Final Exam of ECE 301-004, (CRN: 17102)
7–9pm, Thursday, December 16, 2021, PHYS 114.

1. **<u>Do not write answers on the back of pages!</u>**
2. **After the exam ended, you will have 5 additional minutes to write down your name and Purdue ID on each of the pages.**
3. **If you need additional sheets to write down your answers, please let the instructor/TA know. We will hand out additional answer sheets then.**
4. Enter your student ID number, and signature in the space provided on this page.
5. This is a closed book exam.
6. This exam contains multiple-choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have two hours to complete it. The students are suggested not spending too much time on a single question, and first working on those that you know how to solve.
7. The instructor/TA will hand out loose sheets of paper for the rough work.
8. Neither calculators nor help sheets are allowed.

Name:

Student ID:

As a Boiler Maker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

Signature: Date:

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Question 1: [20%, Work-out question]

1. [1%] What does the acronym AM-SSB stand for? amplitude - modulation single- sideband

Prof. Wang wanted to transmit an AM-SSB signal. To that end, he wrote the following MATLAB code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read two different .wav files
[x1, f_sample, N]=audioread('x1.wav');
x1=x1';
[x2, f_sample, N]=audioread('x2.wav');
x2=x2';

% Step 0: Initialize several parameters
W_1=????;
W_2=4500*pi;
W_3=????;
W_4=????;
W_5=????;
W_6=2000*pi;
W_7=7000*pi;


% Step 1: Make the signals band-limited.
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);

% Step 2: Multiply x1_new and x2_new with a sinusoidal wave.
x1_h=x1_new.*cos(W_2*t);
x2_h=x2_new.*sin(W_3*t);

% Step 3: Keep one of the two side bands
h_one=1/(pi*t).*(sin(W_4*t)-sin(W_5*t));
h_two=1/(pi*t).*(sin(W_6*t).*(2*cos(W_7*t)));
x1_sb=ece301conv(x1_h, h_one);
```

0 W_2 W_3 Hz

2.25K

```
x2_sb=ece301conv(x2_h, h_two);

