Final Exam of ECE301, Section 3 (CRN 17102-004)

8–10am, Wednesday, December 12, 2018, WTHR200 and WTHR320.

1. Please make sure that it is your name printed on the exam booklet. *Enter your student ID number, and signature in the space provided on this page,* **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out *questions. For multiple* choice questions, there is no need to justify your answers. You have ~~one hour~~ to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve. two hours
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

As a Boiler Maker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

Signature: Date:

Question 1: [20%, Work-out question] Amplitude Modulation

1. [1%] What does the acronym AM-DSB stand for? Double Side Band.
2. [1%] What is AM asynchronous demodulation? You can either describe how to implement an AM asynchronous demodulator or you can describe the difference between an asynchronous versus a synchronous demodulator. You will receive full points either way.

Envelope detector

Prof. Wang wanted to transmit an AM-SSB signal. To that end, he wrote the following MATLAB code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read two different .wav files
[x1, f_sample, N]=audioread('x1.wav');
x1=x1';
[x2, f_sample, N]=audioread('x2.wav');
x2=x2';

% Step 0: Initialize several parameters
W_1=????;
W_2=pi*3000;
W_3=pi*6000;
W_4=????;
W_5=????;
W_6=pi*12000;
W_7=pi*6000;

% Step 1: Make the signals band-limited.
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);

% Step 2: Multiply x1_new and x2_new with a sinusoidal wave.
x1_h=x1_new.*sin(W_2*t);
x2_h=x2_new.*sin(W_3*t);

% Step 3: Keep one of the two side bands
h_one=1/(pi*t).*(sin(W_4*t)-sin(W_5*t));
```

```
h_two=1/(pi*t).*(sin(W_6*t)-sin(W_7*t));
x1_sb=ece301conv(x1_h, h_one);
x2_sb=ece301conv(x2_h, h_two);

