Question 1:

- Q1.1: The main difference is that one uses the synchronous carrier and the other does not.
- Q1.2: 3000 Hz.
- Q1.3: $W_4 = 6000\pi$.
- Q1.4: No impact. It is because the bandwidth we are transmitting is 4000π rad/sec (2000Hz)
- Q1.5: $W_3 = 11000\pi$ and $W_6 = 11000\pi$.
- Q1.6: $W_8 = 4000\pi$, $W_9 = 6000\pi$, and $W_{10} = 11000\pi$.
- Q1.7: (i) x2_hat is too weak; (ii) We use the following code instead

  ```
  y2=4*y.*sin(W_10*t);
  x2_hat=ece301conv(y2,h_M);
  ```

- Q1.8: (i) x1_hat is corrupted with the x2 signal; (ii) We should insert the following code

  ```
  h_BPF1=1/(pi*t).*(sin(6000*pi*t)-sin(2000*pi*t));
  y1_BPF=ece301conv(y,h_BPF1);
  y1=4*y1_BPF.*sin(W_9*t);
  x1_hat=ece301conv(y1,h_M);
  ```

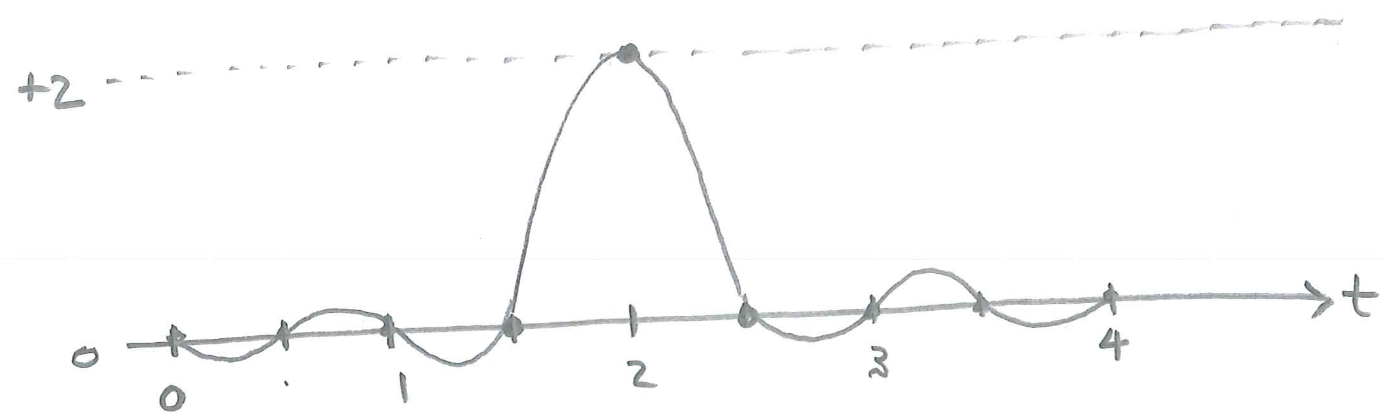
Q2 Orig. Solution

#1 $x(t) = \dfrac{\sin(2\pi(t-2))}{\pi(t-2)}$ Plot for $0 \le t \le 4$

$$= 2 \cdot \frac{\sin(2\pi(t-2))}{2\pi(t-2)} \Rightarrow x(2) = 2$$

The zeros then occur at values st $2\pi(t-2) = \pi k \quad k \neq 0$

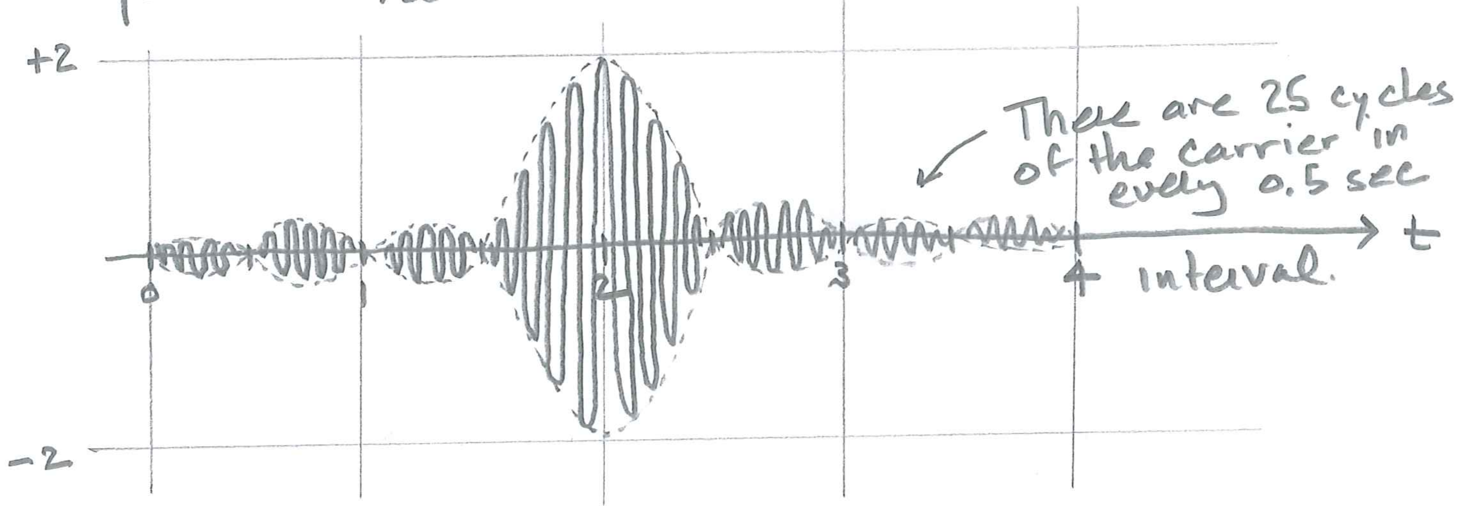
$$\Rightarrow 2(t-2) = k$$
$$t-2 = \frac{k}{2}$$
$$t = 2 + k/2 \quad k \neq 0$$



#2 $y(t) = x(t)\cos 100\pi t$ ie AM DSB-SC @ 50 Hz.

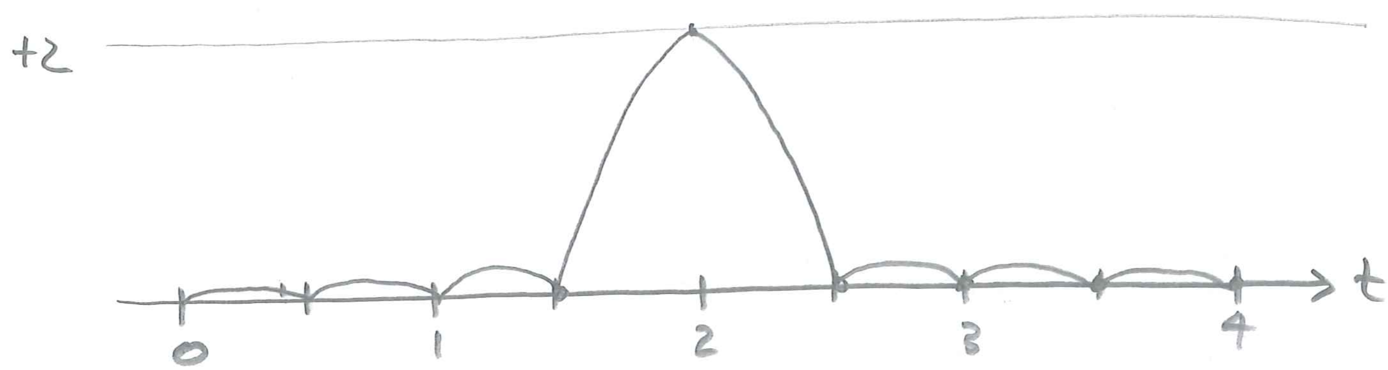
Plot $y(t)$ over range $0 \le t \le 4$

$\Rightarrow$ There will be a lot of cycles of the 50 Hz cosine relative to the oscillations in the envelope since the period is $1/50$ sec.