% Step 4: Create the transmitted signal
y=x1_sb+x2_sb;
audiowrite('y.wav', y, f_sample);
```

2. [1%] What is the carrier frequency (Hz) of the signal x1_new? 2.25K Hz

3. [2%] Our goal is to transmit the "lower-side bands" of the x2 signal. What should be the value of W_3 in the MATLAB code? 9000π,

4. [2%] At the same time, I would also like to "maximize the quality" of the x2 signal. That is, I would like to choose the largest possible W_1 value so that I can keep as much initial frequency as possible. What is the largest W_1 value that is consistent with the choices of W_6 and W_7 values? 4000π

5. [4%] Our goal is to also transmit the "lower-side bands" for the x1 signal. What should be the values of W_4 and W_5 in the MATLAB code, assuming the same W_1 used in the previous sub-question? $W_4 = 4500\pi$, $W_5 = 500\pi$

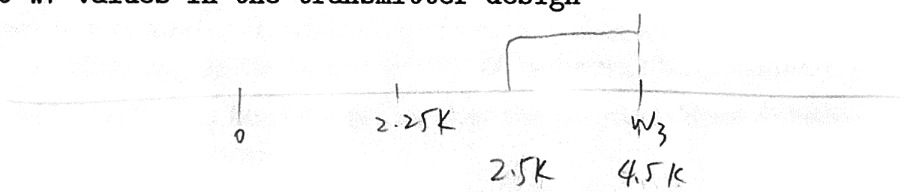
6. [1.5%] Suppose I would like to "pack the two radio stations as closely together as possible", but I must not change the W_6 and W_7 given in the above code and also must not change the W_1 value given in Q1.4. What is the largest W_2 value I can choose without worry about any issue of frequency overlap?

5000π

Prof. Wang decided to use the lower-side-band transmission with W_2 set to 4500π and used the code in the previous page to generate the "y.wav" file. (That is, we do not pack the two radio stations as closely as possible and simply ignore Q1.7.)

A student tried to demodulate the output waveform "y.wav" by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[y, f_sample, N]=audioread('y.wav');
y=y';

% Initialize several parameters


W_8=????;
W_9=????;
W_10=????;



% We reuse the W4 to W7 values in the transmitter design

W_4=????;
W_5=????;
W_6=2000*pi;
W_7=7000*pi;
```




```
% Create the low-pass filter.
h_M=1/(pi*t).*(sin(W_8*t));

% We reuse
h_one=1/(pi*t).*(sin(W_4*t)-sin(W_5*t));
h_two=1/(pi*t).*(sin(W_6*t).*(2*cos(W_7*t)));

% demodulate signal 1
y11=ece301conv(y, h_one);
y1=y11.*sin(W_9*t);
x1_hat=4*ece301conv(y1,h_M);
```

```
sound(x1_hat,f_sample)

% demodulate signal 2
y21=ece301conv(y, h_two);
y2=y21.*sin(W_10*t);
x2_hat=4*ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```

7. [4.5%] Continue from the previous questions. What should the values of W_8 to W_10 be in the MATLAB code? 4000π, 4500π, 9000π

