Midterm #1 of ECE 301-004, (CRN: 17102)
8–9pm, Wednesday, September 15, 2021, FRNY G140.

1. Enter your student ID number, and signature in the space provided on this page **now!**
2. This is a closed book exam.
3. This exam contains multiple-choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and first working on those that you know how to solve.
4. The instructor/TA will hand out loose sheets of paper for the rough work.
5. **If you need additional sheets to write down your answers, please let the instructor/TA know. We will hand out additional answer sheets then.**
6. Neither calculators nor help sheets are allowed.

Name:

Student ID:

As a Boiler Maker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

Signature: Date:

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Question 1: [15%, Work-out question] Consider a DT signal:

$$x[n] = 4^{-|n|} \tag{1}$$

We construct another signal

$$y(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \tag{2}$$

1. [12%] What is the value of $y(\frac{\pi}{3})$? For this question, you do not need to simplify your answer. Your answer can be something like $\frac{\sin(0.25\pi)}{4} + e^{j\pi/3}$.
2. [3%] What is the value of $y(\frac{7\pi}{3}) - y(\frac{\pi}{3})$?
   For this question, you need to simplify your answer. For example, if you see something like $\cos(\frac{\pi}{3})$, you need to write it as 0.5.
   Hint: You may like to think about what is the period of $e^{j\omega}$ when answering Q1.2.

Hint 1: Even if you do not know the answer to Q1.1, you can still try to answer Q1.2 by writing down your reasoning. If your reasoning is correct, you will receive full credit for Q1.2.

Hint 2: If $|r| < 1$, then we have the following formulas for computing the infinite sum of a geometric sequence.

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$$

$$\sum_{k=1}^{\infty} kar^{k-1} = \frac{a}{(1-r)^2}.$$

If $r \neq 1$, then we have the following formula for computing the finite sum of a geometric sequence.

$$\sum_{k=1}^{K} ar^{k-1} = \frac{a(1-r^K)}{1-r}.$$

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This sheet is for Question 1.

Answer

1. $$y(\omega) = \sum_{n=-\infty}^{\infty} 4^{-|n|} e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{0} 4^{n} e^{-j\omega n} + \sum_{n=1}^{\infty} 4^{-n} e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{0} (4e^{-j\omega})^n + \sum_{n=1}^{\infty} (4^{-1}e^{-j\omega})^n$$

$n' = -n$

$$= \sum_{n'=0}^{+\infty} (4^{-1}e^{+j\omega})^{n'} + \frac{4^{-1}e^{-j\omega}}{1-4^{-1}e^{-j\omega}}$$

$$= \frac{1}{1-4^{-1}e^{j\omega}} + \frac{1}{4e^{j\omega}-1}$$

When $\omega = \frac{\pi}{3}$, $y(\frac{\pi}{3}) = \frac{1}{1-\frac{1}{4}e^{j\frac{\pi}{3}}} + \frac{1}{4e^{j\frac{\pi}{3}}-1}$

2. $\because e^{j\frac{7\pi}{3}} = e^{j\frac{\pi}{3}}$ $\quad y(\frac{7\pi}{3}) = y(\frac{\pi}{3})$

$\therefore y(\frac{7\pi}{3}) - y(\frac{\pi}{3}) = 0$

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Question 2: [16%, Work-out question]
Define two CT signals:

$$x(t) = \begin{cases} e^{-2t}\cos(5t) - j \cdot e^{-2t}\sin(5t) & \text{if } t \geq \pi \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

and

$$h(t) = \begin{cases} e^{t} & \text{if } t \leq -2 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Compute the expression of the following integral

$$y(t) = \int_{s=-\infty}^{\infty} x(s)h(2t-s)ds \tag{5}$$

Answer:

$$h(2t-s) = \begin{cases} e^{2t-s} & 2t-s \leq -2 \Leftrightarrow s \geq 2t+2 \\ 0 & \text{otherwise} \end{cases}$$

1) When $2t+2 \leq \pi \Leftrightarrow t \leq \frac{\pi}{2} - 1$

$$y(t) = \int_{s=\pi}^{\infty} e^{2t-s}\left(e^{-2s}\cos(5s) - je^{-2s}\sin(5s)\right)ds$$

$$= \int_{s=\pi}^{\infty} e^{2t-s} \cdot e^{-2s} e^{-j5s} ds$$

$$= e^{2t}\int_{s=\pi}^{\infty} e^{-3s-j5s} ds$$

$$= e^{2t} \left.\frac{e^{-3s-j5s}}{(-3-j5)}\right|_{s=\pi}^{\infty}$$

$$= e^{2t}\left(0 - \frac{e^{-3\pi-j5\pi}}{-3-j5}\right) = \frac{1}{3+5j} e^{-(3+5j)\pi+2t}$$

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This sheet is for Question 2.