(#3) AM asychronous demodulation produces at its output the envelope of its input. This will amount to the absolute value of $x(t)$ ie

$$\hat{x}(t) = |x(t)| \quad 0 \le t \le 4$$



Q3 Original

#1 $w(t) = \cos(5t)\cos(2t)$ ---> find minimum sampling rate.

$$= \left(\frac{e^{j5t}+e^{-j5t}}{2}\right)\left(\frac{e^{j2t}+e^{-j2t}}{2}\right)$$

$$= \frac{e^{j7t}+e^{j3t}+e^{-j3t}+e^{-j7t}}{4}$$

$$= \frac{1}{2}\cos(7t) + \frac{1}{2}\cos(3t)$$

Highest radian freq is $\omega_h = 7$ rad/sec

$\Rightarrow 7 = 2\pi f_h$

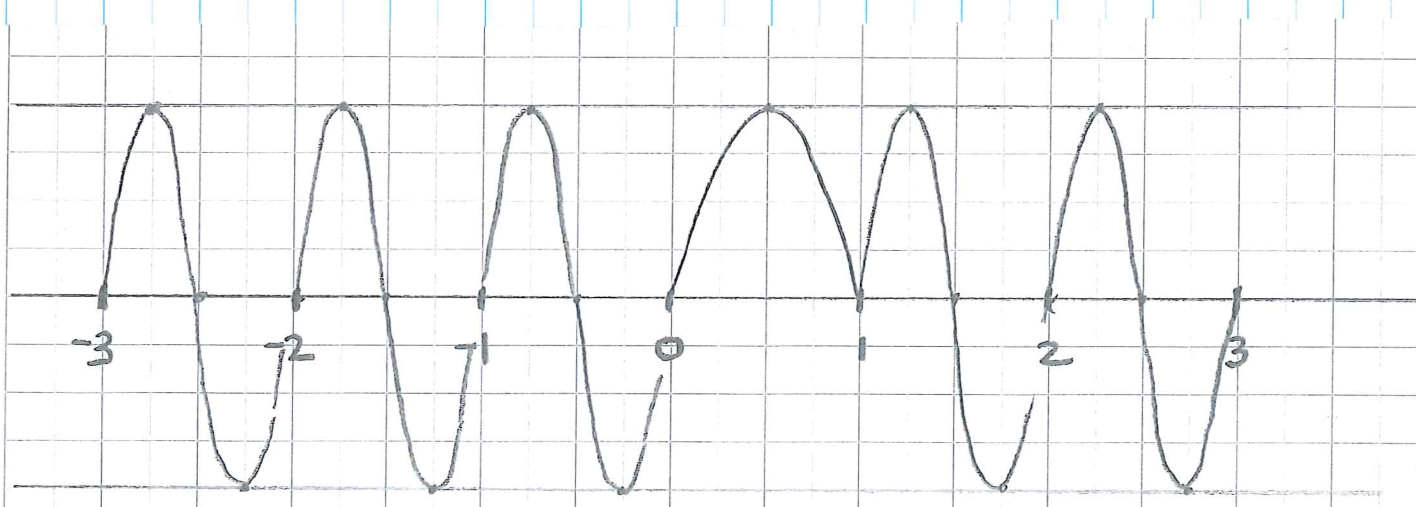
$\Rightarrow f_h = \frac{7}{2\pi} \approx 1.11$ Hz

Min rate $= 2 \times f_h$

$\cong 2.22$ Hz (actually need $f_s > 2.22$ Hz)

