

Question 1: [12%, Work-out question, Learning Objectives 4, 5] Consider the following continuous-time periodic signal $x(t)$:

$$x(t) = \begin{cases} |t| & \text{if } -3 \leq t \leq 3 \\ 3 & \text{if } 3 < t \leq 6 \\ \text{periodic with period 9} \end{cases} \quad (1)$$

Let a_k denote the Continuous-Time Fourier Series coefficients of $x(t)$.

Consider another signal $y(t)$:

$$y(t) = \begin{cases} -1 & \text{if } 0 < t \leq 3 \\ 1 & \text{if } 3 < t \leq 6 \\ 0 & \text{if } 6 < t \leq 9 \\ \text{periodic with period 9} \end{cases} \quad (2)$$

Let b_k denote the Continuous-Time Fourier Series coefficients of $y(t)$.

- [12%] Express the value of b_k in terms of a_k .

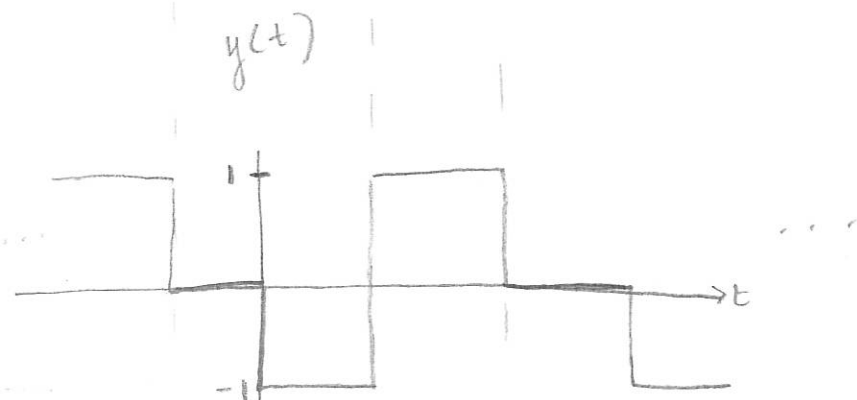
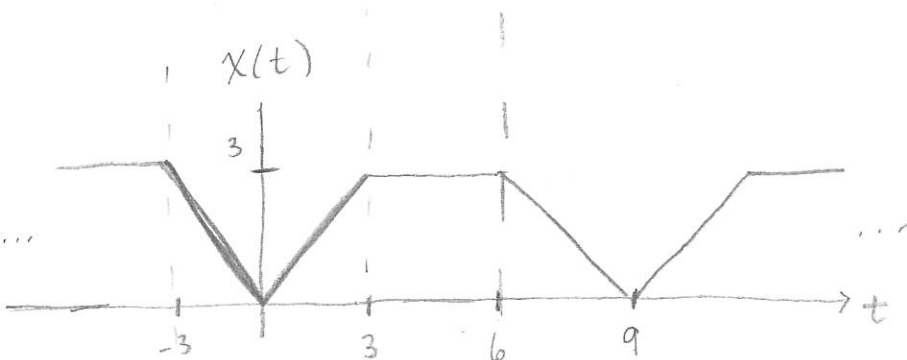
Hint 1: It will be useful if you can plot $x(t)$ and $y(t)$ first.

Hint 2: You may need to combine more than one properties.