% Step 4: Create the transmitted signal
y=x1_sb+x2_sb;
audiowrite('y.wav', y, f_sample);
```

1500 Hz

3. [1.5%] What is the carrier frequency (Hz) of the signal x1_new?

4. [1.5%] For the second signal x2_new, is this AM-SSB transmitting an upper-side-band signal or a lower-side-band signal? USB

5. [1.5%] What should the values of W_4 and W_5 be in the MATLAB code, if we decide to use a lower-side-band transmission for the first signal x1_new? $W_4 = 3000\pi$ $W_5 = 0\pi$.

6. [1.5%] What should the values of W_4 and W_5 be in the MATLAB code, if we decide to use an upper-side-band transmission for the first signal x1_new? $W_4 = 6000\pi$ $W_5 = 3000\pi$

7. [2%] Continue from the previous sub-question. Suppose upper-side-band transmission is used for the first signal x1_new. To ensure that the receiver side can have the best possible quality, it is important for the transmitter to choose the largest W_1 value when possible. What is the largest W_1 value that can be used without significantly degrading the quality of any of the two transmitted signals?

$W_1 = 3000\pi$

Knowing that Prof. Wang decided to use an upper-side-band transmission for the first signal x1_new and he chose the W_1 value to be $W_1 = 2000 \times \pi$. He then used the code in the previous page to generate the "y.wav" file. A student tried to demodulate the output waveform "y.wav" by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[y, f_sample, N]=audioread('y.wav');
y=y';

% Initialize several parameters
W_8=????;
W_9=????;
W_10=????;


% Create a low-pass filter.
h_M=1/(pi*t).*(sin(W_8*t));

% demodulate signal 1
y1=2*y.*sin(pi*W_9*t);
x1_hat=ece301conv(y1,h_M);

sound(x1_hat,f_sample)

% demodulate signal 2
y2=2*y.*sin(pi*W_10*t);