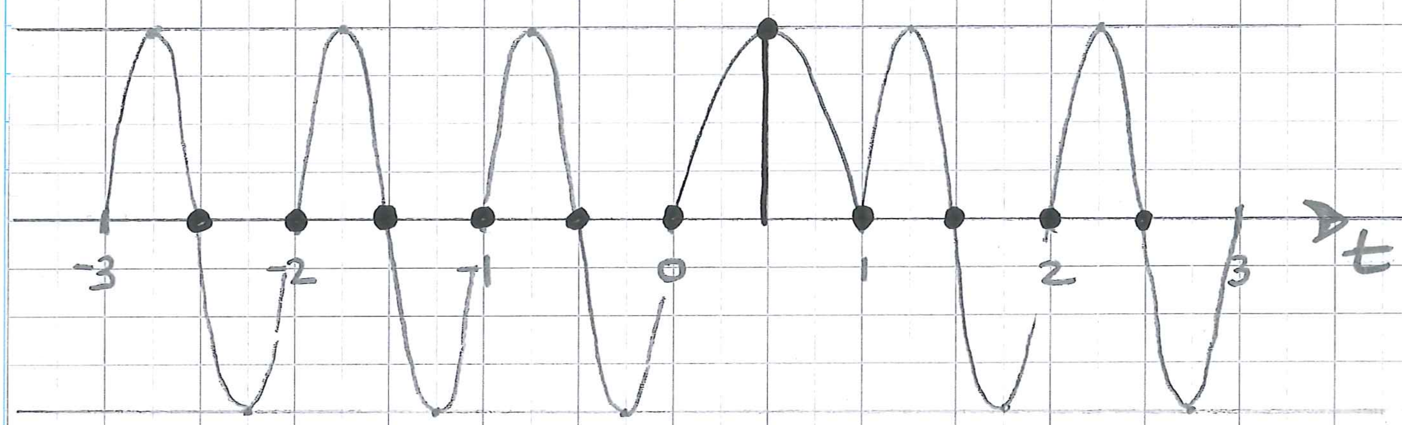
#2

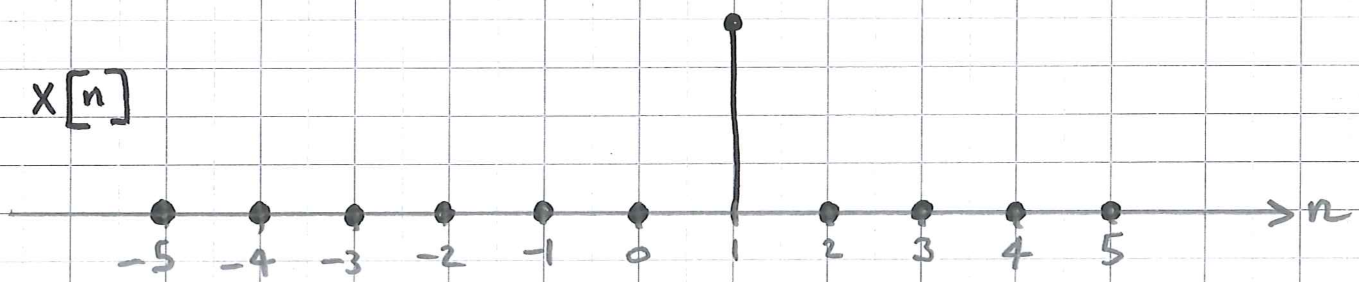
$$x(t) = \begin{cases} \sin(\pi t) & 0 \le t < 1 \longrightarrow \frac{1}{2}\text{ Hz} \\ \sin(2\pi t) & \text{else} \longrightarrow 1\text{ Hz} \end{cases}$$



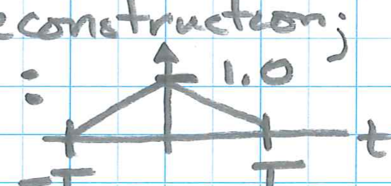
(#3) $x[n] = x(t)\Big|_{t=nT}$ $\quad T = 1/2$ sec

Plot for $-5 \le n \le 5$ (ie $-2.5 \le t \le 2.5$)

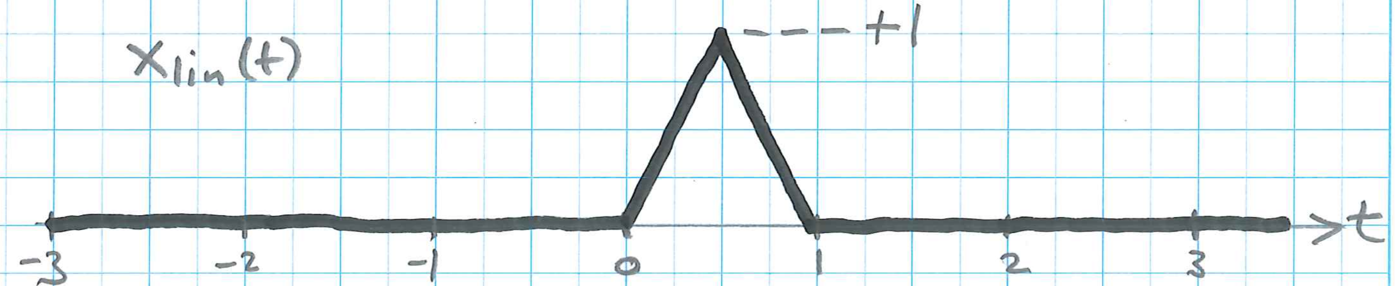




(#4) $x_{lin}(t)$ is the linear interpolation reconstruction; it corresps to an impulse response:



and therefore simply joins points with straight lines



(#5) $x_{opt}(t)$ corresp. to opt. bandlimited reconstruction of $x(t)$. In this case the interpolation filter is

$$h_{opt}(t) = \frac{\sin(\pi t/T)}{\pi t/T} \qquad T = \tfrac{1}{2}\text{ sec}$$