time shift
+ derivative

$$x(t-t_0) \rightarrow a_k e^{-jk\omega_0 t_0}$$

$$\frac{d}{dt} x(t) \rightarrow jk\omega_0 a_k$$



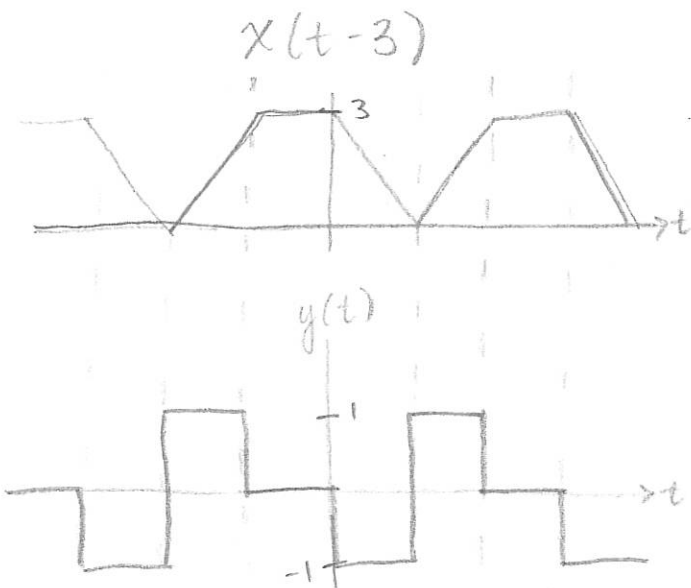
$$y(t) = \frac{d}{dt} x(t-3)$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{9}$$

$$b_k = jk\omega_0 a_k e^{-jk\omega_0 t_0}$$

$$b_k = jk \frac{2\pi}{9} a_k e^{-jk \frac{2\pi}{9} \cdot 3}$$

$$b_k = \frac{2\pi j k}{9} a_k e^{-jk \frac{2\pi}{3}}$$



Question 2: [20%, Work-out question, Learning Objectives 2, 3, 4, and 5] Consider the following discrete-time signal:

$$x[n] = e^{j0.1\pi n} + e^{j0.2\pi n} \quad (3)$$

and a DT-LTI system with the following impulse response:

$$h[n] = \begin{cases} e^{-n} & \text{if } 0 \leq n \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

1. [8%] Find the DTFS of $x[n]$.
2. [12%] Let $y[n]$ denote the output of this LTI system when the input is $x[n]$. Find the DTFS of $y[n]$.

Hint 1: You may need the following formula:

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r} \text{ if } |r| < 1. \quad (5)$$

Hint 2: If you do not know the answer to the previous subquestion, you can assume the following:

- (a) The period of $x[n]$ is 100.
- (b) The DTFS coefficients of $x[n]$ are

$$a_k = \begin{cases} k & \text{if } 1 \leq k \leq 20 \\ 0 & \text{if } 21 \leq k \leq 100 \end{cases} \quad (6)$$

You will receive full mark if your answer is correct. However, please indicate explicitly whether you are solving the original question or you are solving the alternative question.

$$1. \quad N = \text{LCM} \left(\frac{2\pi}{0.1\pi}, \frac{2\pi}{0.2\pi} \right) = \text{LCM}(20, 10) \\ = 20$$

$$\omega_0 = \frac{2\pi}{20} = 0.1\pi$$

$$a_k = \begin{cases} 1, & k = 1, 2 \\ 0, & \text{else} \end{cases}$$

$$2. h[n] = e^{-n} u[n]$$

$$H(j\omega) = \sum_{n=0}^{\infty} e^{-n} e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(e^{-(1+j\omega)} \right)^n$$

$$= \frac{1}{1 - e^{-(1+j\omega)}}$$

$$b_k = H(j\omega) \cdot a_k$$

$$b_1 = H(j0.1\pi) \cdot a_1 \quad b_2 = H(j0.2\pi) \cdot a_2$$

$$b_1 = \frac{1}{1 - e^{-(1+j0.1\pi)}} \quad b_2 = \frac{1}{1 - e^{-(1+j0.2\pi)}}$$

($b_k = 0$ for
all other k)

