Final Exam of ECE301-003, 004 (CRN 17101, 17102)
3:30–5:30pm, Wednesday, December 11, 2019, WTHR 200.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have two hours to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name: Solution

Student ID:

As a Boiler Maker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

Signature: Date:

Question 1: [20%, Work-out question]

1. [1%] What does the acronym FDM stand for? Freq. Division Multiplexing

Prof. Wang wanted to transmit an AM-SSB signal. To that end, he wrote the following MATLAB code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read two different .wav files
[x1, f_sample, N]=audioread('x1.wav');
x1=x1';
[x2, f_sample, N]=audioread('x2.wav');
x2=x2';

% Step 0: Initialize several parameters
W_1=????;
W_2=pi*5000;
W_3=pi*7000;
W_4=pi*8000;
W_5=????;
W_6=????;
W_7=????;


% Step 1: Make the signals band-limited.
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);

% Step 2: Multiply x1_new and x2_new with a sinusoidal wave.
x1_h=x1_new.*sin(W_2*t);
x2_h=x2_new.*cos(W_3*t);

% Step 3: Keep one of the two side bands
h_one=1/(pi*t).*(sin(W_4*t)-sin(W_5*t));
h_two=1/(pi*t).*(sin(W_6*t)-sin(W_7*t));
x1_sb=ece301conv(x1_h, h_one);
x2_sb=ece301conv(x2_h, h_two);

% Step 4: Create the transmitted signal
```

```
y=x1_sb+x2_sb;
audiowrite('y.wav', y, f_sample);
```

2. [1.5%] What is the carrier frequency (Hz) of the signal x2_new? 3500 Hz

3. [3%] Our goal is to transmit the "upper-side bands" for both x1 and x2 signals. What should the values of W_5 and W_7 be in the MATLAB code?

4. [1.5%] Continue from the previous sub-question. To ensure that the receiver side can have the best possible quality, it is important for the transmitter to choose the largest W_1 value when possible. What is the largest W_1 value that can be used without significantly degrading the quality of any of the two transmitted signals?

5. [1.5%] Continue from the previous sub-question. What is the smallest W_6 value we can use without degrading the quality of x2 signal?

3. $W_5 = 5000\pi$ ~~$W_7 = 7000\pi$~~ $W_7 = 7000\pi$ ~~$W_7 = 6500\pi$~~

4. $W_1 = 2000\pi$

5. $W_6 = 9000\pi$

Knowing that Prof. Wang decided to use an upper-side-band transmission for the first signal x1_new and he chose the W_1 value to be $W_1 = 1500 \times \pi$. He then used the code in the previous page to generate the "y.wav" file. A student tried to demodulate the output waveform "y.wav" by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[y, f_sample, N]=audioread('y.wav');
y=y';

% Initialize several parameters
W_8=????;
W_9=????;
W_10=????;
W_11=????;
W_12=????;


% Create the low-pass filter.
h_M=1/(pi*t).*(sin(W_8*t));

% demodulate signal 1
h_BPF1=1/(pi*t).*(sin(W_9*t));
y1_BPF=ece301conv(y,h_BPF1);
y1=y1_BPF.*cos(pi*5000*t);
x1_hat=ece301conv(y1,h_M);

sound(x1_hat,f_sample)

% demodulate signal 2
h_BPF2=1/(pi*t).*(sin(W_10*t)*2cos(W_11*t));
y2_BPF=ece301conv(y,h_BPF2);
y2=y2_BPF.*cos(pi*W_12*t);
x2_hat=ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```

$W_8 = 1500\pi$

$W_9 = 6500\pi$

$W_{10} = 750\pi$

$W_{11} = 7750\pi$

$W_{12} = 7000$

6. [7.5%] Continue from the previous questions. What should the values of W_8 to W_12 be in the MATLAB code? When answering this question, please assume

$W_1 = 1500 \times \pi$.

7. [2%] It turns out that the above MATLAB code is not written correctly and neither signal x1_new nor signal x2_new can be correctly demodulated. Please use 2 to 3 sentences to (i) what kind of problem does x1_new have, i.e., how does the problem impact the sound quality of "sound(x1_hat,f_sample)"? (ii) how can the MATLAB code be corrected so that the playback/demodulation can performed successfully?

8. [2%] Please use 2 to 3 sentences to (i) what kind of problem does x2_new have, i.e., how does the problem impact the sound quality of "sound(x2_hat,f_sample)"? (ii) how can the MATLAB code be corrected so that the playback/demodulation can performed successfully?

Hint: If you do not know the answers of Q1.2 to Q1.8, please simply draw the AMSSB modulation (using lower side band) and demodulation diagrams and mark carefully all the parameter values. You will receive 12 points for Q1.2 to Q1.8.