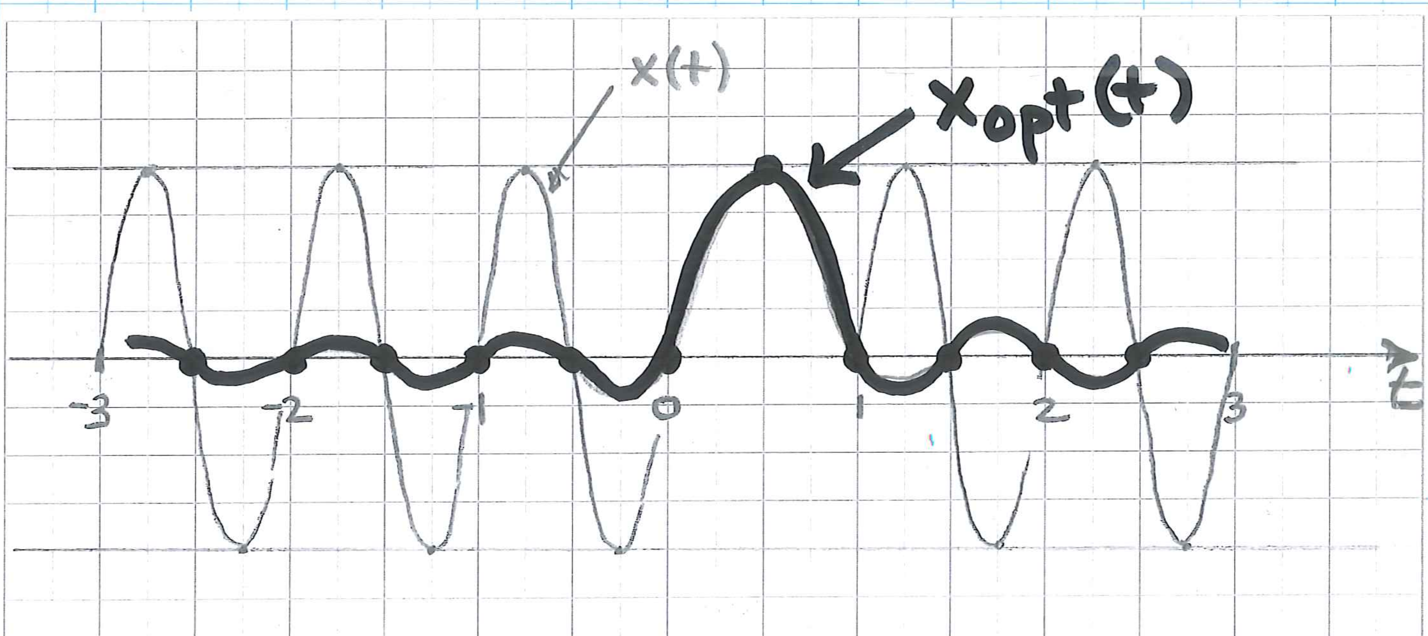
and

$$x_{opt}(t) = \sum_n x[n]\, h_{opt}(t-nT)$$

But only one sample is non-zero, $x[1] = +1$

$$\therefore\ x_{opt}(t) = h_{opt}(t-T)$$

$$= \frac{\sin(\pi(t-T)/T)}{\pi(t-T)/T}$$

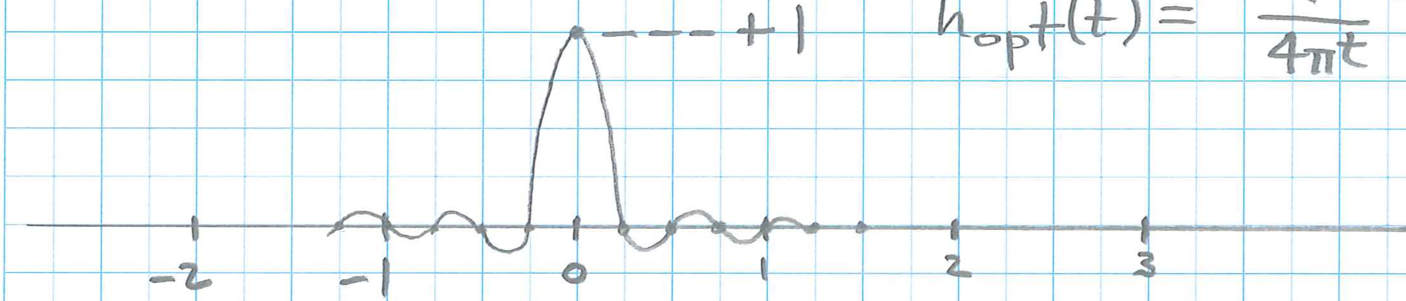


#6 Now instead use $\tilde{h}_{opt}(t) = \dfrac{\sin(\pi t/\tilde{T})}{\pi t/\tilde{T}}$

where

$$\tilde{T} = 0.25\text{ sec} = T/2$$

$$h_{opt}(t) = \frac{\sin(4\pi t)}{4\pi t}$$



There is still only one non-zero sample and therefore

$$\tilde{x}_{opt}(t) = \frac{\sin(4\pi(t-1/4))}{4\pi(t-1/4)}$$

$\Longrightarrow$ Now the sinc mainlobe is half as wide <u>and</u> is centered at $t = 0.25$ sec rather than at a half-second as before