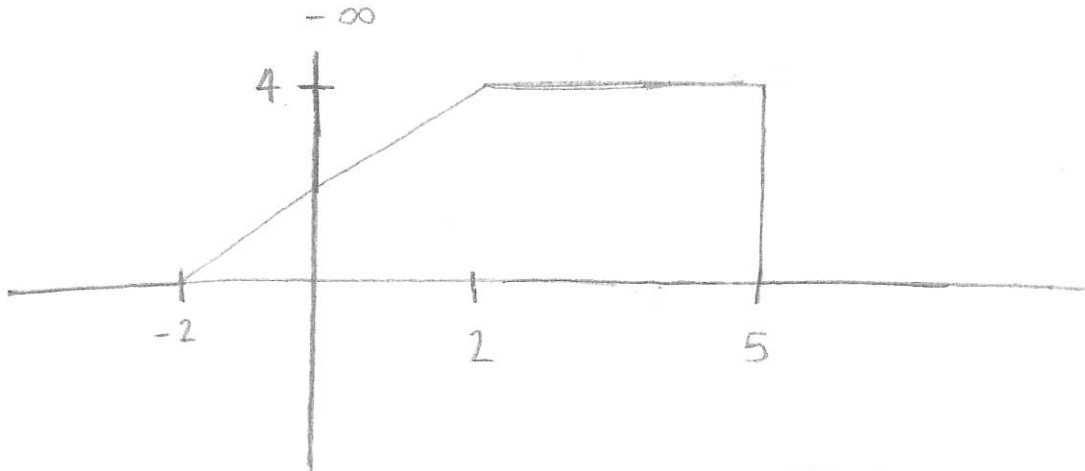
Question 3: [14%, Work-out question, Learning Objectives 4 and 5] Consider a continuous time signal $x(t)$:

$$x(t) = \begin{cases} 2+t & \text{if } -2 \leq t < 2 \\ 4 & \text{if } 2 \leq t < 5 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

Let $X(j\omega)$ denote the CTFT of $x(t)$.

- [6%] Find the value of $X(j0)$.
- [8%] Find the value of $\int_{-\infty}^{\infty} X(j\omega) e^{j2\omega} d\omega$.

$$1. X(j0) = \int_{-\infty}^{\infty} x(t) e^{-j \cdot 0 \cdot t} dt$$



$$X(j0) = \frac{1}{2}(4)(4) + 3(4) = \boxed{20}$$

2. Inverse CTFT:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

$$x(2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j2\omega} d\omega$$

$$x(2) = 4$$

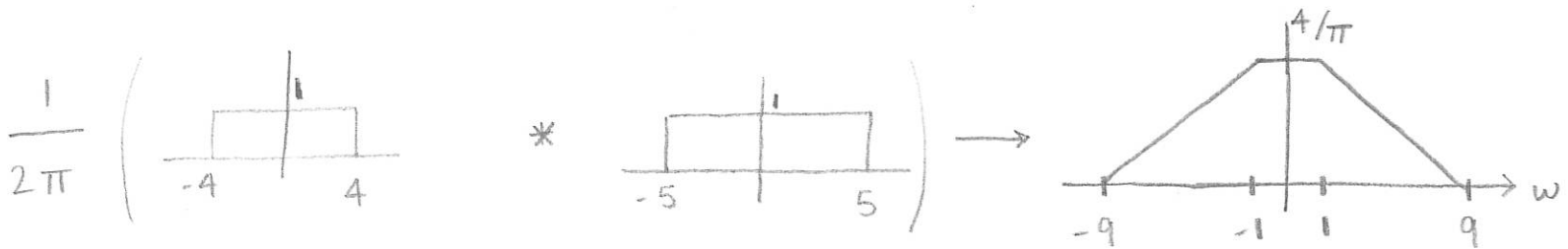
$$\int_{-\infty}^{\infty} X(j\omega) e^{j2\omega} d\omega = 2\pi x(2) = \boxed{8\pi}$$

Question 4: [15%, Work-out question, Learning Objectives 2, 3, 4 and 5] Consider the following signal $x(t)$

$$x(t) = \left(\frac{\sin(4t) \sin(5t)}{(\pi t)^2} \right) * \frac{\sin(6t)}{\pi t} \quad (8)$$