x2_hat=ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```

$W_8 = 2000\pi$

8. [2%] Continue from the previous questions. What should the value of W_8 be in the MATLAB code? When answering this question, please assume that the first radio x1_new was transmitted using the *upper side-band* and $W_1 = 2000 \times \pi$.

9. [2%] The student did not know how to choose the W_9 value. Instead, he/she played with different nine choices: i.e., $W_9 = 0$, 1000π, 2000π, 3000π, 4000π, 5000π, 6000π, 7000π, or 8000π. One of them gave him/her perfect results. Namely, when playing x1_hat, the resulting sound is identical to playing the original signal x1_new.

$W_9 = 3000$

Question: Which choice of W_9 is the one that gave the perfect result?

10. [2%] The student did not know how to choose the W_{10} value. Instead, he/she played with different nine choices: i.e., $W_{10} = 0, 1000\pi, 2000\pi, 3000\pi, 4000\pi, 5000\pi, 6000\pi, 7000\pi$, or 8000π. Unfortunately, none of these nine choices gave him/her the perfect result when playing x2_hat. That is, when playing x2_hat, the quality of the resulting sound is always significantly worse than playing the original signal x2_new.

    Question: Please use one sentence to describe what kind of sound quality problem that the student is experiencing when playing x2_hat? One can hear a bit about $x_1(t)$

11. [3%] It turns out that some important commands are missing when the student wrote the demodulation MATLAB code.

    Question: What are the missing commands when demodulating x2_new? (Hint: If you do not know how to write the MATLAB code, you can also describe by words what "function block" is missing in the above MATLAB code. You will receive 2 points if your answer is correct.)

