Q 2.

Consider a DT signal: $x[n] = \sum_{k=5}^{\infty} e^{-(2+2j)k} \cdot \delta[n-k]$

2.1 $x[0] = \sum_{k=5}^{\infty} e^{-(2+2j)k} \delta[0-k] = \sum_{k=5}^{\infty} e^{-(2+2j)k} \cdot 0 = 0$

2.2 $x[10] = \sum_{k=5}^{\infty} e^{-(2+2j)k} \delta[10-k] = e^{-(2+2j)\cdot 10} \delta[10-10] = e^{-(2+2j)10} = e^{-20-20j}$

2.3 $x[n] = \sum_{k=5}^{\infty} e^{-(2+2j)k} \delta[n-k] = \begin{cases} e^{-(2+2j)n} & n \geq 5 \\ 0 & \text{otherwise} \end{cases}$

Total Energy $E_{x_1} = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=5}^{\infty} |e^{-(2+2j)n}|^2$

$= \sum_{n=5}^{\infty} |e^{-2n} \cdot e^{-j2n}|^2 = \sum_{n=5}^{\infty} |e^{-2n}|^2 |e^{-j2n}|^2$

$= \sum_{n=5}^{\infty} e^{-4n} = \frac{e^{-20}}{1-e^{-4}} = \frac{e^{-16}}{e^{4}-1}$

Back-up: $x_2[n] = (\sin(0.7n) - j\cos(0.7n))\, u[n-3]$

Average power $P_{x_2} = \frac{1}{41} \sum_{n=-20}^{20} |x_2[n]|^2 = \frac{1}{41} \sum_{n=-20}^{20} |(\sin(0.7n) - j\cos(0.7n))\, u[n-3]|^2$

$= \frac{1}{41} \sum_{n=3}^{20} |\sin(0.7n) - j\cos(0.7n)|^2 = \frac{1}{41} \sum_{n=3}^{20} 1$

$= \frac{18}{41}$

Q3) $y(t) = \int_{-\infty}^{\infty} x(s)\, h(2t-s)\, ds$

$s < 2t$

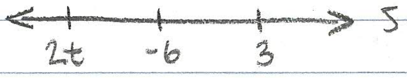
$x(s) = \begin{cases} 3^s & \text{if } -6 < s < 3 \\ 0 & \text{otherwise} \end{cases}$ $\quad h(2t-s) = \begin{cases} 3^{2t-s} & \text{if } 0 < 2t-s \\ 0 & \text{otherwise} \end{cases}$

We want to redefine the limits of the integral in $y(t)$ based on when the expression $x(s)h(2t-s)$ is nonzero. This expression is nonzero when both $x(s)$ and $h(2t-s)$ are nonzero. In order for this to be true, we need to meet 2 conditions:

Condition ①: $-6 < s < 3$
Condition ②: $s < 2t$

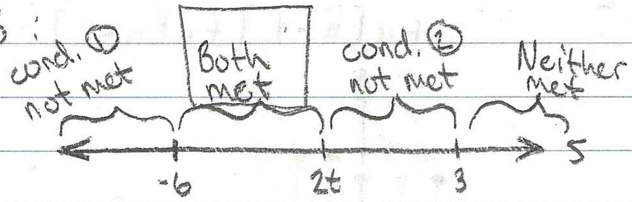
In $y(t)$, we integrate over $s$, so we want to consider for what ranges of $s$ these 2 conditions hold true. We consider 3 cases based on where $2t$ might fall relative to $-6$ and $3$:

Case ① $2t \le -6$:



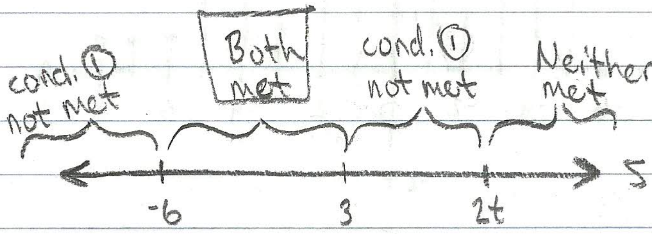

There is no range of $s$ for which the two conditions are true.

Case ② $-6 < 2t < 3$:




Both conditions are met if $-6 < s < 2t$

Case ③ $2t \ge 3$:




Both conditions are met if $-6 < s < 3$

We now break up $y(t)$ based on the ranges of $t$ from our 3 cases and limit the range of integration based on when both conditions are true.