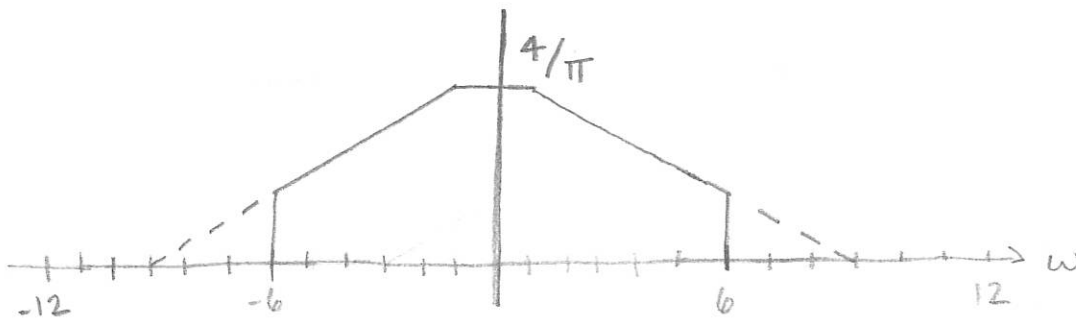
where the notation "*" means convolution. Find the corresponding Fourier transform $X(j\omega)$ and plot it for the range of $-12 < \omega < 12$.

$$X(t) = \left(\left(\frac{\sin(4t)}{\pi t} \right) \left(\frac{\sin(5t)}{\pi t} \right) \right) * \frac{\sin(6t)}{\pi t}$$



$X(j\omega)$

(LPF w/ cutoff frequency $\omega = 6$)



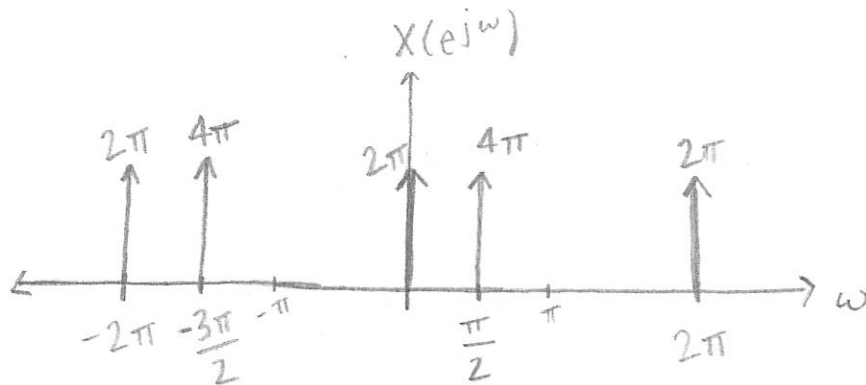
Question 5: [16%, Work-out question, Learning Objectives 4, 5, and 6] Consider a discrete time signal

$$x[n] = \sum_{k=0}^1 (k+1)e^{j2.5k\pi n} \quad (9)$$

Find the corresponding DTFT $X(e^{j\omega})$ and plot it for the range $-2\pi < \omega < 2\pi$.

$$\begin{aligned} x[n] &= (1)e^{j2.5(0)\pi n} + 2e^{j2.5\pi n(1)} \\ &= 1 + 2e^{j2.5\pi n} \end{aligned}$$

$$X(j\omega) = 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - 2\pi l) + (2)2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - 2.5\pi - 2\pi l)$$



Question 6: [23%, Learning Objectives 3, 4, 5, and 6] Consider the following AM transmission system. The input signal is $x(t) = \frac{\sin(t)}{\pi t}$. We first multiply $x(t)$ by $\cos(4t)$. That is,

$$y(t) = x(t) \cdot \cos(4t).$$

The transmitter then transmits signal $y(t)$ through the antenna.