8. [4%] It turns out that the above MATLAB code is not written correctly and the end results do not sound right. Answer the following questions

   (a) Is signal x1_new correctly/perfectly demodulated? If yes, then go to subquestion (d). If no, then continue answering the following sub-questions. No

   (b) Use 2 to 3 sentences to answer (i) what kind of problem does x1_new have, i.e., how does the problem impact the sound quality of "sound(x1_hat,f_sample)"?

   (c) How can the MATLAB code be corrected so that the playback/demodulation can be successful? y1 = y11. * cos (W_9 * t) ;

   (d) Is signal x2_new correctly/perfectly demodulated? If yes, then your answer to Q1.8 is complete. If no, then continue answering the following sub-questions. Yes

   (e) Use 2 to 3 sentences to answer (i) what kind of problem does x2_new have, i.e., how does the problem impact the sound quality of "sound(x2_hat,f_sample)"?

   (f) How can the MATLAB code be corrected so that the playback/demodulation can be successful?

Hint: If you do not know the answers of Q1.2 to Q1.8, please simply draw the AMSSB modulation (using lower side band) and demodulation diagrams and mark carefully all the parameter values. You will receive 10 points for Q1.2 to Q1.8 if your system diagrams are correct and all parameter values are marked correctly.

b) x1-hat is distorted compare to x1-new. Because the positive frequency component is multiplied by (-j) and the negative frequency component is multiplied by j (If AM-DSB is applied, x1-hat is silent because of the mismatch of cosine and sine.)

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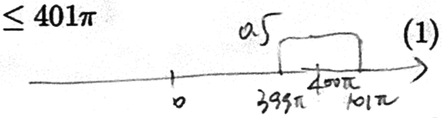
Question 2: [8%, Work-out question]

Consider a continuous time AM signal $y(t) = x(t)\cos(400\pi t)$. Namely the carrier frequency is $\omega_0 = \cancel{4\pi}$. We also know that the frequency domain of the AM signal $y(t)$ is

400π

$$Y(j\omega) = \begin{cases} 0.5 & \text{if } 399\pi \leq |\omega| \leq 401\pi \\ 0 & \text{otherwise} \end{cases} \tag{1}$$



1. [2.5%] Suppose we use the *synchronous demodulation* to reconstruct the original signal $x(t)$ from $y(t)$ and we denote the output by $x_{\text{syn}}(t)$. Plot $x_{\text{syn}}(t)$ for the range of $-4 \leq t \leq 4$.

2. [5.5%] Suppose we use the *asynchronous demodulation* to reconstruct the original signal $x(t)$ from $y(t)$ and we denote the output by $x_{\text{asynch}}(t)$. Plot $x_{\text{asynch}}(t)$ for the range of $-4 \leq t \leq 4$.

