Midterm #1 of ECE 301-002, 003, 004, 005 (CRN: 17727, 17101, 17102, 25618)
8–9pm, Wednesday, September 16, 2020, Online Exam.

1. Enter your student ID number, and signature in the space provided on this page **now!**
2. This is a closed book exam.
3. This exam contains multiple-choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and first working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed. No earphones are allowed either.

Name:

Student ID:

As a Boiler Maker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

Signature: Date:

Question 1: [18%, Work-out question] Consider a DT signal:

$$x[n] = \sum_{k=5}^{\infty} e^{-(2+2j)k} \cdot \delta[n-k]. \tag{1}$$

1. [3%] What is the value of $x[0]$?
2. [3%] What is the value of $x[10]$?
3. [12%] Find out the total energy of $x[n]$.

   Hint 1: If $|r| < 1$, then we have the following formulas for computing the infinite sum of a geometric sequence.

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$$
$$\sum_{k=1}^{\infty} kar^{k-1} = \frac{a}{(1-r)^2}.$$

   If $r \neq 1$, then we have the following formula for computing the finite sum of a geometric sequence.

$$\sum_{k=1}^{K} ar^{k-1} = \frac{a(1-r^K)}{1-r}.$$

   Hint 2: If you do not know the answer to this question, please find the average power of the signal $x_2[n] = (\sin(0.7n) - \cos(0.7n) \cdot j) \cdot U[n-3]$ in the interval $-20 \leq n \leq 20$. You will receive 7 points if your answer is correct.

Question 2: [17%, Work-out question]

Define two CT signals:

$$x(t) = \begin{cases} 3^t & \text{if } -6 < t < 3 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

and

$$h(t) = \begin{cases} 3^t & \text{if } 0 < t \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Compute the expression of the following integral

$$y(t) = \int_{s=-\infty}^{\infty} x(s)h(2t-s)ds \tag{4}$$

Hint: You can leave (part of) your answer to be of the form like $\frac{e^{3jt}-2^{-12t}}{3+10j}$. There is no need to further simplify the expression.

Question 3: [15%, Work-out question] Consider a DT signal

$$x[n] = \begin{cases} 5^n & n \leq 10 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

We use $x[n]$ to create another function $f(z)$ by

$$f(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \tag{6}$$

Find out the value of $f(2+2j)$.

Hint 1: You can leave (part of) your answer to be of the form like $\frac{e^{3jt}-2^{-12t}}{3+10j}$. There is no need to further simplify the expression.

Hint 2: The formulas in the hints of Question 1 may also be useful for this question.

Question 4: [12%, Work-out question]

Consider the following DT signals.

$$x[n] = e^{j\frac{\sqrt{\pi}}{5}n} \tag{7}$$

$$h[n] = U[n-30] - U[n-100]. \tag{8}$$

Define

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \tag{9}$$

Find the expression of $y[n]$.

Hint 1: You may want to use the change-of-variable formula to simplify the computation.

Hint 2: You can leave (part of) your answer to be of the form like $\frac{e^{3jt}-2^{-12t}}{3+10j}$. There is no need to further simplify the expression.

Hint 3: The formulas in the hints of Question 1 may also be useful for this question.

Question 5: [18%, Work-out question]

Consider the following linear system. We know for a fact that if the input signal is an *even* signal $x[n]$, the output signal $y[n]$ of this linear system is

$$y[n] = 3x[n]. \tag{10}$$

For example, we know that if the input signal is $x[n] = n^4$, then the output signal is $y[n] = 3n^4$. If the input signal is $x[n] = |n|$, then the output signal is $y[n] = 3|n|$. And so on so forth.

We also know that if the input signal is an *odd* signal $x[n]$, the output signal $y[n]$ of this linear system is

$$y[n] = x[n-3]. \tag{11}$$

For example, we know that if the input signal is $x[n] = n^3$, then the output signal is $y[n] = (n-3)^3$.

Suppose we are encountering a new input signal $x[n] = U[n+2] - U[n-6]$. Since this new $x[n]$ is neither even nor odd, we cannot directly multiply-by-3 or shift-by-3 to find the output $y[n]$. Answer the following questions:

1. [4%] Plot $x[n]$ for the range of $-8 \leq n \leq 8$.

2. [14%] What is the output $y[n]$. In your answers, please plot $y[n]$ for the range of $-8 \leq n \leq 8$.

   Hint: If you do not know the answer to this question, please plot the odd part of the signal $x[n]$ for the range of $-8 \leq n \leq 8$. You will receive 8 points if your answer is correct.

Question 6: [20%, Multiple Choices]

The following questions are multiple-choice questions and there is no need to justify your answers. Consider two continuous-time signals:

$$x_1(t) = \sum_{n=-\infty}^{\infty} 2^{-|n|} e^{jtn} \tag{12}$$

$$x_2(t) = \cos(3\sqrt{2}t) + \sin(\sqrt{8}t - 0.5\pi) \tag{13}$$

and two discrete-time signals:

$$x_3[n] = \sum_{k=-|n|}^{3|n|} k \cdot \cos(0.5\pi k) \tag{14}$$

$$x_4[n] = \sum_{k=-\infty}^{\infty} \sin(2n) \cdot \delta[n - 5k]. \tag{15}$$

1. [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not, *respectively.* If it is periodic, write down the fundamental period. Please state explicitly which signal is periodic and which is not.

2. [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them, respectively. Please state explicitly which signal is even, which is odd, and which is neither.