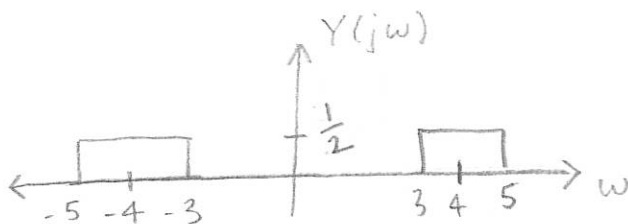
At the receiver side, we first multiply $y(t)$ by $2 \sin(4t)$. That is $z(t) = y(t) \cdot 2 \sin(4t)$ and then pass $z(t)$ through a low pass filter with cutoff frequency $W = 7.5$ rad/sec. Denote the final output by $w(t) = z(t) * h_{LPF}(t)$.

- [18%] Plot the CTFT $W(j\omega)$ of $w(t)$ for the range of $-10 < \omega < 10$.
- [5%] Find the expression of $w(t)$.

Hint 1: This is a more advanced question. Please work on it after you have finished other questions.

Hint 2: If you do not know how to solve this question, you can write down the transmitter and receiver diagrams of AM-DSB when (i) filtering a radio signal to make it of bandwidth 10k Hz; (ii) then sending the filtered signal using carrier frequency 950kHz; and (iii) the receiver needs to demodulate the original signal. You need to carefully mark all the frequency parameters used in your scheme with the correct unit. If your answer is correct, you will receive 13 points.

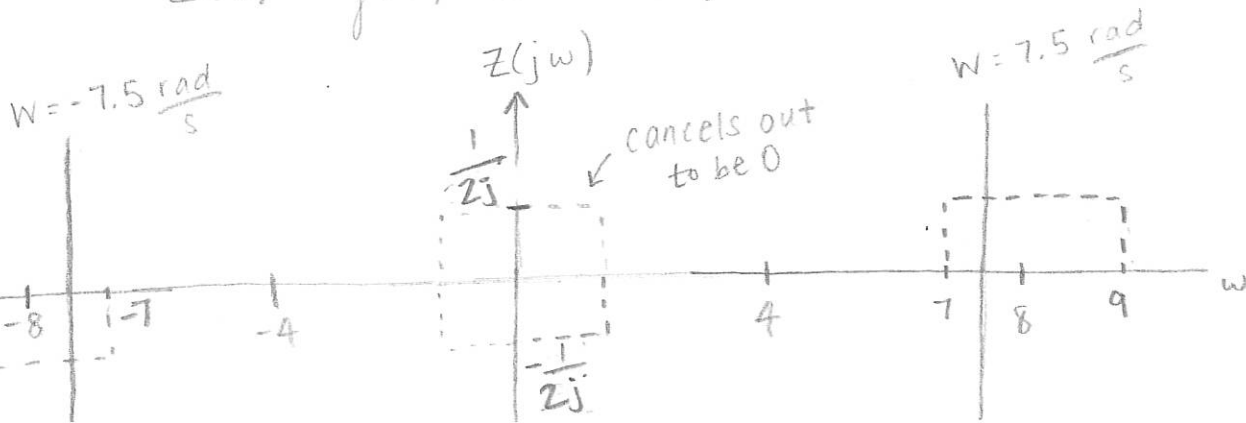
$$1. \quad y(t) = \left(\frac{\sin(t)}{\pi t} \right) \cos(4t)$$