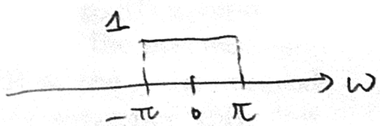
   Hint 1: You may want to draw $y(t)$ first, which may help your answer this sub-question.

   Hint 2: If you do not know the answer to the previous sub-question, please answer the following instead (i) Plot $\frac{\sin(2t)}{4t}$ for the range of $-5\pi \leq t \leq 5\pi$. Please carefully marked all the critical points; and (ii) What is the main difference between *synchronous* demodulation and *asynchronous* demodulation. You will receive 3 points if both your answers of (i) and (ii) are correct.

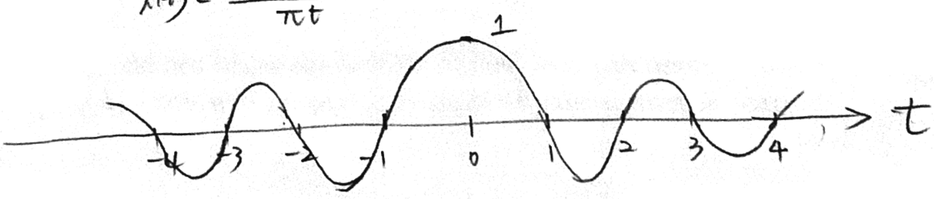
Answer:

1. The spectrum of x(t):

$X(j\omega)$



$$x(t) = \frac{\sin(\pi t)}{\pi t}$$



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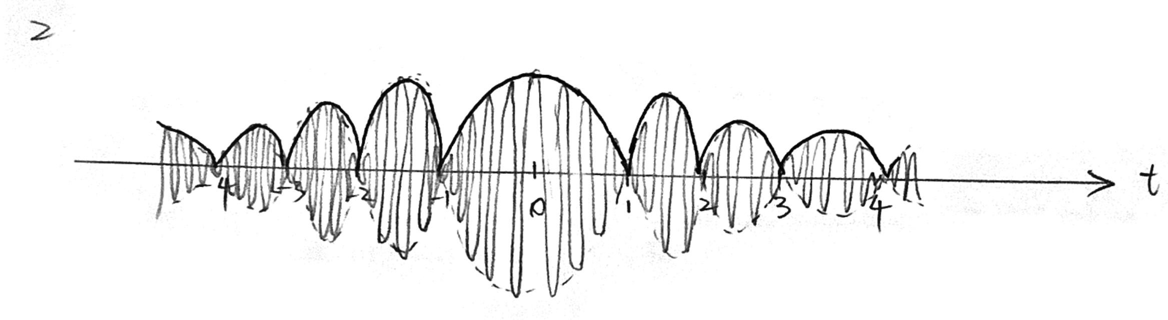
This sheet is for Question 2.



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Question 3: [14%, Work-out question]

1. [4%] Consider a continuous time signal

   0   0.1 $\cos(\frac{\pi}{2})$ 0.3 $\cos(\frac{3}{2}\pi)$

$$x(t) = \begin{cases} \cos(5\pi t) & \text{if } 0.1 \le |t| \le 0.3 \\ 0 & \text{otherwise} \end{cases} \qquad T = 0.1s \qquad (2)$$

   We sample $x(t)$ with the sampling frequency 10Hz and denote the sampled values by $x[n]$. Plot $x[n]$ for the range of $-5 \le n \le 5$.

2. [5%] Find the expression of the DTFT $X(e^{j\omega})$ of $x[n]$. Plot $X(e^{j\omega})$ for the range of $-2\pi < \omega < 2\pi$.

   Hint: If you do not know the answer of $x[n]$, you can assume that $x[n] = \sum_{k=2}^{3} \delta[n-k] + \delta[n+k]$. You will receive full points if your answer is correct.

3. [3%] We use $x_{\text{ZOH}}(t)$ to represent the reconstructed signal using "zero-order hold". Plot $x_{\text{ZOH}}(t)$ for the range of $-1 \le t \le 1$.

   Hint: If you do not know the answer of $x[n]$, you can assume that $x[n] = \sum_{k=2}^{3} \delta[n-k] + \delta[n+k]$. You will receive full points if your answer is correct.