7. (1) silent

(2) $\cdot 4\sin(5000\pi t)$

8. (1) Too weak

(2) $\cdot 4\cos(4000\pi t)$

Question 2: [11%, Work-out question]

1. [1.5%] Consider a continuous time signal $x(t)$

$$x(t) = \frac{\sin(3\pi t)}{\pi t}. \tag{1}$$

Plot the CTFT $X(j\omega)$ of $x(t)$ for the range of $-4\pi \leq \omega \leq 4\pi$.

2. [2%] We then construct $y(t)$ by

$$y(t) = x(t) \cdot \cos(100\pi t). \tag{2}$$

Plot the CTFT $Y(j\omega)$ of $y(t)$ for the range of $-104\pi \leq \omega \leq 104\pi$.

3. [3%] Finally we construct $z(t)$ by

$$z(t) = y(t) * \frac{\sin(101\pi t)}{\pi t} \tag{3}$$

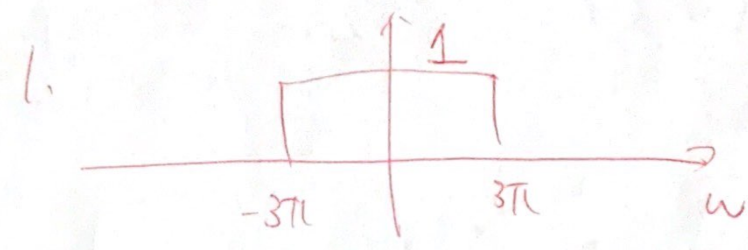
Plot the CTFT $Z(j\omega)$ of $z(t)$ for the range of $-104\pi \leq \omega \leq 104\pi$.

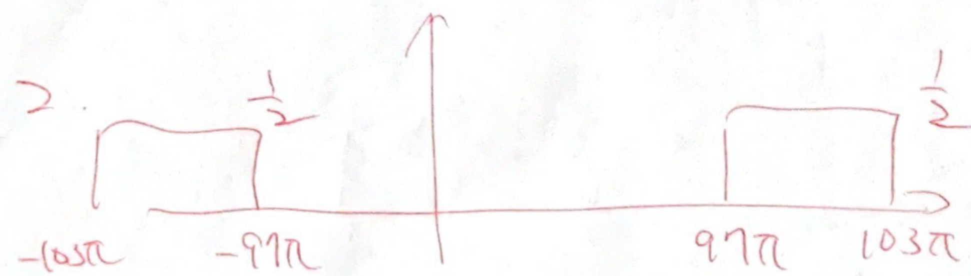
Hint: If you do not know how to solve this question, you can solve the following alternative question instead. You will receive 2 points if your answer is correct.

Suppose $h(t) = \left(\frac{\sin(5t)}{t}\right)^2$. Find the CTFT $H(j\omega)$.

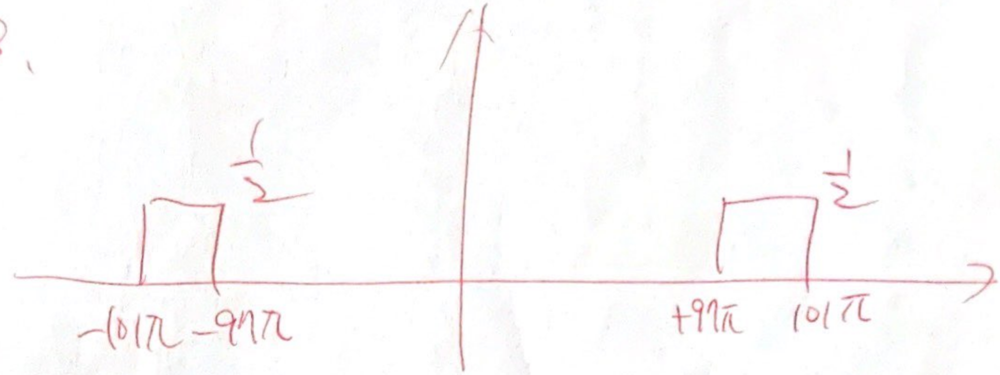
4. [4.5%] Continue from the previous sub-question. Plot $z(t)$ for the range of ~~-5~~ -2.5 $< t <$ ~~5~~ 2.5. → Find the expression first.

Hint: If you do not know how to solve this question, please write down what is the definition of *AM asynchronous demodulation* and give a detailed example how to use AM asynchronous demodulation to demodulate an AM signal. If your answer is correct, you will receive 2.5 points for this sub-question.