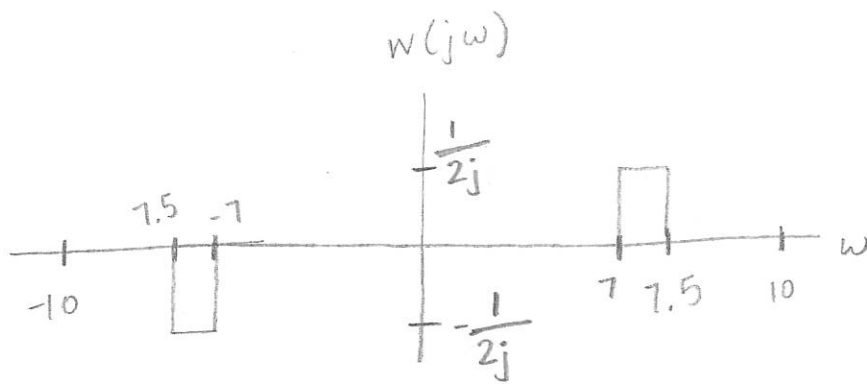


$$\frac{\sin Wt}{\pi t} \xrightarrow{\mathcal{F}} \begin{cases} 1, & |\omega| < W \\ 0, & |\omega| > W \end{cases}$$

$$x(t)y(t) \xrightarrow{\mathcal{F}} \frac{1}{2\pi} (x(t) * y(t))$$

$$z(t) = y(t) \cdot 2 \sin(4t)$$





2. $W(j\omega)$ is a rect function with a width of 0.5,
convolved with a sine wave with $\omega_0 = 7.25$

$$w(t) = \left(\frac{\sin(t/4)}{\pi t} \right) \sin(7.25t)$$

Discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \quad (1)$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n} \quad (2)$$

Continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \quad (3)$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt \quad (4)$$

Continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \quad (5)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad (6)$$

Discrete-time Fourier transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(j\omega) e^{j\omega n} d\omega \quad (7)$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \quad (8)$$

Laplace transform

$$x(t) = \frac{1}{2\pi} e^{\sigma t} \int_{-\infty}^{\infty} X(\sigma + j\omega) e^{j\omega t} d\omega \quad (9)$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \quad (10)$$

Z transform

$$x[n] = r^n \mathcal{F}^{-1}(X(re^{j\omega})) \quad (11)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \quad (12)$$

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$x(t)$ } Periodic with period T and $y(t)$ } fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} x(t) = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a_{-k}^*
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t - \tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk \frac{2\pi}{T} a_k$
Integration		$\int_{-\infty}^t x(t) dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a_{-k}^* \\ \Re\{a_k\} = \Re\{a_{-k}\} \\ \Im\{a_k\} = -\Im\{a_{-k}\} \\ a_k = a_{-k} \\ \angle a_k = -\angle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\begin{cases} \Re\{a_k\} \\ \Im\{a_k\} \end{cases}$
Parseval's Relation for Periodic Signals			
$\frac{1}{T} \int_T x(t) ^2 dt = \sum_{k=-\infty}^{+\infty} a_k ^2$			

three examples, we illustrate this. The last example in this section then demonstrates how properties of a signal can be used to characterize the signal in great detail.

Example 3.6

Consider the signal $g(t)$ with a fundamental period of 4, shown in Figure 3.10. We could determine the Fourier series representation of $g(t)$ directly from the analysis equation (3.39). Instead, we will use the relationship of $g(t)$ to the symmetric periodic square wave $x(t)$ in Example 3.5. Referring to that example, we see that, with $T = 4$ and $T_1 = 1$,

$$g(t) = x(t - 1) - 1/2. \quad (3.10)$$

Thus, in general, *none* of the finite partial sums in eq. (3.52) yield the exact values of $x(t)$, and convergence issues, such as those considered in Section 3.4, arise as we consider the problem of evaluating the limit as the number of terms approaches infinity.

3.7 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

There are strong similarities between the properties of discrete-time and continuous-time Fourier series. This can be readily seen by comparing the discrete-time Fourier series properties summarized in Table 3.2 with their continuous-time counterparts in Table 3.1.