Question 4: [8%, Work-out question]

Consider two continuous time signals $x(t)$ and

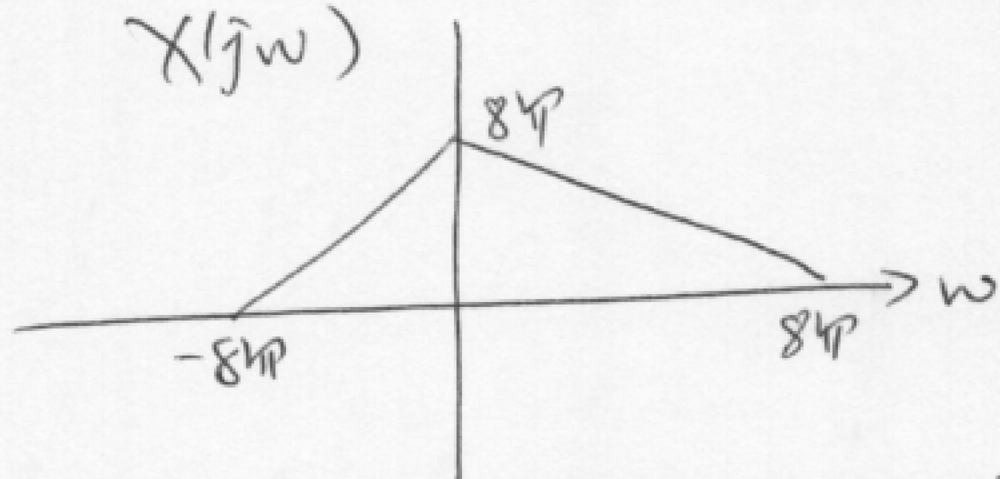
$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - \frac{k}{6}) \tag{4}$$

and we know that the CTFT $X(j\omega)$ of $x(t)$ is

$$X(j\omega) = \begin{cases} 8\pi - \omega & \text{if } 0 \le \omega < 8\pi \\ 8\pi + \omega & \text{if } -8\pi \le \omega < 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Define $y(t) = x(t) \cdot p(t)$. Plot the CTFT $Y(j\omega)$ for the range of $-12\pi \le \omega \le 12\pi$.

Hint: If you do not know the answer to this question, please plot $z(t) = \cos(\pi t) \cdot p(t)$ for the range of $-1 \le t \le 1$. You will receive 4 points if your answer is correct.