For $2t \leq -6$ $(\Rightarrow t \leq -3)$ : $y(t) = \int_{-\infty}^{\infty} 0 \, ds = 0$

For $-6 < 2t < 3$ $(\Rightarrow -3 < t < 3/2)$ : $y(t) = \int_{-6}^{2t} x(s) h(2t-s) \, ds$

$$= \int_{-6}^{2t} 3^s \cdot 3^{2t-s} \, ds$$

$$= \int_{-6}^{2t} 3^{s+2t-s} \, ds = \int_{-6}^{2t} 3^{2t} \, ds$$

$$= 3^{2t} \cdot s \Big|_{-6}^{2t} = (2t+6) \, 3^{2t}$$

For $2t \geq 3$ $(\Rightarrow t \geq 3/2)$ : $y(t) = \int_{-6}^{3} x(s) h(2t-s) \, ds = \int_{-6}^{3} 3^{2t} \, ds$

$$= 9 \cdot 3^{2t} = 3^2 \cdot 3^{2t} = 3^{2t+2}$$

$$\boxed{y(t) = \begin{cases} 0 & \text{if } t \leq -3 \\ (2t+6) \, 3^{2t} & \text{if } -3 < t < 3/2 \\ 3^{2t+2} & \text{if } t \geq 3/2 \end{cases}}$$

Q4. Consider a DT signal $x[n] = \begin{cases} 5^n & n \le 10 \\ 0 & \text{otherwise} \end{cases}$

$$f(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{10} 5^n z^{-n} = \sum_{n=-\infty}^{10} \left(\frac{5}{z}\right)^n \quad \text{Let } n=-m$$

$$= \sum_{-m=-\infty}^{-m=10} \left(\frac{5}{z}\right)^{-m} = \sum_{m=-10}^{\infty} \left(\frac{5}{z}\right)^{-m} = \sum_{m=-10}^{\infty} \left(\frac{z}{5}\right)^m$$

$$f(2+2j) = \sum_{m=-10}^{\infty} \left(\frac{2+2j}{5}\right)^m \quad \text{where } |r| = \left|\frac{2+2j}{5}\right| = |0.4+0.4j| = 0.4\sqrt{2} < 1$$

$$= \frac{\left(\frac{2+2j}{5}\right)^{-10}}{1-\frac{2+2j}{5}} = \frac{5\left(\frac{2\sqrt{2}}{5}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}j\right)\right)^{-10}}{3-2j} = \frac{\frac{5^{11}}{2^{15}}\left(e^{j\frac{\pi}{4}}\right)^{-10}}{3-2j} = \frac{\frac{5^{11}}{2^{15}} e^{-j\frac{5\pi}{2}}}{3-2j}$$

$$= \frac{5^{11}}{2^{15}} \frac{e^{-j\frac{\pi}{2}}}{3-2j} = \frac{5^{11}}{2^{15}} \frac{-j}{3-2j}$$

Q5 Consider the following DT signals

$$x[n] = e^{j(\sqrt{\pi}/5)n} \qquad h[n] = U[n-30] - U[n-100]$$

$$h[n] = U[n-30] - U[n-100] = \begin{cases} 1 & 30 \le n \le 99 \\ 0 & \text{otherwise} \end{cases}$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \qquad \text{where } h[n-k] = 1 \text{ when } 30 \le n-k \le 99$$

$$= \sum_{k=n-99}^{n-30} x[k] \qquad \text{or } n-99 \le k \le n-30$$

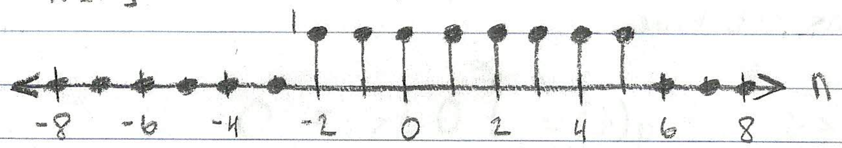
$$h[n-k] = 0 \text{ otherwise}$$

$$= \sum_{k=n-99}^{n-30} e^{j\frac{\sqrt{\pi}}{5}k}$$

$$= \frac{e^{j\frac{\sqrt{\pi}}{5}(n-99)} \cdot \left(1 - e^{j\frac{\sqrt{\pi}}{5}\cdot 70}\right)}{1 - e^{j\sqrt{\pi}/5}} = \frac{e^{j\frac{\sqrt{\pi}}{5}(n-99)} \cdot \left(1 - e^{j14\sqrt{\pi}}\right)}{1 - e^{j\sqrt{\pi}/5}}$$

Q6.1) $x[n] = u[n+2] - u[n-6]$

$x[n]$:




Q6.2) Because the system is linear, we know that if input $x_1[n]$ produces output $y_1[n]$, and if input $x_2[n]$ produces output $y_2[n]$, then the input $(x_1[n] + x_2[n])$ will produce output $(y_1[n] + y_2[n])$.