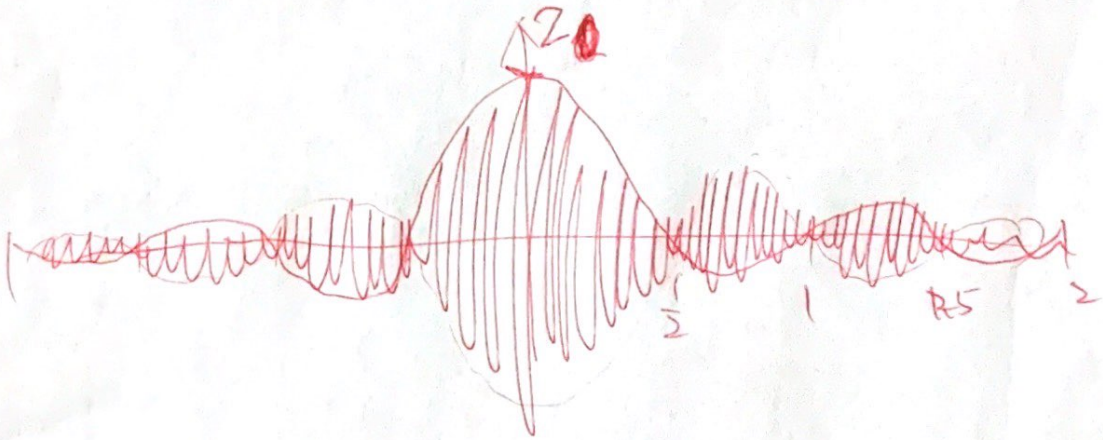


3.



4.

$$z(t) = \frac{\sin(2\pi t)}{\pi t} \cdot \cos(99\pi t)$$



Question 3: [16%, Work-out question]

1. [1.5%] Consider the following continuous time signal

$$x(t) = \sin(1.25\pi t) \tag{4}$$

   Plot $x(t)$ for the range of $-4 < t < 4$.

2. [3%] We sample $x(t)$ with the sampling frequency 2.5Hz and denote the sampled values by $x[n]$. Plot $x[n]$ for the range of $-5 \leq n \leq 5$.

3. [3%] We use $x_{\text{ZOH}}(t)$ to represent the reconstructed signal using "zero-order hold". Plot $x_{\text{ZOH}}(t)$ for the range of $-5 \leq t \leq 5$. -2 2

   Hint: if you do not know the answer of $x[n]$, you can assume that $x[n] = 2(-1)^{n+1} - 1$ and the sampling frequency is 2.5Hz. You will receive full points if your answer is correct.

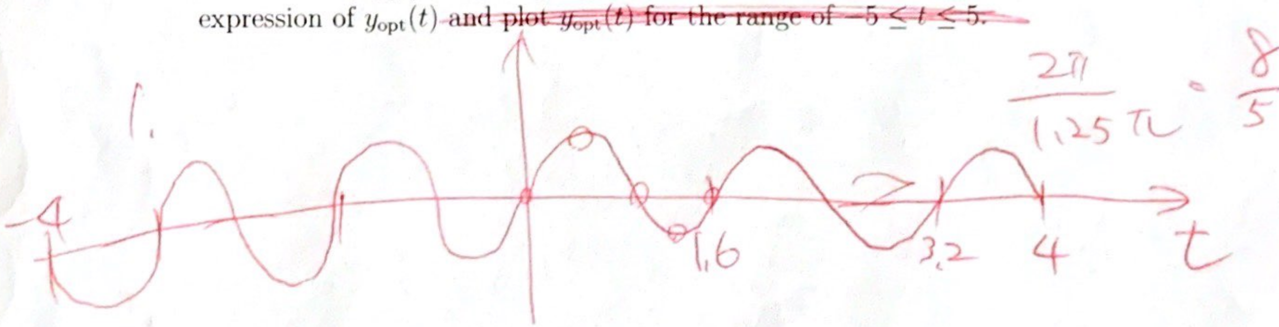
4. [2%] We use $x_{\text{opt}}(t)$ to represent the optimal band-limited reconstruction of $x(t)$. Question: Are $x_{\text{opt}}(t)$ and $x(t)$ identical? Hint: This is not a yes-no question. Please carefully write down your reasonings. (1 to 3 sentences should suffice.) You won't receive any point if there is no justification.

5. [1.5%] Suppose we construct another array $y[n] = x[n] + x[n-1]$. Plot $y[n]$ for the range of $-5 \leq n \leq 5$.

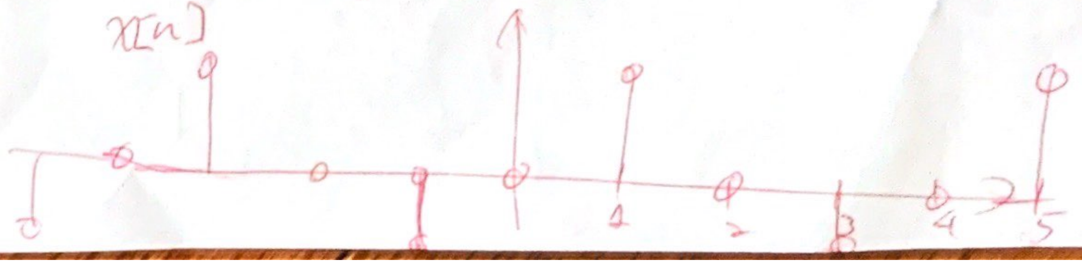
   Hint: If you do not know the answer to Q3.2, you can assume $x[n] = \cos(0.5\pi n)$. You will receive full credit if your answer is correct.