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2) When $2t+2 > \pi \iff t > \frac{\pi}{2} - 1$

$$y(t) = e^{2t} \int_{s=2t+2}^{\infty} e^{-3s-j5s} \, ds$$

$$= e^{2t} \left. \frac{e^{-3s-j5s}}{-3-j5} \right|_{s=2t+2}^{\infty}$$

$$= \frac{e^{2t}}{-3-5j} \left(0 - e^{-(3+5j)(2t+2)} \right)$$

$$= \frac{1}{3+5j} e^{-(3+5j)(2t+2)+2t}$$

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Question 3: [15%, Work-out question] Consider three CT signals

$$x(t) = e^{jw \cdot t} \tag{6}$$

$$y(t) = \begin{cases} \pi & -1 \leq t \leq 4 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Find out the expression of $z(t) = \int_{-\infty}^{\infty} x(s)y(t-s)ds$.

Hint 1: It may be easier to apply some change of variables.

Hint 2: You can leave (part of) your answer to be of the form like $\frac{e^{3jt}-2^{-12t}}{3+10j}$. There is no need to further simplify the expression.

Answer

$$y(t-s) = \begin{cases} \pi & -1 \leq t-s \leq 4 \iff t-4 \leq s \leq t+1 \\ 0 & \text{otherwise} \end{cases}$$

$$Z(t) = \int_{s=t-4}^{t+1} e^{jws} \pi \, ds$$

$$= \pi \left. \frac{e^{jws}}{jw} \right|_{s=t-4}^{t+1}$$

$$= \pi \frac{e^{jw(t+1)} - e^{jw(t-4)}}{jw}$$

$$= \frac{\pi e^{jwt}}{jw} \left(e^{jw} - e^{-4jw} \right)$$

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Question 4: [20%, Work-out question]
Consider the following DT signals.

$$x[n] = \begin{cases} n+2 & \text{if } -1 \le n \le 2 \\ 6-n & \text{if } 3 \le n \le 6 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

1. [12%] Plot the odd part of $x[n]$ for the range of $-8 \le n \le 8$. That is, we use $x_{\text{odd}}[n]$ to represent the odd part of $x[n]$ and your answer will be a plot of $x_{\text{odd}}[n]$ for the range of $-8 \le n \le 8$.

2. [8%] Define

$$h[n] = e^{(1+2j)n} \cdot U[-n+1]. \tag{9}$$
$$y[n] = x[n] \cdot h[n] \tag{10}$$

Find the *total energy* of $y[n]$.

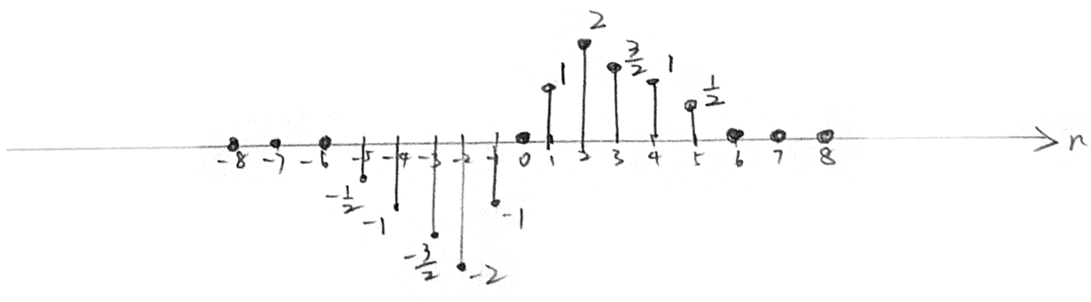
Note that we are looking at the product of $x[n]$ and $h[n]$. It has no relationship to the previous sub-question, which is about $x_{\text{odd}}[n]$ instead.

Answer :

1.

$$x[-n] = \begin{cases} -n+2 & \text{if } -2 \le n \le 1 \\ 6+n & \text{if } -6 \le n \le -3 \\ 0 & \text{otherwise} \end{cases}$$

$$x_{odd}[n] = \frac{x[n] - x[-n]}{2} = \begin{cases} -3 - \frac{n}{2} & \text{if } -6 \le n \le -3 \\ -2 & \text{if } n = -2 \\ n & \text{if } -1 \le n \le 1 \\ 2 & \text{if } n = 2 \\ 3 - \frac{n}{2} & \text{if } 3 \le n \le 6 \\ 0 & \text{otherwise} \end{cases}$$



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This sheet is for Question 4.