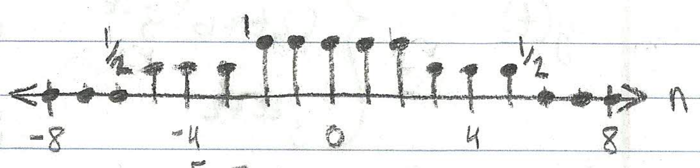
We also know how the system responds to even and odd functions, so we want to rewrite $x[n]$ as a sum of an even and an odd function.

Even part of $x[n]$ is given by:

$$x_E[n] = \tfrac{1}{2}(x[n] + x[-n])$$

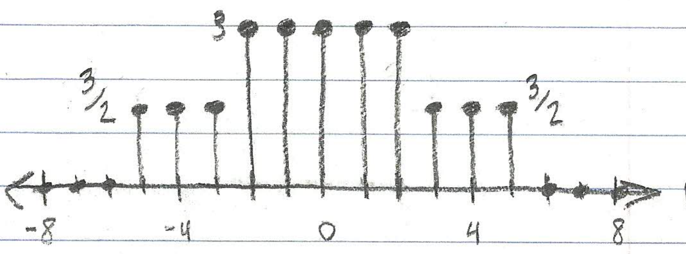
$$= \tfrac{1}{2}(u[n+2] + u[n-6] + u[-n+2] + u[-n-6])$$

Sketch of $x_E[n]$:




Since $x_E[n]$ is even, response $y_E[n]$ to input $x_E[n]$ is $3x_E[n]$

$y_E[n]$:



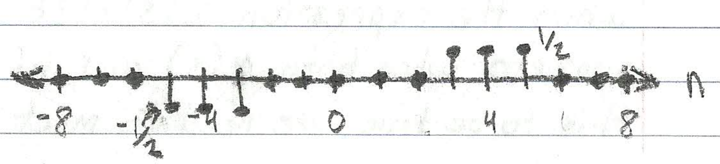
Odd part of $x[n]$ is given by:

$$x_o[n] = \tfrac{1}{2}(x[n] - x[-n])$$

$$= \tfrac{1}{2}(u[n+2] + u[n-6] - u[-n+2] - u[-n-6])$$

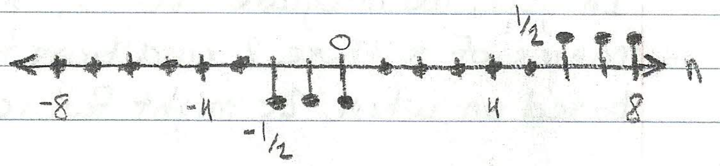
Sketch of $x_o[n]$:

* (Answer to Hint) *




Since $x_o[n]$ is odd, response $y_o[n]$ to input $x_o[n]$ is $x_o[n-3]$

$y_o[n]$:

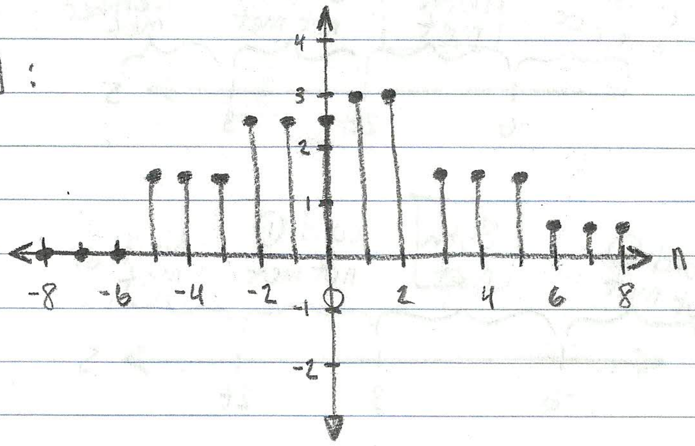



Because system is linear:

$$y[n] = y_E[n] + y_o[n]$$

$$= \tfrac{1}{2}\Big[3(u[n+2] + u[n-6] + u[-n+2] + u[-n-6]) + u[n-1] + u[n-9] - u[-n+5] - u[-n-3]\Big]$$

$y[n]$:



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Q7) $x_1(t) = \sum_{n=-\infty}^{\infty} 2^{-|n|} e^{jtn} = \sum_{n=-\infty}^{\infty} 2^{-|n|}(\cos(tn) + j\sin(tn))$

Periodic?

Consider $2^{-|n|}(\cos(tn) + j\sin(tn))$, $n$ is an integer

$$\cos((t+N)n) = \cos(tn + Nn)$$

Check: $Nn = 2\pi K \Rightarrow N = \frac{2\pi K}{n}$

$\Rightarrow N = 2\pi/n$ for any $n$ (except $n=0$, but in that case $2^{-|0|}e^0 = 1$)

So for all $n$, the LCM of $N$ is $2\pi$ (because we can always multiply $N$ by $n$ and get $2\pi$)

Same result $N$ for $j\sin(tn) \Rightarrow$ Periodic w/ period $2\pi$

Odd or Even?