Lower percentage

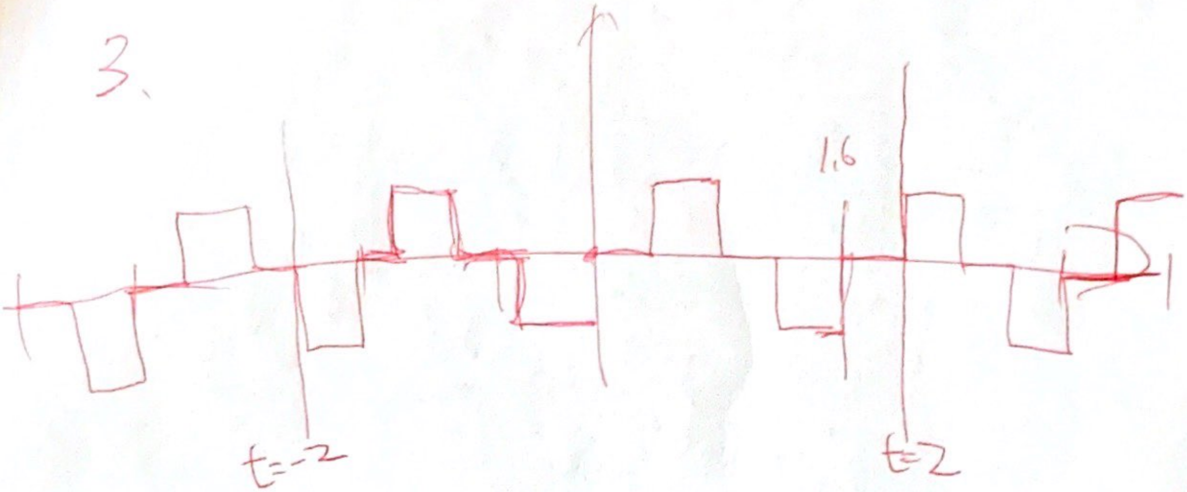
6. [5%] We use $y_{\text{opt}}(t)$ to represent the optimal band-limited reconstruction based on the new array $y[n]$ with sampling frequency 2.5Hz. Question: Write down the exact expression of $y_{\text{opt}}(t)$ and plot $y_{\text{opt}}(t)$ for the range of $-5 \leq t \leq 5$.



2. $T = \frac{1}{2.5} = 0.4$



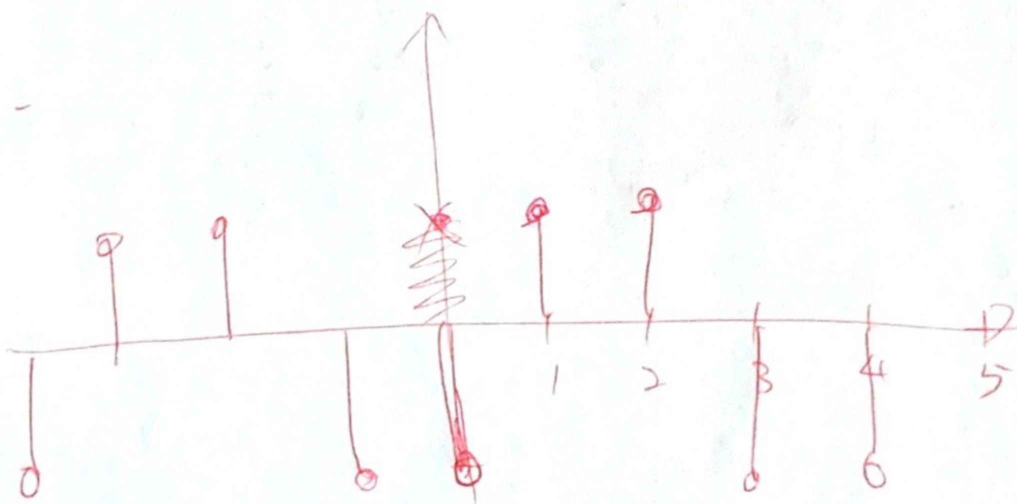
3.



4. Yes. sampling freq = 5π

$> 2 \cdot 1.25\pi$.

5.