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$x[n]$ } Periodic with period N and $y[n]$ } fundamental frequency $\omega_0 = 2\pi/N$	a_k } Periodic with b_k } period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n} x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a_{-k}^*
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m} a_k$ (viewed as periodic with period mN)
Periodic Convolution	$\sum_{r=(N)} x[r]y[n-r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=(N)} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^n x[k]$ (finite valued and periodic only) (if $a_0 = 0$)	$\left(\frac{1}{1 - e^{-jk(2\pi/N)}}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a_{-k}^* \\ \Re\{a_k\} = \Re\{a_{-k}\} \\ \Im\{a_k\} = -\Im\{a_{-k}\} \\ a_k = a_{-k} \\ \angle a_k = -\angle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{O}\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\begin{cases} \Re\{a_k\} \\ j\Im\{a_k\} \end{cases}$
Parseval's Relation for Periodic Signals		
$\frac{1}{N} \sum_{n=(N)} x[n] ^2 = \sum_{k=(N)} a_k ^2$		

f eqs. periodic h $M = 1; = 4.$
sequence in (3.106), the ns, we have
(3.10)
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4.6 TABLES OF FOURIER PROPERTIES AND OF BASIC FOURIER TRANSFORM PAIRS

In the preceding sections and in the problems at the end of the chapter, we have considered some of the important properties of the Fourier transform. These are summarized in Table 4.1, in which we have also indicated the section of this chapter in which each property has been discussed.

In Table 4.2, we have assembled a list of many of the basic and important Fourier transform pairs. We will encounter many of these repeatedly as we apply the tools of

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$

4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t} x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta)Y(j(\omega - \theta))d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt} x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^t x(t)dt$	$\frac{1}{j\omega} X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j \frac{d}{d\omega} X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \text{Re}\{X(j\omega)\} = \text{Re}\{X(-j\omega)\} \\ \text{Im}\{X(j\omega)\} = -\text{Im}\{X(-j\omega)\} \\ X(j\omega) = X(-j\omega) \\ \angle X(j\omega) = -\angle X(-j\omega) \end{cases}$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{O}\{x(t)\}$ [$x(t)$ real]	$\text{Re}\{X(j\omega)\}$ $j\text{Im}\{X(j\omega)\}$

4.3.7	Parseval's Relation for Aperiodic Signals		
		$\int_{-\infty}^{+\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) ^2 d\omega$	

FORM PAIRS

we have consid-
re summarized in
which each prop-
important Fourier
apply the tools of

transform

(ω)

(θ) $d\theta$

(0) $\delta(\omega)$

($j\omega$)

$\Re\{X(-j\omega)\}$

$-\Im\{X(-j\omega)\}$

($j\omega$)

$\Re\{X(-j\omega)\}$

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imaginary and odd

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi \delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$, otherwise
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, otherwise
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, otherwise
$x(t) = 1$	$2\pi \delta(\omega)$	$a_0 = 1$, $a_k = 0$, $k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
Periodic square wave		
$x(t) = \begin{cases} 1, & t < T_1 \\ 0, & T_1 < t \leq \frac{T}{2} \end{cases}$	$\sum_{k=-\infty}^{+\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
and $x(t + T) = x(t)$		
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k
$x(t) \begin{cases} 1, & t < T_1 \\ 0, & t > T_1 \end{cases}$	$\frac{2 \sin \omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \omega < W \\ 0, & \omega > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi \delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at} u(t)$, $\Re\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$te^{-at} u(t)$, $\Re\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!} e^{-at} u(t)$, $\Re\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—