12. [1%] In addition to adding new commands, the W_{10} value also needs to be carefully chosen.

    Question: What is the value of W_{10} needed for perfect demodulation?

    Hint: If you do not know the answers of Q1.3 to Q1.12, please simply draw the AMSSB modulation and demodulation diagrams and mark carefully all the parameter values. You will receive 11 points for Q1.3 to Q1.12.

change

$y_2 = 2 * \text{ece301conv}\left(y, \frac{1}{\pi t}\left(\sin(9000\pi t) - \sin(6000\pi t)\right)\right)$

$\times \sin(6000\pi t)$

12. $W_{10} = 6000$.

Question 2: [9%, Work-out question]

1. [6%] Consider a discrete time signal $x[n]$

$$x[n] = \begin{cases} 3 & \text{if } n = 1 \text{ or } n = -1 \\ 0 & \text{if } n = 0 \text{ or } n = 2 \\ \text{periodic with period 4} & \end{cases} \tag{1}$$

Find the DTFS of $x[n]$ and plot the DTFS coefficients a_k for the range of $k = 0$ to 3.

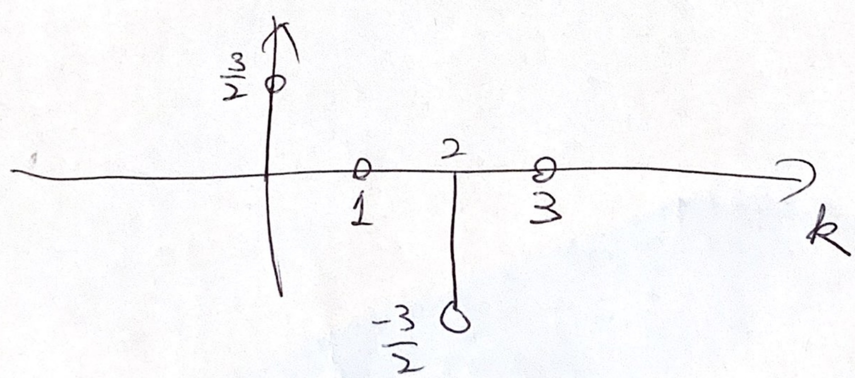
2. [3%] Find the DTFT of $x[n]$ and plot the DTFT $X(e^{j\omega})$ for the range of $-\overset{2{,}5}{\cancel{2}}\pi < \omega < \cancel{2}\pi$ 3.5

Hint: If you do not know the answer to the previous subquestion, you can assume $a_k = 2\sin(\frac{k\pi}{2})$. You will receive full credit if your answer is correct.

1.

$$a_k = \frac{1}{4}\left(3 \cdot e^{-jk\frac{2\pi}{4}\times 1} + 3\, e^{-jk\frac{2\pi}{4}\cdot(-1)}\right)$$

$$= \frac{3}{2}\cos\left(k \cdot \frac{\pi}{2}\right).$$



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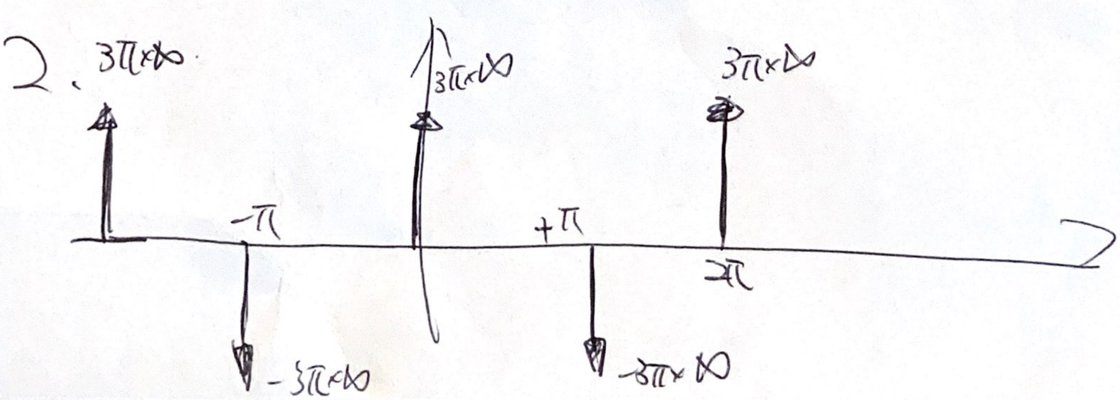


Question 3: [12%, Work-out question]

1. [2%] Suppose $w(t) = \sin(3t) + \cos(5t)$. Question: According to the *sampling theorem*, what is the smallest sampling frequency (Hz) needed in order to for perfectly reconstruct $w(t)$?