6. $y(t) = \sin(1.25\pi t) + \sin(1.25\pi(t-0.4))$

$= \sin(1.25\pi t) + (-1)\cos(1.25\pi t)$

$= \sqrt{2}\sin(1.25\pi t - \frac{\pi}{4}) = \sqrt{2}\cos(1.25\pi t - \frac{3\pi}{4})$

Question 4: [11%, Work-out question]

Consider three continuous time signals

$$x(t) = \frac{\sin(2\pi t)}{\pi t} \tag{5}$$

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - k/4) \tag{6}$$

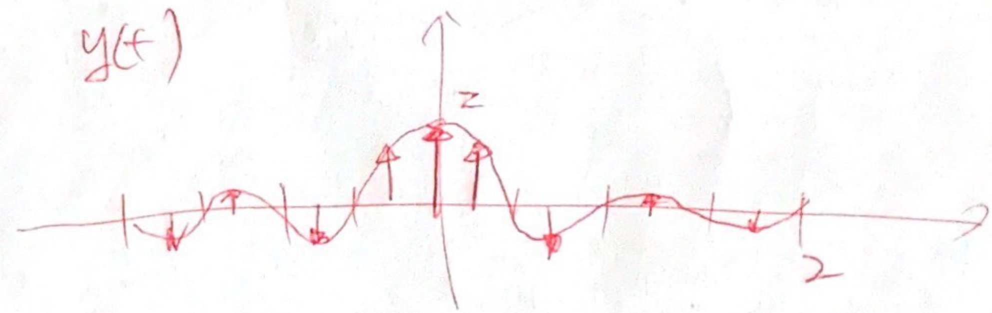
$$h(t) = \frac{\sin(5\pi t)}{\pi t} \tag{7}$$

1. [3%] Define $y(t) = x(t) \cdot p(t)$. Plot $y(t)$ for the range of $-2 \le t \le 2$.
2. [4%] Plot the CTFT $Y(j\omega)$ for the range of $-10\pi \le \omega \le 10\pi$.
3. [4%] Define $z(t) = y(t) * h(t)$. Find the exact expression of $z(t)$.

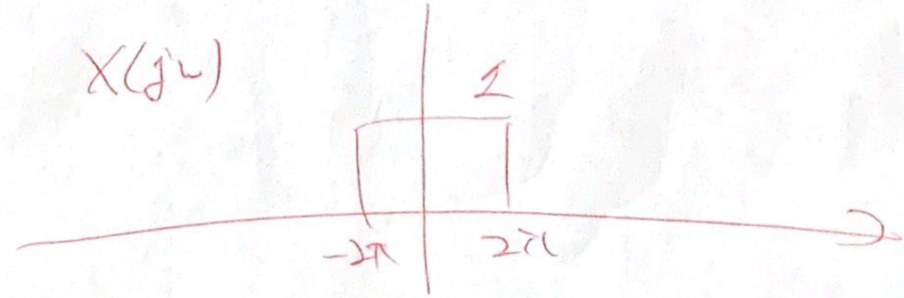
   Hint: If you do not know the answer to the previous sub-questions, please solve the following question instead.

   Define $z(t) = \left(\sum_{k=-\infty}^{\infty} \delta(t - 3k)\right) * (U(t+1) - U(t-1))$ and plot $z(t)$ for the range of $-10 < t < 10$. You will receive 3 points if your answer is correct.

1.

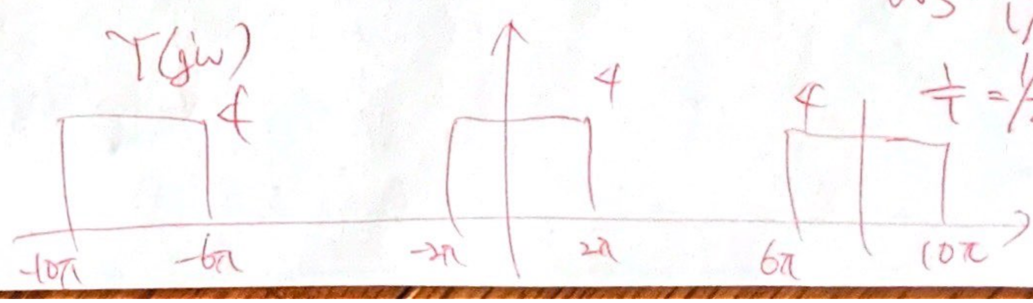


2.