4. [2%] A student tried to duplicate the results of the previous sub-question using the same sampled array $x[n]$. However, when he/she tried to reconstruct the signal, he/she mistakenly set the sampling frequency to be 5Hz. We use $\hat{x}_{\text{ZOH}}(t)$ to represent the latest reconstructed signal using by him/her. That is, both the previous sub-question $x_{\text{ZOH}}(t)$ and this sub-question $\hat{x}_{\text{ZOH}}(t)$ apply ZOH to the same sampled array $x[n]$. However, the former uses the correct sampling frequency 10Hz but the latter uses a wrong sampling frequency 5Hz. Plot $\hat{x}_{\text{ZOH}}(t)$ for the range of $-1 \le t \le 1$.

   Hint: If you do not know the answer of $x[n]$, you can assume that $x[n] = \sum_{k=2}^{3} \delta[n-k] + \delta[n+k]$. You will receive full points if your answer is correct.

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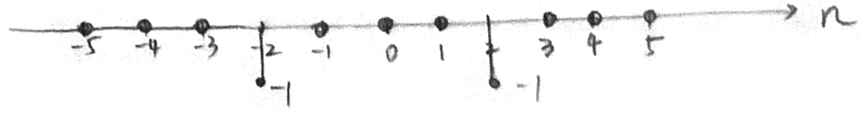
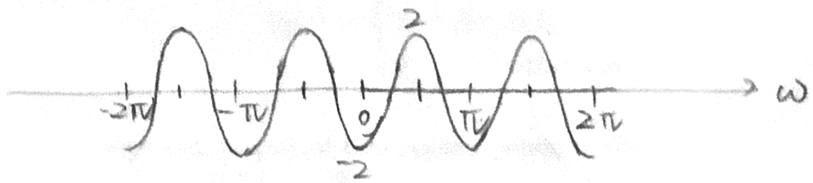
This sheet is for Question 3.

Answer:

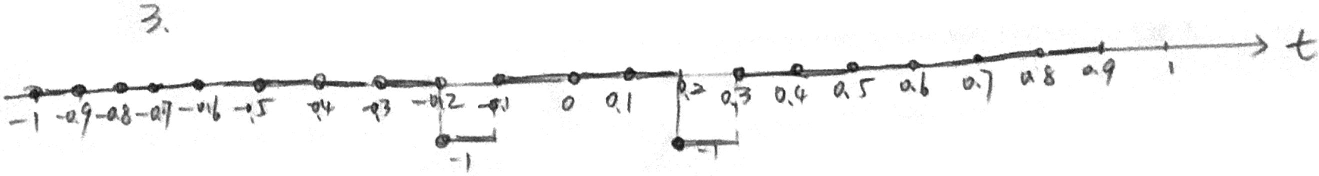
1. $x[n]$



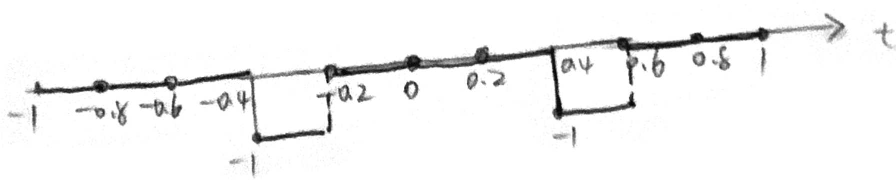
2. $X(e^{jw}) = -e^{j2w} - e^{-j2w} = -2\cos(2w)$



3.



4.



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Question 4: [16%, Work-out question]

1. [1%] Consider a continuous time signal $x(t) = 2 + \cos(\frac{5\pi t}{3})$. Plot $x(t)$ for the range of $-1.2 < t < 1.2$.

2. [5%] We sample $x(t)$ via *impulse train sampling* with sampling period 0.3. Denote the final *impulse-train-sampled* signal by $x_p(t)$. Plot $x_p(t)$ for the range of $-1.2 < t < 1.2$;

3. [5%] Consider an LTI system S1 with impulse response

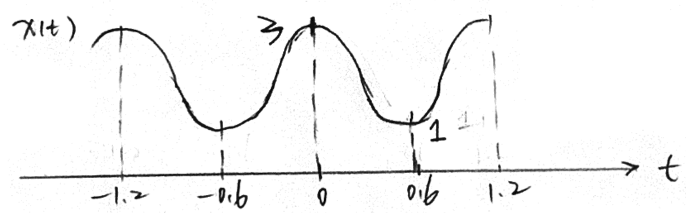
$$h_1(t) = \begin{cases} \frac{t}{0.3} + 1 & \text{if } -0.3 < t \leq 0 \\ 1 - \frac{t}{0.3} & \text{if } 0 < t \leq 0.3 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

   Let $y_1(t)$ denote the output of this system when using $x_p(t)$ as the input. Plot $y_1(t)$ for the range of $-1.2 < t < 1.2$.

4. [5%] Plot the Fourier transform $X_p(j\omega)$ for the range of $-\frac{20\pi}{3} < \omega < \frac{20\pi}{3}$.
   Hint: You can still answer Q4.4 even if you do not know the answer to Q4.3.

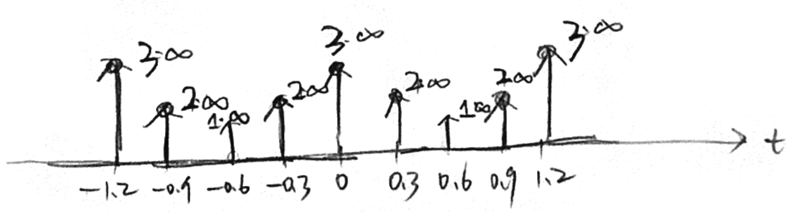
Answer:

1.



$T = \frac{2\pi}{w_0} = \frac{6}{5} = 1.2\,s$

2. $T_s = 0.3\,s$



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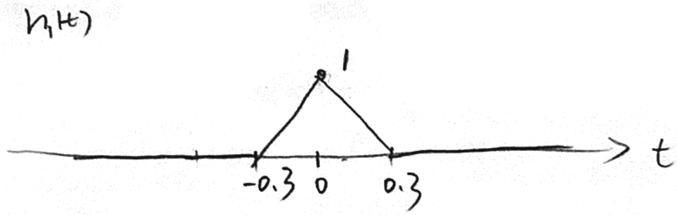
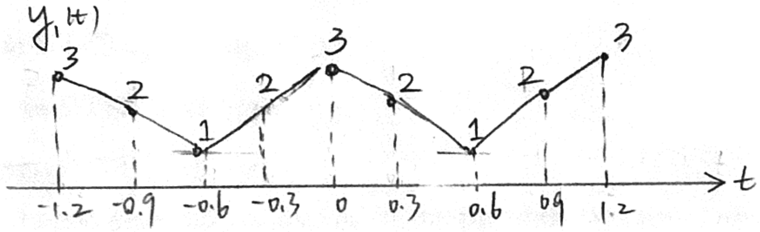
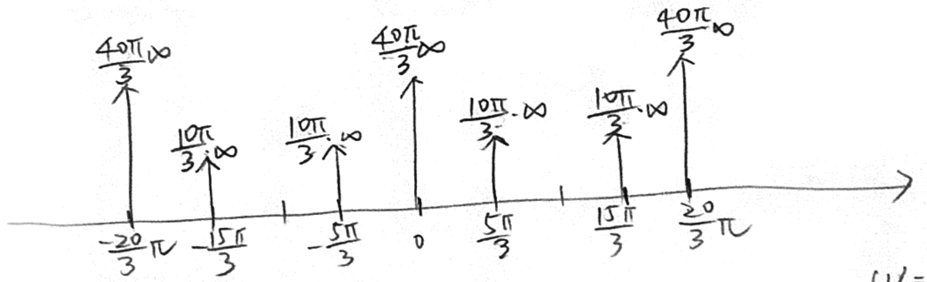
This sheet is for Question 4.

3.





4.



$\omega_s = \frac{2\pi}{T_s} = \frac{20}{3}\pi$

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Question 5: [10%, Work-out question]