   $\frac{2\times5}{2\pi} = \frac{5}{\pi}$ Hz.

2. [1.5%] Consider the following continuous time signal

$$x(t) = \frac{2\sin(\pi t)}{\pi t} \tag{2}$$

   Plot $x(t)$ for the range of $-4 < t < 4$.

3. [1.5%] Plot the CTFT $X(j\omega)$ of $x(t)$ for the range of $-5\pi < \omega < 5\pi$.

4. [3%] If we perform impulse train sampling on $x(t)$ with the sampling frequency 2Hz and denote the impulse train sampled signal by $x_p(t)$. Plot its CTFT $X_p(j\omega)$ for the range of $-5\pi \leq \omega \leq 5\pi$.

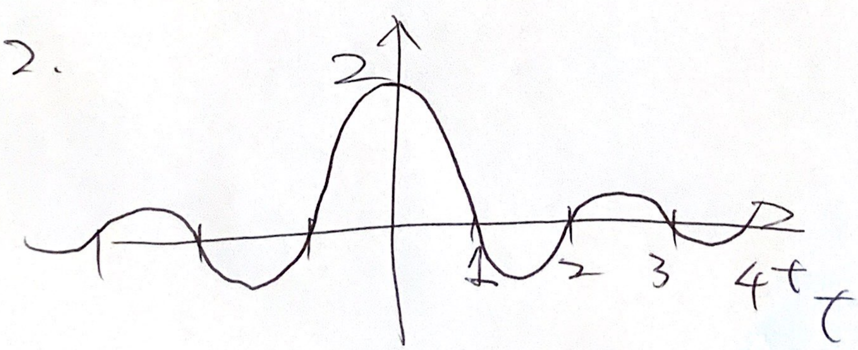
   Hint: If you do not know the answer to the previous sub-questions, please write down the relationship between $X(j\omega)$ and $X_p(j\omega)$. You will receive 1.5 points if your answer is correct.

5. [3%] We use $x_{\text{band}}(t)$ to represent the reconstructed signal using "band-limited reconstruction". Plot $x_{\text{band}}(t)$ for the range of $-4 < t < 4$.

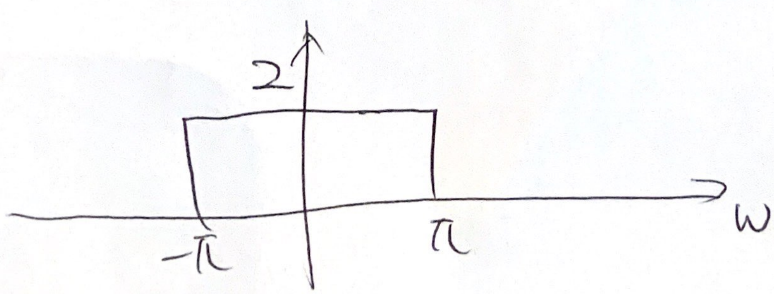
   Hint: if you do not know the answer of $x[n]$, you can assume that $x[n] = \delta[n-6] - \delta[n+6]$ and the sampling frequency is 2Hz. You will receive full points if your answer is correct.

6. [1%] Write down the exact expression of $x_{\text{band}}(t)$.

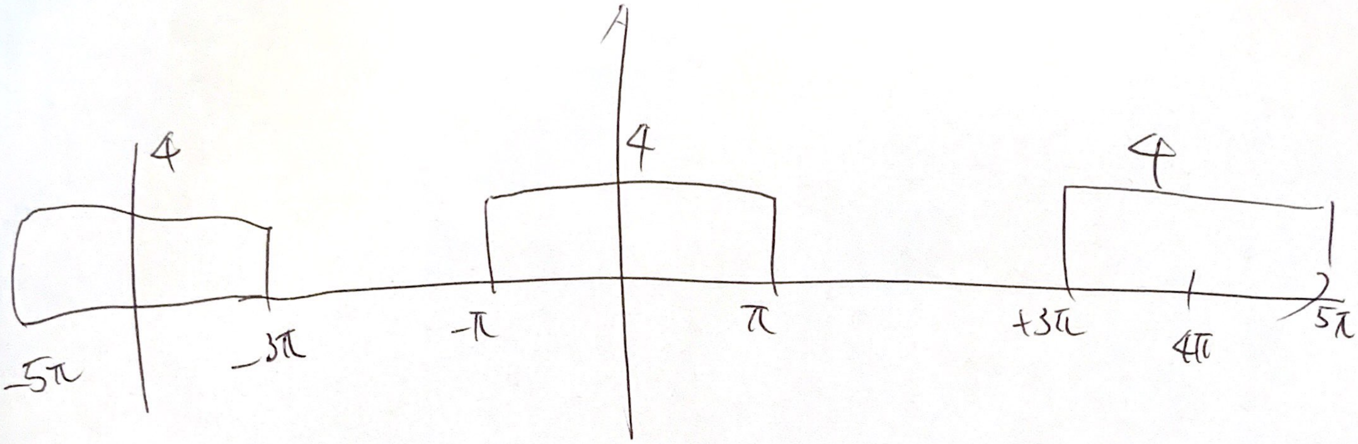
2.



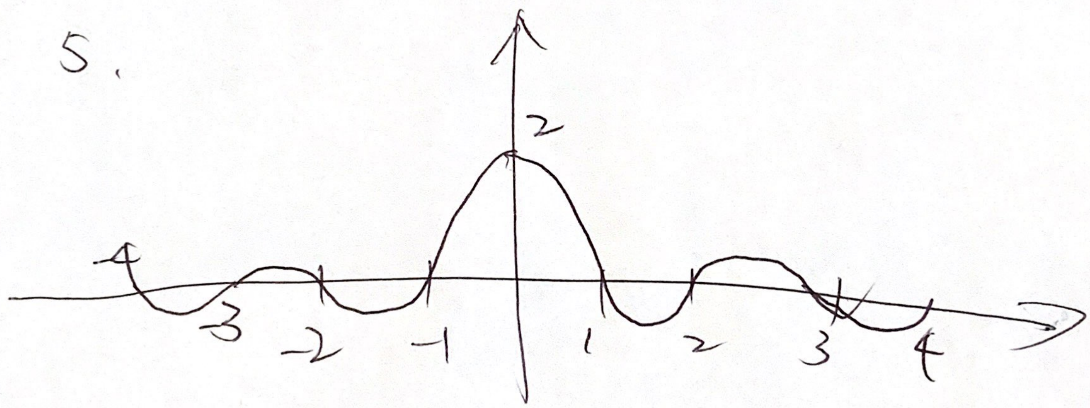
3.



4. $\omega_s = 4\pi$ $T = 0.5$ $\frac{1}{T} = 2$



5.



6. $\dfrac{2\sin(\pi t)}{\pi t}$

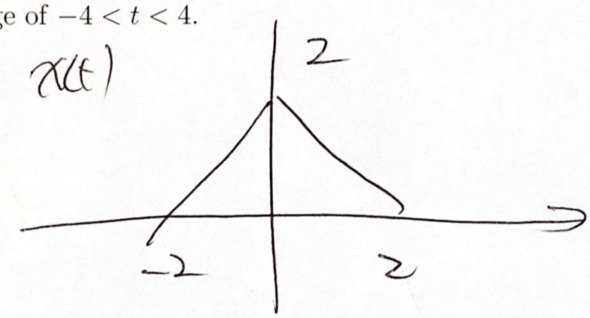
Question 4: [10%, Work-out question]

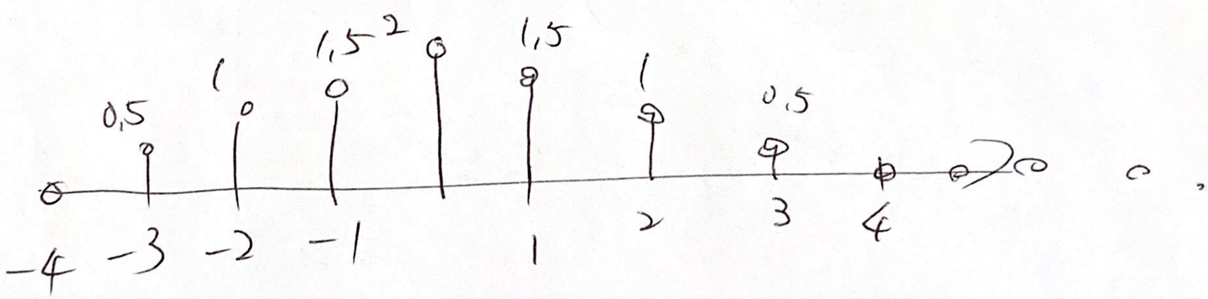
Consider a continuous time signal

$$x(t) = \begin{cases} t-2 & \text{if } 2 < t < 0 \\ 2-t & \text{if } 0 \le t < 2 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

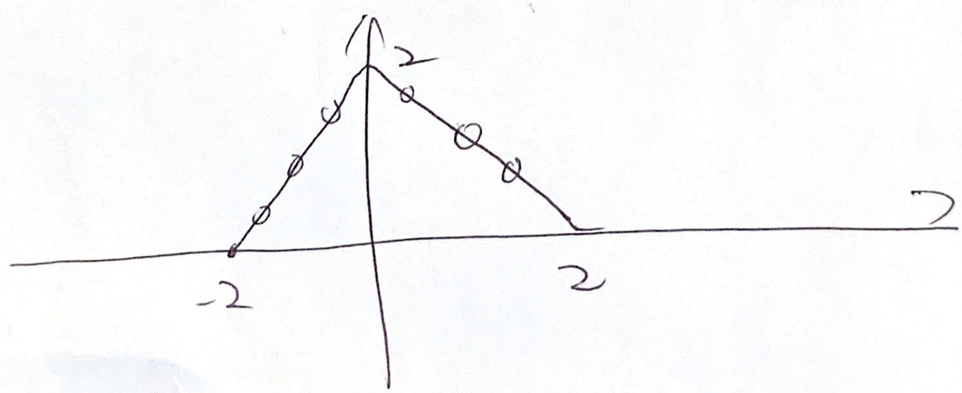