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2. $$y[n] = \begin{cases} x[n] \cdot e^{(1+2j)n} & n \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$E_\infty = \sum_{n=-\infty}^{+\infty} |y[n]|^2 = \sum_{n=-\infty}^{1} |y[n]|^2$$

$$= \sum_{n=-1}^{1} |x[n] \cdot e^{(1+2j)n}|^2$$

$$= \sum_{n=-1}^{1} |x[n] e^{n}|^2$$

$$= |1 \cdot e^{-1}|^2 + |2e^{0}|^2 + |3e^{1}|^2$$

$$= e^{-2} + 4 + 9e^{2}$$

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Question 5: [14%, Work-out question]

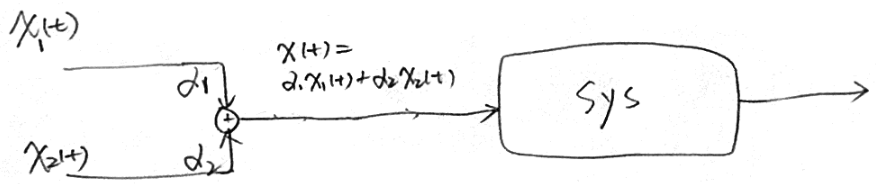
Consider the following CT system.

$$y(t) = \int_{s=-t^2}^{t^2+1} x(s^3)ds \tag{11}$$

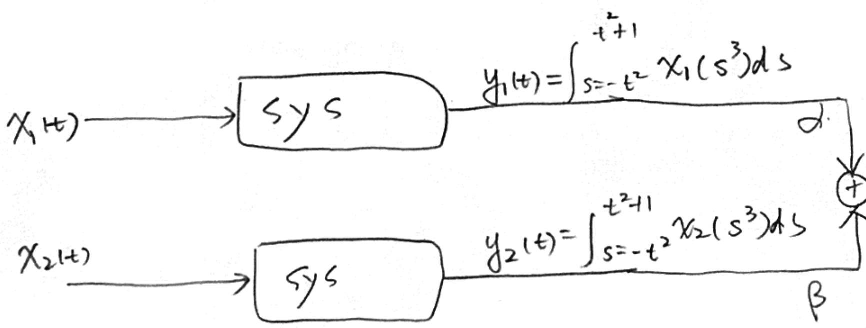
Is such a system *linear*? You need to carefully write down your reasonings. An answer without justification will not receive any point.

Answer:

Compare the output of the two configurations



$$y(t) = \int_{s=-t^2}^{t^2+1} (\alpha x_1(s^3) + \beta x_2(s^3)) ds$$



$$\bar{y}(t) = \alpha y_1(t) + \beta y_2(t)$$

$$= \int_{s=-t^2}^{t^2+1} (\alpha x_1(s^3) + \beta x_2(s^3)) ds$$

$\because$ $y(t) = \bar{y}(t)$

The system is linear.

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Question 6: [20%, Multiple Choices]

The following questions are multiple-choice questions and there is no need to justify your answers. Consider two continuous-time signals:

$$x_1(t) = \cos(\sqrt{2}\cdot t)\sin(\sqrt{3}\cdot t)\sin(\sqrt{6}\cdot t) \quad (12)$$

$$x_2(t) = \sum_{k=2}^{10}\sin(3k|t| + \frac{\pi}{2}) \quad (13)$$

and two discrete-time signals:

$$x_3[n] = e^{jn}\sin(2n-1) - e^{-jn}\sin(2n+1) \quad (14)$$

$$x_4[n] = (U[n+41] - U[n-41])\cdot\cos(0.25\pi n). \quad (15)$$

1. [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not, *respectively.* If it is periodic, write down the fundamental period. Please state explicitly which signal is periodic and which is not.

2. [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them, respectively. Please state explicitly which signal is even, which is odd, and which is neither.

Answer:

1. $x_1(t)$ is aperiodic

$x_{11}(t) = \cos(\sqrt{2}t)$ periodic $T_{11} = \frac{2\pi}{\sqrt{2}}$

$x_{12}(t) = \sin(\sqrt{3}t)$ periodic $T_{12} = \frac{2\pi}{\sqrt{3}}$

$x_{13}(t) = \sin(\sqrt{6}t)$ periodic $T_{13} = \frac{2\pi}{\sqrt{6}}$

LCM(T_{11}, T_{12}, T_{13}) does not exist because $\sqrt{2}, \sqrt{3}, \sqrt{6}$. have no LCM.

$x_2(t)$ is periodic with $T_2 = \frac{2\pi}{3}$

$x_{2k} = \sin(3k|t| + \frac{\pi}{2}) = \cos(3k|t|) = \cos(3kt)$

$T_{2k} = \frac{2\pi}{3k}$, $k = 2, \dots, 10$

$\text{LCM}(T_{2k}) = \frac{2\pi}{3}$

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This sheet is for Question 6.

$x_3[n]$ is aperiodic.

e^{jn}, e^{-jn} are not periodic, either do $\sin(2n-1)$, $\sin(2n+1)$

$x_4[n]$ is aperiodic

$\cos(\frac{\pi}{4}n)$ is periodic with $N=8$, but it is supressed to zero for $|n|>41$.

2. $x_1(t)$ is even.

$x_2(t)$ is even.

$x_3[n]$ is even.

$x_4[n]$ is neither

(Check by plugging in definition of even signals: $x(t) = x(-t)$ and odd signals: $x(t) = -x(-t)$)