Consider the following discrete time signals

$$x[n] = \cos(\frac{13\pi n}{5}) \tag{4}$$

$$y[n] = e^{\frac{3\pi n}{5}} \tag{5}$$

Define $z[n] = x[n] \cdot y[n]$.

1. [4%] Find the DTFS a_k of $x[n]$.
2. [1.5%] Find the DTFS b_k of $y[n]$.
3. [4.5%] Find the DTFS c_k of $z[n]$.

   Hint: If you do not know the a_k and b_k of the previous two sub-questions, you can express c_k as an expression of a_k and b_k. You will receive 2 points if your answer is correct.

Answer: 1. $x[n] = \frac{1}{2}e^{j\frac{13\pi}{5}n} + \frac{1}{2}e^{-j\frac{13\pi}{5}n}$

$N = \min\left(\frac{2\pi}{13\pi/5}m\right), m \in \mathbb{Z}$

$= 10.$

$\omega_0 = \frac{2\pi}{N} = \frac{\pi}{5}$

$a_{13} = a_3 = \frac{1}{2}$

$a_{-13} = a_7 = \frac{1}{2}$

$a_k = 0$ for $k = 0, 1, 2, 4, 5, 6, 8, 9$

a_k is periodic with $k = 10$.

2. $N = \min\left(\frac{2\pi}{3\pi/5}\right), m \in \mathbb{Z}$

$= 10$ $\quad \omega_0 = \frac{2\pi}{N} = \frac{\pi}{5}$

$b_3 = 1$, $b_k = 0$ for $k = 0, 1, 2, 4 \sim 9$

b_k is periodic with $k = 10$.

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This sheet is for Question 5.

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3. By the frequency shifting property

$$c_k = a_{k-3}$$

$c_0 = c_6 = \frac{1}{2}$, $c_k = 0$ for $k = 1 \sim 3, 5 \sim 9$.

c_k is periodic with $k = 10$.

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Question 6: [8%, Work-out question]

Consider the following continuous time input and continuous time output system.

$$y(t) = \int_{t-2}^{t+1} (s - t - 1)x(s - 3)ds \tag{6}$$

1. ~~[8%]~~ Find the impulse response $h(t)$ of this system and plot it for the range of $-6 < t < 6$.

Answer: let $s' = s-3$ $\quad \frac{ds'}{ds} = 1$

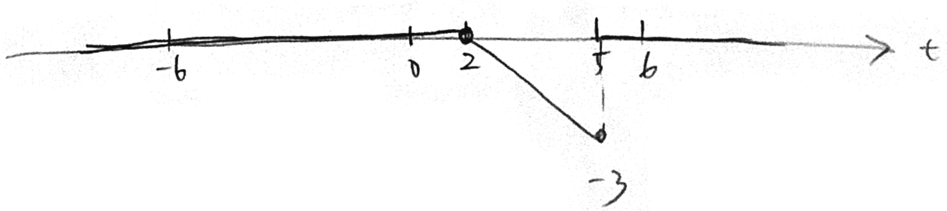
$$y(t) = \int_{s'+3=t-2}^{t+1} (s'-t+2)\, x(s')\, ds'$$

$$= \int_{-\infty}^{+\infty} (s'-t+2)\, x(s') \Big(u(s'-(t-5)) - u(s'-(t-2)) \Big)\, ds'$$

$$h(t-s') = (s'-t+2)\big(u(s'-t+5) - u(s'-t+2) \big)$$

$$\Downarrow$$

$$h(t) = (-t+2)\big(u(-t+5) - u(-t+2) \big)$$



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Question 7: [9%, Work-out question]

Consider a CT signal $x(t)$ with the corresponding Fourier transform being

$$X(j\omega) = \frac{4}{-3 + j\omega} \tag{7}$$

The following sub-questions aim to find the $x(t)$.

1. [3%] Consider a new signal $y(t) = x(-t)$. Find the CTFT $Y(j\omega)$;
2. [2%] Find the expression of $y(t)$.