1. [3%] We sample $x(t)$ with the sampling frequency 2Hz and denote the sampled values by $x[n]$. Plot $x[n]$ for the range of $\cancel{-10}\ 5 \le n \le \cancel{10}\ 5$.
2. [2%] We use $x_{\text{lin}}(t)$ to represent the reconstructed signal using "linear interpolation". Plot $x_{\text{lin}}(t)$ for the range of $-4 < t < 4$.
3. [5%] Suppose we pass $x[n]$ through a discrete-time LTI system with $h[n] = \delta[n] + \delta[n-4]$. Denote the new discrete-time array by $y[n]$. We then again apply linear interpolation to $y[n]$ and construct the corresponding $y_{\text{lin}}(t)$. Plot $y_{\text{lin}}(t)$ for the range of $-4 < t < 4$.

1.

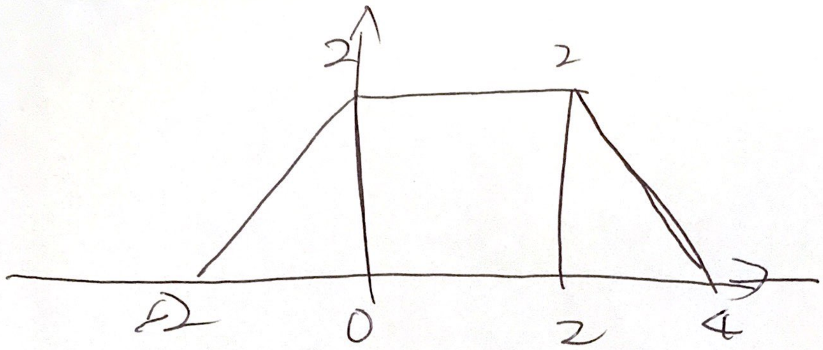




2.



3. $y_{lin}(t)$



Question 5: [10%, Work-out question]

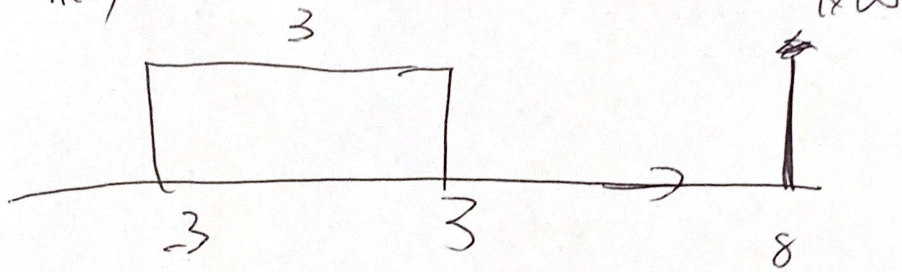
Consider the following continuous time signals

$$x(t) = \begin{cases} 2 & \text{if } -2 < t < 2 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

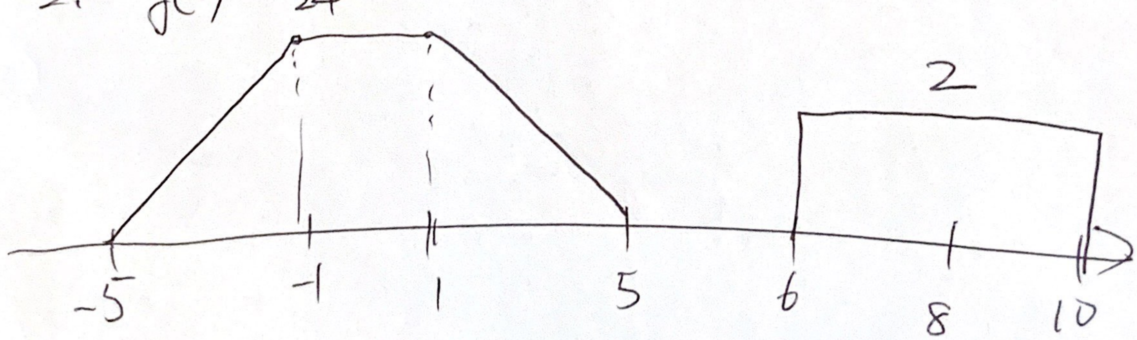
$$h(t) = \begin{cases} 3 & \text{if } -3 < t < 3 \\ \delta(t-8) & \text{if } 3 \le t < 10 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

1. [2%] Plot $h(t)$ for the range of $-10 < t < 10$.
2. [8%] Plot the signal $y(t) = x(t) * h(t)$ for the range of $-10 < t < 10$.

1. h(t)



2. y(t)



Question 6: [14%, Work-out question]
Consider the differential equation.

$$y(t) + 3\frac{d}{dt}y(t) + 2\frac{d^2}{dt^2}y(t) = 2x(t) \tag{6}$$

1. [7%] Find the impulse response $h(t)$ and the corresponding frequency response $H(j\omega)$.

2. [7%] Suppose the input is $x(t) = \sum_{k=1}^{6} e^{j(k-2)\pi t}$. Find the expression of $y(t)$.

   Hint: If you do not know the answer to the previous question, you can assume that $h(t) = \left(\frac{\sin(\pi t)}{\pi t}\right)^2$. You will receive full credit if your answer is correct.