Break up summation into $n = (-\infty, -1], 0, [1, \infty)$

$$x_1(t) = \sum_{n=-\infty}^{-1} 2^{-|n|} e^{jtn} + 2^0 e^0 + \sum_{n=1}^{\infty} 2^{-|n|} e^{jtn}$$

$$= \sum_{n=1}^{\infty} 2^{-|n|} e^{-jtn} + 1 + \sum_{n=1}^{\infty} 2^{-|n|} e^{jtn}$$

$$x_1(-t) = \sum_{n=-\infty}^{-1} 2^{-|n|} e^{-jtn} + 2^0 e^0 + \sum_{n=1}^{\infty} 2^{-|n|} e^{-jtn}$$

$$= \sum_{n=1}^{\infty} 2^{-|n|} e^{jtn} + 1 + \sum_{n=1}^{\infty} 2^{-|n|} e^{-jtn}$$

$x_1(t) = x_1(-t) \Rightarrow$ Even

$$\boxed{x_1(t) \text{ is Periodic, } T_1 = 2\pi \text{, and Even}}$$

(2)

$$x_2(t) = \cos(3\sqrt{2}t) + \sin(\sqrt{8}t - 0.5\pi)$$

Periodic?

$$\sin(a - \pi/2) = -\cos(a) \Rightarrow x_2(t) = \cos(3\sqrt{2}t) - \cos(\sqrt{8}t)$$
$$= \cos(3\sqrt{2}t) - \cos(2\sqrt{2}t)$$

$$\cos(3\sqrt{2}(t+N)) = \cos(3\sqrt{2}t + 3\sqrt{2}N) \qquad \cos(2\sqrt{2}(t+N)) = \cos(2\sqrt{2}t + 2\sqrt{2}N)$$
$$3\sqrt{2}N = 2\pi K_1 \qquad 2\sqrt{2}N = 2\pi K_2$$
$$\Rightarrow N = \frac{2\pi}{3\sqrt{2}} \qquad \Rightarrow N = \frac{2\pi}{2\sqrt{2}} = \frac{\pi}{\sqrt{2}}$$

$$\text{LCM}\left\{\frac{2\pi}{3\sqrt{2}}, \frac{\pi}{\sqrt{2}}\right\} = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$$
$\rightarrow$ Periodic

Odd or Even?

$\cos(3\sqrt{2}t) = \text{Even}$ $\Rightarrow$ $x_2(t)$ is Even
$\cos(2\sqrt{2}t) = \text{Even}$

$$\boxed{x_2(t) \text{ is Periodic, } T_2 = \sqrt{2}\pi\text{, and Even}}$$

(3)

$$x_3[n] = \sum_{k=-|n|}^{3|n|} k \cdot \cos(0.5\pi k)$$

Periodic?

Total range of summation is $4 \cdot |n|$
$\cos(\frac{\pi}{2} \cdot k)$ must be equal to 0, 1, or −1, but is multiplied by k, which grows as n grows.

$\Rightarrow$ Possible magnitude of $x_3[n]$ gets bigger as n gets bigger
$\Rightarrow$ Not periodic

Odd or Even?

$$x_3[-n] = \sum_{k=-|-n|}^{3|-n|} k \cdot \cos(0.5\pi k) = \sum_{k=-|n|}^{3|n|} k \cdot \cos(0.5\pi k) = x_3[n]$$

$$\boxed{x_3[n] \text{ is Not Periodic, and Even}}$$

(4)

$$x_4[n] = \sum_{k=-\infty}^{\infty} \sin(2n) \cdot \delta[n-5k]$$

Periodic?

$$\sin(2(n+N)) = \sin(2n + 2N) \rightarrow \cancel{2}N = \cancel{2}\pi K$$

$$N = \pi K$$

No values of N and K allow both to be integers

$\Rightarrow$ Not Periodic

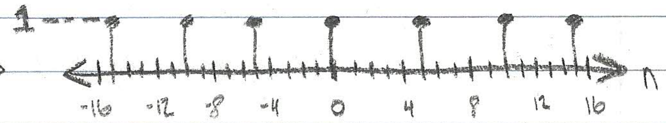
Odd or Even?

$$x_4[n] = \sin(2n) \cdot \underbrace{\sum_{k=-\infty}^{\infty} \delta[n-5k]}$$

$\hookrightarrow$ = 1 when $n = 5k$

Sketch of $\sum_{k=-\infty}^{\infty} \delta[n-5k]$:

Is an even function $\rightarrow$




$\sin(2n)$ is an odd function

$x_4$ = odd $\cdot$ even = odd

$$\boxed{x_4[n] \text{ is Not Periodic, and Odd}}$$