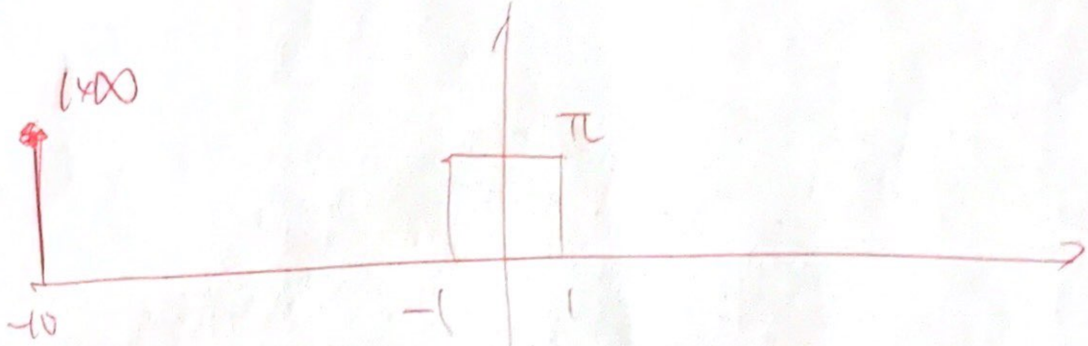


$W_S = \frac{2\pi}{1/4} = 8\pi$

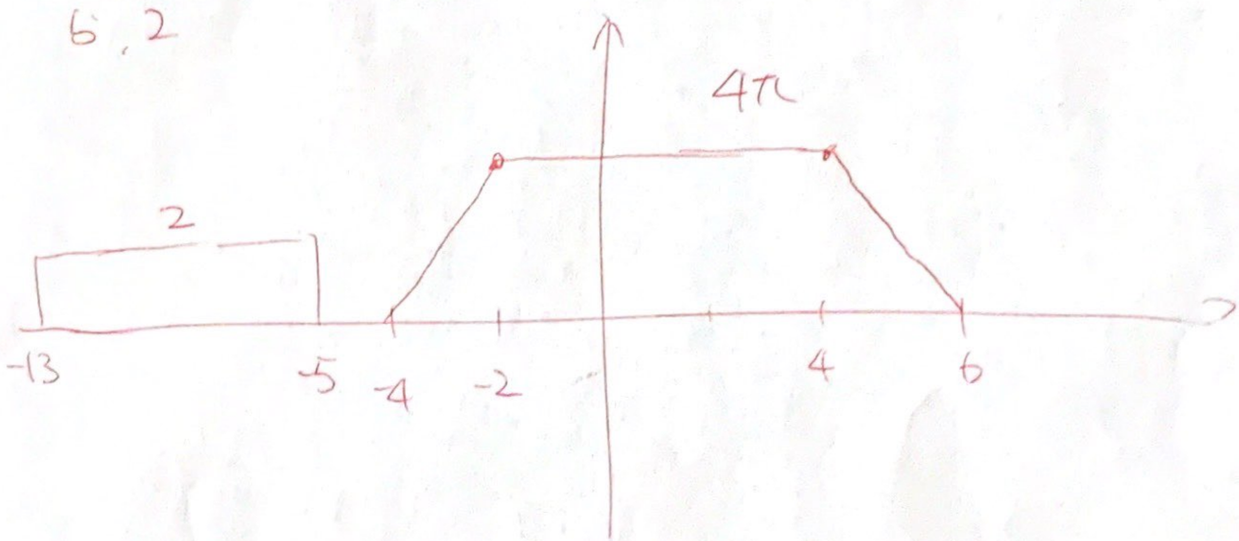
$\frac{1}{T} = \frac{1}{1/4} = 4$



5.1



5.2



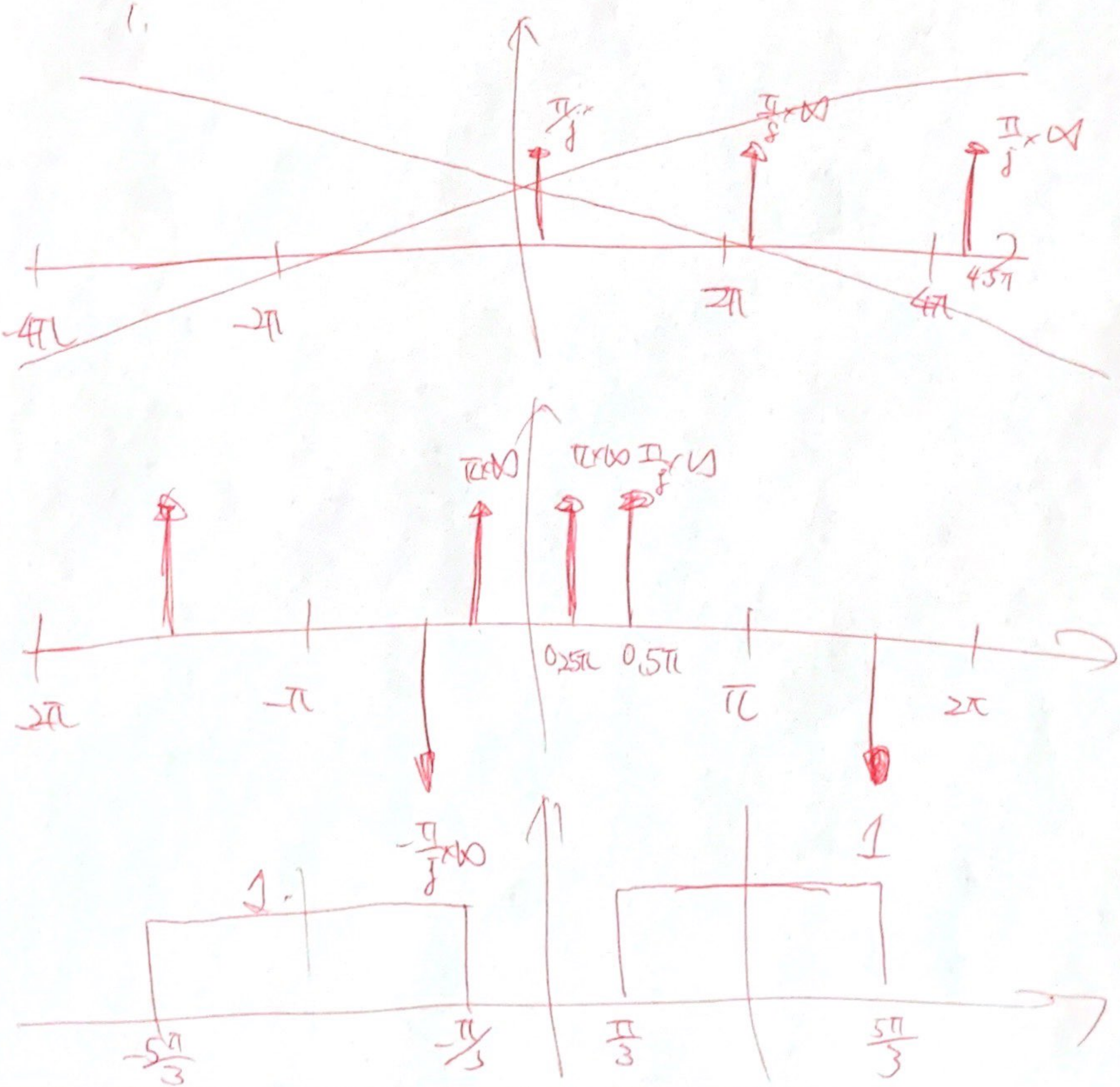
Question 6: [10%, Work-out question]

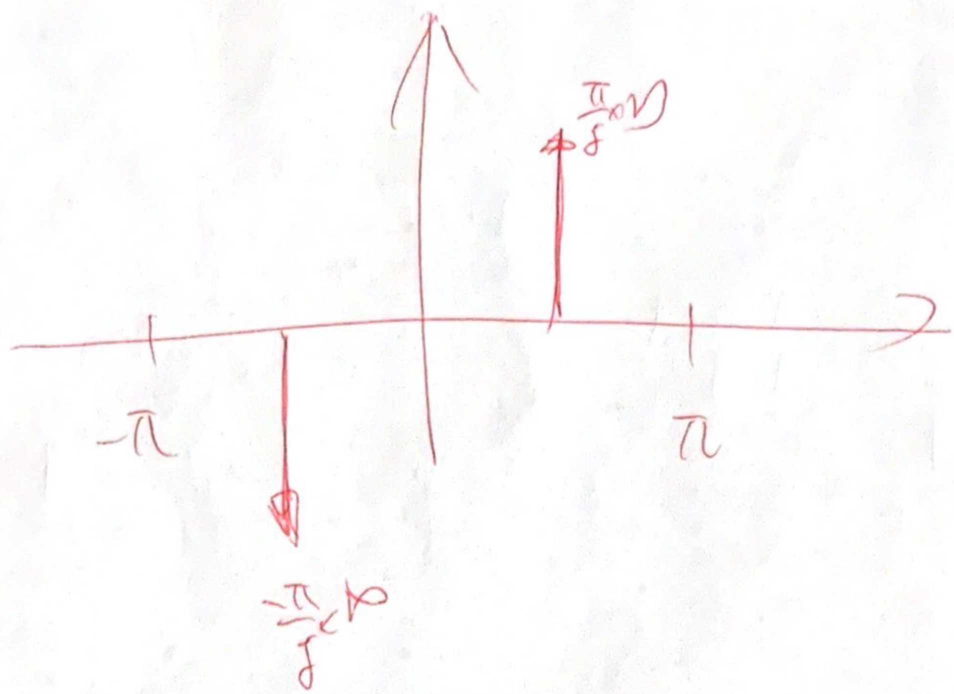
Consider the following discrete time signals

$$x[n] = \sin(1.5\pi n) + \cos(0.25\pi n) \tag{10}$$

$$h[n] = \frac{\sin(\frac{2\pi n}{3})}{\pi n} e^{j\pi n} \tag{11}$$

1. [4%] Plot the DTFT $X(e^{j\omega})$ of $x[n]$ for the range of $-2\pi < \omega < 2\pi$.
2. [3%] Plot the DTFT $H(e^{j\omega})$ of $h[n]$ for the range of $-2\pi < \omega < 2\pi$.
3. [3%] Let $y[n] = x[n] * h[n]$. Plot the DTFT $Y(e^{j\omega})$ of $y[n]$ for the range of $-2\pi < \omega < 2\pi$.