TABLE 5.1 PROPERTIES OF THE DISCRETE-TIME FOURIER TRANSFORM

Section	Property	Aperiodic Signal	Fourier Transform
		$x[n]$	$X(e^{j\omega})$ periodic with
		$y[n]$	$Y(e^{j\omega})$ period 2π
5.3.2	Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
5.3.3	Time Shifting	$x[n - n_0]$	$e^{-j\omega n_0} X(e^{j\omega})$
5.3.3	Frequency Shifting	$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$
5.3.4	Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
5.3.6	Time Reversal	$x[-n]$	$X(e^{-j\omega})$
5.3.7	Time Expansion	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
5.4	Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
5.5	Multiplication	$x[n]y[n]$	$\frac{1}{2\pi} \int_{2\pi} X(e^{j\theta})Y(e^{j(\omega - \theta)})d\theta$
5.3.5	Differencing in Time	$x[n] - x[n - 1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
5.3.5	Accumulation	$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1 - e^{-j\omega}} X(e^{j\omega})$ $+ \pi X(e^{j0}) \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$
5.3.8	Differentiation in Frequency	$nx[n]$	$j \frac{dX(e^{j\omega})}{d\omega}$
5.3.4	Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \Re\{X(e^{j\omega})\} = \Re\{X(e^{-j\omega})\} \\ \Im\{X(e^{j\omega})\} = -\Im\{X(e^{-j\omega})\} \\ X(e^{j\omega}) = X(e^{-j\omega}) \\ \angle X(e^{j\omega}) = -\angle X(e^{-j\omega}) \end{cases}$
5.3.4	Symmetry for Real, Even Signals	$x[n]$ real and even	$X(e^{j\omega})$ real and even
5.3.4	Symmetry for Real, Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
5.3.4	Even-odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}\{x[n]\}$ [$x[n]$ real] $x_o[n] = \mathcal{O}\{x[n]\}$ [$x[n]$ real]	$\Re\{X(e^{j\omega})\}$ $j\Im\{X(e^{j\omega})\}$
5.3.9	Parseval's Relation for Aperiodic Signals	$\sum_{n=-\infty}^{+\infty} x[n] ^2 = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) ^2 d\omega$	

a duality relationship between the discrete-time Fourier transform and the continuous-time Fourier series. This relation is discussed in Section 5.7.2.

5.7.1 Duality in the Discrete-Time Fourier Series

Since the Fourier series coefficients a_k of a periodic signal $x[n]$ are themselves a periodic sequence, we can expand the sequence a_k in a Fourier series. The duality property for discrete-time Fourier series implies that the Fourier series coefficients for the periodic sequence a_k are the values of $(1/N)x[-n]$ (i.e., are proportional to the values of the original

TABLE 5.2 BASIC DISCRETE-TIME FOURIER TRANSFORM PAIRS

Signal	Fourier Transform	Fourier Series Coefficients (if periodic)
$\sum_{k=(N)} a_k e^{jk(2\pi N)n}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	a_k
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} 1, & k = m, m \pm N, m \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational \Rightarrow The signal is aperiodic
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2}, & k = \pm m, \pm m \pm N, \pm m \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational \Rightarrow The signal is aperiodic
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$ $a_k = \begin{cases} \frac{1}{2j}, & k = r, r \pm N, r \pm 2N, \dots \\ -\frac{1}{2j}, & k = -r, -r \pm N, -r \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational \Rightarrow The signal is aperiodic
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, & n \leq N_1 \\ 0, & N_1 < n \leq N/2 \end{cases}$ and $x[n + N] = x[n]$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N \sin[2\pi k/2N]}, k \neq 0, \pm N, \pm 2N, \dots$ $a_k = \frac{2N_1 + 1}{N}, k = 0, \pm N, \pm 2N, \dots$
$\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N}$ for all k
$a^n u[n], a < 1$	$\frac{1}{1 - ae^{-j\omega}}$	—
$x[n] = \begin{cases} 1, & n \leq N_1 \\ 0, & n > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$	—
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} \text{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \leq \omega \leq W \\ 0, & W < \omega \leq \pi \end{cases}$ $X(\omega)$ periodic with period 2π	—
$\delta[n]$	1	—
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi \delta(\omega - 2\pi k)$	—
$\delta[n - n_0]$	$e^{-j\omega n_0}$	—
$(n + 1)a^n u[n], a < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$	—
$\frac{(n + r - 1)!}{n!(r - 1)!} a^n u[n], a < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$	—