [14%, Work-out question] Consider a CT system with the following input/output relationship:

$$y(t) = \int_{-4}^{2} x(t-s)e^{-s}e^{js}ds. \tag{5}$$

Find the expression of the impulse response $h(t)$.

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Question 3: [14%, Work-out question]

Consider a CT-LTI system with the impulse response being $h(t) = e^{(-j+5)t}U(-t)$. Let $y(t)$ denote the output when the input is $x(t) = e^{j4t}$. Find the absolute value of $y(3)$, i.e., find $|y(3)|$, and also find the angle of $y(3)$, i.e., find $\angle y(3)$.

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Question 4: [18%, Work-out question] Consider a periodic CT signals

$$x(t) = \begin{cases} -2+j & \text{if } 5 \le t < 25 \\ 0-2j & \text{if } 25 \le t < 35 \\ 1 & \text{if } 35 \le t < 45 \\ \text{periodic with period } T=40 & \end{cases} \tag{6}$$

We denote the Fourier series of $x(t)$ by (a_k, ω_0) where $\omega_0 = \frac{2\pi}{40}$.

1. [6%] Find the value of a_0.
2. [6%] Find the value of $\sum_{k=-\infty}^{\infty} a_k$.
   Hint: We know that the CTFS synthesis formula is $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$.
3. [6%] Find the value of $\sum_{k=-\infty}^{\infty} |a_k|^2$.

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Question 5: [14%, Work-out question] Consider a DT periodic signal $x[n]$. We know that

$$x[n] = \sin(\frac{7\pi}{3}n) \tag{7}$$

Define a_k as the DTFS coefficients of $x[n]$.

1. [8%] Find the period N and the DTFS coefficients a_k of $x[n]$.

Consider another signal $y[n]$. We do not know the values of $y[n]$ explicitly. But we do know that $y[n]$ has period 6 and its DTFS coefficients are b_0 to b_5 being $b_k = k$. Namely, $b_0 = 0$, $b_1 = 1$, $b_2 = 2$, $b_3 = 3$, $b_4 = 4$, and $b_5 = 5$. Define $z[n] = x[n] \cdot y[n]$.

2. [6%] Find the DTFS coefficient c_2 of $z[n]$. Namely, you do not need to find the value for general c_k. Instead, we only need to find the c_k when $k = 2$.

   Hint 1: You would need the multiplication property of DTFS.

   Hint 2: If you do not know the answer to the previous question, you can assume $a_2 = a_4 = 3$, and $a_0 = a_1 = a_3 = a_5 = 0$. You will receive 5 points if your answer is correct.

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Question 6: [20%, Multiple Choices]

The following questions are multiple-choice questions and there is no need to justify your answers. Consider the following two systems:

System 1: When the input is $x_1(t)$, the output is

$$y_1(t) = \begin{cases} \int_0^{20} e^{-x_1(t-s)} ds & \text{if } x_1(t+1) > 0 \\ \int_0^{20} e^{+x_1(t-s)} ds & \text{if } x_1(t+1) \leq 0 \end{cases} \tag{8}$$

System 2: When the input is $x_2[n]$, the output is

$$y_2[n] = \sum_{k=0}^{\infty} \cos(k-1) x_2[2n-k] \tag{9}$$

Answer the following questions

1. [4%] Is System 1 memoryless? Is System 2 memoryless?
2. [4%] Is System 1 causal? Is System 2 causal?
3. [4%] Is System 1 stable? Is System 2 stable?
4. [4%] Is System 1 linear? Is System 2 linear?
5. [4%] Is System 1 time-invariant? Is System 2 time-invariant?

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This sheet is for Question 6.

Discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \tag{1}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n} \tag{2}$$

Continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \tag{3}$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt \tag{4}$$

Continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \tag{5}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{6}$$

Discrete-time Fourier transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega \tag{7}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \tag{8}$$

Laplace transform

$$x(t) = \frac{1}{2\pi} e^{\sigma t} \int_{-\infty}^{\infty} X(\sigma + j\omega) e^{j\omega t} d\omega \tag{9}$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \tag{10}$$

Z transform

$$x[n] = r^n \mathcal{F}^{-1}(X(re^{j\omega})) \tag{11}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{12}$$

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{matrix} x(t) \\ y(t) \end{matrix}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} x(t) = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau) y(t-\tau) d\tau$	$T a_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T} a_k$
Integration		$\int_{-\infty}^{t} x(t)\, dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right) a_k = \left(\frac{1}{jk(2\pi/T)}\right) a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} \quad [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} \quad [x(t) \text{ real}] \end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

three examples, we illustrate this. The last example in this section then demonstrates how properties of a signal can be used to characterize the signal in great detail.

Example 3.6

Consider the signal $g(t)$ with a fundamental period of 4, shown in Figure 3.10. We could determine the Fourier series representation of $g(t)$ directly from the analysis equation (3.39). Instead, we will use the relationship of $g(t)$ to the symmetric periodic square wave $x(t)$ in Example 3.5. Referring to that example, we see that, with $T = 4$ and $T_1 = 1$,

$$g(t) = x(t-1) - 1/2. \quad (3.6\text{-})$$

Thus, in general, *none* of the finite partial sums in eq. (3.52) yield the exact values of $x(t)$, and convergence issues, such as those considered in Section 3.4, arise as we consider the problem of evaluating the limit as the number of terms approaches infinity.

3.7 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

There are strong similarities between the properties of discrete-time and continuous-time Fourier series. This can be readily seen by comparing the discrete-time Fourier series properties summarized in Table 3.2 with their continuous-time counterparts in Table 3.1.

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{matrix} a_k \\ b_k \end{matrix}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n} x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m} a_k \begin{pmatrix}\text{viewed as periodic} \\ \text{with period } mN\end{pmatrix}$
Periodic Convolution	$\sum_{r=\langle N \rangle} x[r]y[n-r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N \rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k] \begin{pmatrix}\text{finite valued and periodic only} \\ \text{if } a_0 = 0\end{pmatrix}$	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right) a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a^*_{-k} \\ \mathcal{R}e\{a_k\} = \mathcal{R}e\{a_{-k}\} \\ \mathcal{I}m\{a_k\} = -\mathcal{I}m\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}v\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{O}d\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\mathcal{R}e\{a_k\}$ $j\mathcal{I}m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2 = \sum_{k=\langle N \rangle} |a_k|^2$$