-π
π

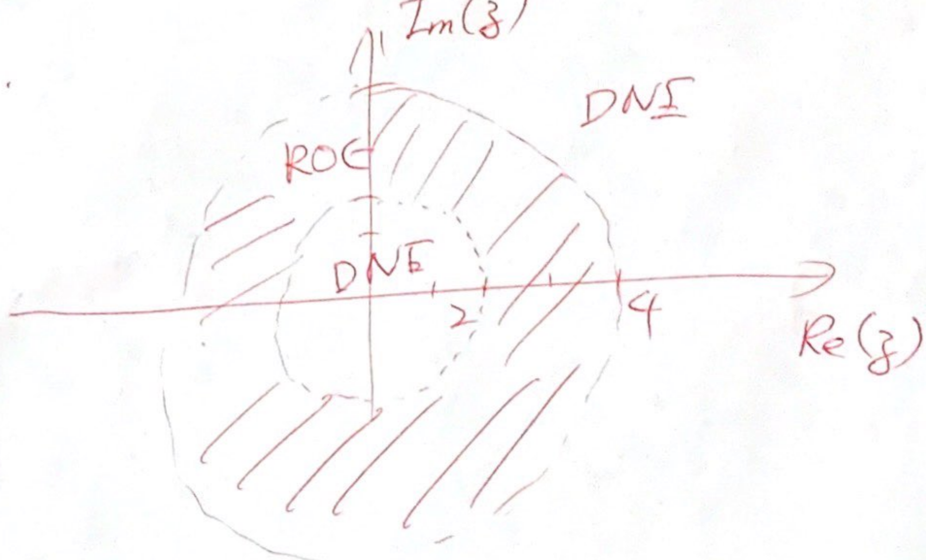
Q7.1 $X(z) = \sum_{n=0}^{\infty} 2^n z^{-n} + \sum_{n=-\infty}^{0} 4^n z^{-n}$

$$= \frac{1}{1-2z^{-1}} + \frac{1}{1+4^{-1}z}$$

iff $|2z^{-1}| < 1$ & $|4^{-1}z| < 1$

$\Leftrightarrow$ $2 < |z| < 4$ ROC

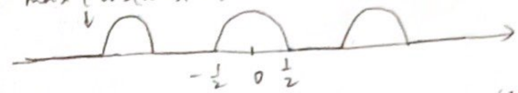
Q7.2.



Q7.3 No: ROC does not contain the unit cycle.

Question 8: [15%, Multiple-choice question] Consider two signals

$$h_1(t) = \int_{s=-t}^{t} \max(\cos(\pi s), 0)ds \tag{14}$$



and

$$h_2[n] = \prod_{k=1}^{50} \cos(0.2k\pi n) \tag{15}$$

$$= \cos(0.2\pi n) \cdot \cos(0.4\pi n) \cdot \cos(0.6\pi n) \cdot \cdots \cos(10\pi n) \tag{16}$$

Hint 1: The notation $\max(x, 0)$ means "taking the maximum." For example, $\max(\pi, 0) = \pi$ and $\max(-\sqrt{2}, 0) = 0$.

Hint 2: The notation $\prod_k$ means "product." For example, $\prod_{k=1}^{4}(2k+1) = 3 \cdot 5 \cdot 7 \cdot 9 = 945$. For comparison, $\sum_k$ means "sum." For example, $\sum_{k=1}^{4}(2k+1) = 3+5+7+9 = 24$.

1. [1.25%] Is $h_1(t)$ periodic? No , It is increasing .
2. [1.25%] Is $h_2[n]$ periodic? Yes, $\cos(0.2k\pi n)$ has common period 10.
3. [1.25%] Is $h_1(t)$ even or odd or neither? Odd , $h_1(-t) = \int_{s=t}^{-t} \max(\cos(\pi s), 0) ds = -\int_{s=-t}^{t} \max(\cos(\pi s), 0) ds = -h_1(t)$
4. [1.25%] Is $h_2[n]$ even or odd or neither? Even , $h_2[-n] = (-1)^{50} \prod_{k=1}^{50} \cos(0.2k\pi n) = h_2[n]$
5. [1.25%] Is $h_1(t)$ of finite power? No.
6. [1.25%] Is $h_2[n]$ of finite power? Yes. $|\cos(0.2\pi n)| < 1$, $h_2[n] = \left|\prod_{k=1}^{50} \cos(0.2k\pi n)\right|^2 < 1$ for $\forall n$.

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless? No
2. [1.25%] Is System 2 memoryless? No
3. [1.25%] Is System 1 causal? No $h_1(t) > 0$ for some $t < 0$.
4. [1.25%] Is System 2 causal? No
5. [1.25%] Is System 1 stable? No
6. [1.25%] Is System 2 stable? No