3. [3%] Find the expression of $x(t)$ and plot it for the range of $-3 < t < 3$.
4. [1%] We know from Table 4.2 Basic Fourier Transform Pairs:

   Time domain: $e^{-at}u(t), \mathrm{Re}(a) > 0$; Frequency domain: $\frac{1}{a+j\omega}$.

   If we apply this formula blindly, we will have $x(t) = 4e^{-(-3)t}u(t)$ by comparing Equation (7) with the above Fourier transform pair.

   Answer this question: Why this direct application of the table <u>does not</u> give the right answer of $x(t)$. Please write a few sentence about why blindly applying the formula does not give us the right answer. Specifically, your answer should be about why we require $\mathrm{Re}(a) > 0$ in the formula table above.

   Hint: you should think about how we find the Fourier transform $e^{-at}u(t)$.

Answer: 1. By the time-reverse property

$$Y(j\omega) = X(-j\omega) = \frac{4}{-3-j\omega}$$

2. $y(t) = -4e^{-3t}u(t)$

according to the transform pair

$e^{-at}u(t),\ \mathrm{Re}\{a\}>0 \leftrightarrow \frac{1}{a+j\omega}$

3. $x(t) = y(-t) = -4e^{3t}u(-t)$

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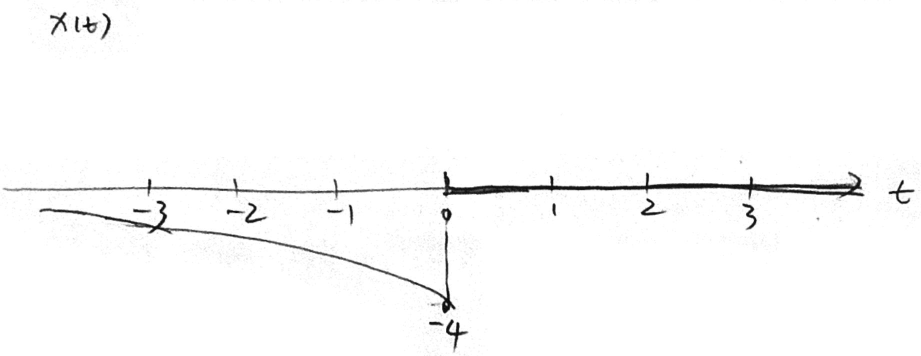
This sheet is for Question 7.



4. Because

$$X(j\omega) = \int_{-\infty}^{+\infty} e^{-at} u(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{+\infty} e^{-(a+j\omega)t}\, dt$$

If $a<0$, $\left|e^{-(a+j\omega)t}\right| \to \infty$ when $t \to \infty$,

It is not integrable.

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Question 8: [15%, Multiple-choice question] Consider two signals

$$h_1(t) = \int_0^{3+\sin(t)} s^2 \cos s ds \tag{8}$$

and

$-k+n+3$

$$h_2[n] = \sum_{k=-\infty}^{\infty} U[(k-3)^3] \cdot e^{-(k-3)\cos^2(\pi(k-3))} \cdot \delta[n-(k-3)] \tag{9}$$

$k \geq 3$ $\quad$ 1

$h_2[n] = \begin{cases} e^{-n} & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$

1. [1.25%] Is $h_1(t)$ periodic? Yes
2. [1.25%] Is $h_2[n]$ periodic? No
3. [1.25%] Is $h_1(t)$ even or odd or neither? Neither
4. [1.25%] Is $h_2[n]$ even or odd or neither? Neither
5. [1.25%] Is $h_1(t)$ of finite power? Yes
6. [1.25%] Is $h_2[n]$ of finite power? Yes

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless? No
2. [1.25%] Is System 2 memoryless? No
3. [1.25%] Is System 1 causal? No
4. [1.25%] Is System 2 causal? Yes
5. [1.25%] Is System 1 stable? No
6. [1.25%] Is System 2 stable? Yes