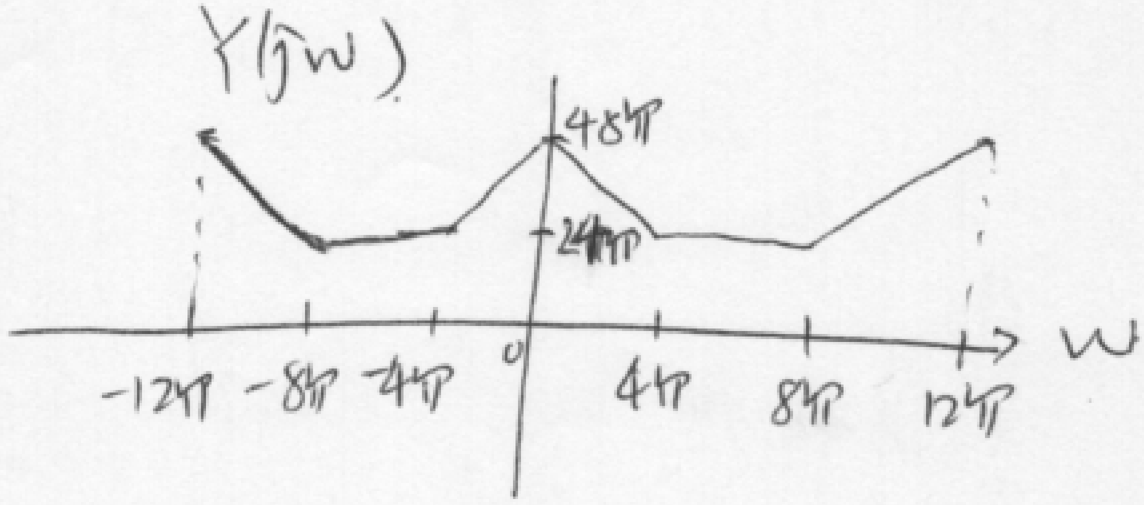
$$y(t) = x(t) \cdot p(t) \xleftrightarrow{F.T.} Y(jw) = \frac{1}{2\pi} X(jw) * P(jw)$$

$$P(jw) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(w - k\frac{2\pi}{T}) \quad T = \frac{1}{6}$$

$$\therefore \; Y(jw) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(w - k\frac{2\pi}{T}))$$

$$= 6 \cdot \sum_{k=-\infty}^{\infty} X(j(w - 12\pi \cdot k))$$

Y(jw) is sum of shifted versions of X(jw)



//

Question 5: [11%, Work-out question]

Consider the following discrete time signals

$$x[n] = e^{j\cdot(10\pi n)} + e^{-j(n-10)} \tag{6}$$

$$h[n] = \delta[n-199] + \begin{cases} 3 & \text{if } 0 \le n \le 99 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Find the expression of $y[n] = x[n] * h[n]$.

Hint 1: The following formulas may be useful:

$$\text{If } |r| < 1, \text{ then } \quad \sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}. \tag{8}$$

$$\text{If } r \neq 1, \text{ then } \quad \sum_{k=1}^{K} ar^{k-1} = \frac{a\cdot(1-r^K)}{1-r}. \tag{9}$$

Hint 2: You may want to consider a more general input $x[n] = e^{j\omega n}$ first.

Hint 3: It would be useful to consider $h[n]$ as a summation of two signals $h_1[n] + h_2[n]$.

$$x[n] = 1 + e^{-j(n-10)}, \quad h[n] = \underset{\uparrow \atop h_1[n]}{\delta[n-199]} + \underbrace{3(u[n] - u[n-100])}_{\uparrow \atop h_2[n]}$$

$y_1[n] = x[n] * h_1[n]$ ← convolve w/ shifted delta

$$= 1 + e^{-j(n-199-10)} = 1 + e^{-j(n-209)}$$

$$y_2[n] = x[n] * h_2[n]$$

$$= \sum_{k=-\infty}^{\infty} x[n-k]\cdot h[k]$$

$$= 3\sum_{k=0}^{99} (1 + e^{-j(n-k-10)})$$

$$= 300 + 3e^{-j(n-10)} \cdot \sum_{k=0}^{99} e^{jk}$$

$$= 300 + 3e^{-j(n-10)} \cdot \frac{1-e^{j100}}{1-e^{j}}$$

$$= 300 + \frac{3e^{-j(n-10)} - 3e^{-j(n-110)}}{1-e^{j}}$$

$$\therefore \; y[n] = y_1[n] + y_2[n] = 301 + e^{-j(n-209)} + \frac{3e^{-j(n-10)} - 3e^{-j(n-110)}}{1-e^{j}} \quad //$$

Q 6 version 1

$$x(t) = \begin{cases} 2 & 0 \le t < 2 \\ 0 & 2 \le t < 6 \\ \text{periodic} & \text{period } 6 \end{cases}$$

a) Find CTFT.

First find CTFS

$$\omega_0 = \frac{2\pi}{T}$$

$$\boxed{\omega_0 = \frac{\pi}{3}}$$

$$a_k = \frac{1}{T} \int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{6} \int_0^2 2\, e^{-jk\omega_0 t}\, dt$$

case 1: $k=0$ $\qquad a_0 = 2/3$

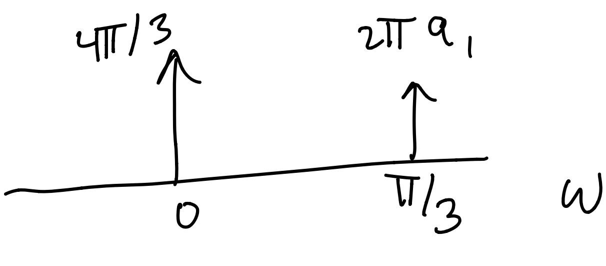
case 2: $k \neq 0$ $\qquad a_k = \frac{1}{3} \left(\frac{e^{-jk\omega_0 t}}{-jk\omega_0} \right) \Big|_0^2$

$$a_k = \frac{1}{jk\pi} \left(1 - e^{-jk\frac{2\pi}{3}} \right)$$

Now CTFT:

$$X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k\, \delta\left(\omega - \frac{2\pi k}{T}\right)$$

b) Sketch CTFT between $-0.1\pi < \omega < 0.3\pi$



(only $k=0$ and $k=1$ lead to impulses inside interval of interest

Q7 version 1

$$x[n] = \sqrt{2}^n \, e^{j\frac{\pi}{4}n} \, u[n-10]$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \, z^{-n}$$

$$= \sum_{n=10}^{\infty} \left(\sqrt{2} \, e^{j\frac{\pi}{4}}\right)^n z^{-n}$$

let $s = \sqrt{2} \, e^{j\pi/4}$ and let $k = n-9$ so $n = k+9$

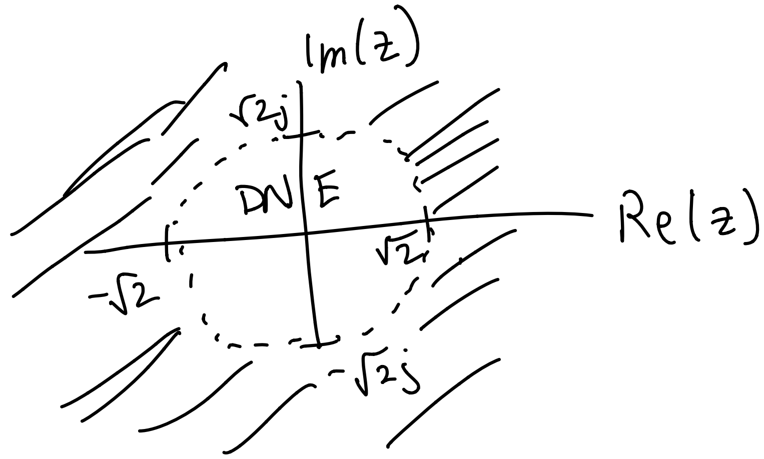
$$= \sum_{k=1}^{\infty} \left(\frac{s}{z}\right)^{k+9} = \left(\frac{s}{z}\right)^{10} \sum_{k=1}^{\infty} \left(\frac{s}{z}\right)^{k-1}$$

$$= \left(\frac{s}{z}\right)^{10} \frac{1}{1-\frac{s}{z}} \quad \text{if } \left|\frac{s}{z}\right| < 1$$

or if $|z| > |s|$

$$= \frac{s^{10}}{z^{10}} \, \frac{z}{z-s} \quad \text{if } |z| > \sqrt{2}$$

ROC:
shaded
outside the
circle



Question 8

- Q8.1: aperiodic.
- Q8.2: aperiodic.
- Q8.3: even.
- Q8.4: neither.
- Q8.5: finite power.
- Q8.6: finite power.
- Q8.7: with memory.
- Q8.8: with memory.
- Q8.9: non-causal.
- Q8.10: causal.
- Q8.11: stable.
- Q8.12: stable.