1.

$$Y(j\omega)\left(1 + 3j\omega + 2(j\omega)^2\right) = 2X(j\omega)$$

$$H(j\omega) = \frac{2}{1+3(j\omega)+2(j\omega)^2} = \frac{4}{1+2(j\omega)} + \frac{-2}{1+(j\omega)}$$

$$h(t) = \qquad\qquad = \frac{2}{\frac{1}{2}+j\omega} + \frac{-2}{(1+j\omega)}$$

$$= 2e^{-\frac{1}{2}t}u(t) - 2e^{-t}u(t).$$

2.

$$y(t) = \sum_{k=1}^{6}\left(\frac{2}{\frac{1}{2}+j(k-2)\pi} - \frac{2}{1+j(k-2)\pi}\right) \cdot e^{j(k-2)\pi t}$$

#

Question 7: [10%, Work-out question]

1. [5%] Consider the following discrete time signal.

$$y[n] = \begin{cases} 1 & \text{if } n \geq 3 \\ 0 & \text{otherwise} \end{cases}. \tag{7}$$

   Derive the Z-transform expression of $y[n]$ and *derive* the corresponding region of convergence. Please carefully write down your reasonings. If you use the table without explanation, then you will receive 3 points instead.

   Hint: You may need the following formulas

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r} \text{ if } |r| < 1 \tag{8}$$

$$\sum_{k=1}^{\infty} akr^{k-1} = \frac{a}{(1-r)^2} \text{ if } |r| < 1 \tag{9}$$

2. [2%] Continue from the previous question. Plot the region of convergence of $y[n]$ in the complex plane. (I.e., the horizontal axis is $\mathrm{Re}(z)$ and the vertical axis is $\mathrm{Im}(z)$.)

3. [3%] Consider the following discrete time signal.

$$x[n] = \begin{cases} 2 & \text{if } 10 \leq n \geq 59 \\ 1 & \text{if } 60 \leq n \geq 79 \\ 0 & \text{if } 80 \leq n \geq 109 \\ \text{periodic with period 100} & \end{cases}. \tag{10}$$

   Let a_k denote the DTFS of $x[n]$. Find the value of $\sum_{k=0}^{99} a_k(-1)^k$

Q $Y(z) = \sum_{n=-\infty}^{\infty} y[n] z^{-n}$

$= \sum_{n=3}^{\infty} z^{-n}$

$\hookrightarrow \dfrac{z^{-3}}{1-z^{-1}}$ ※

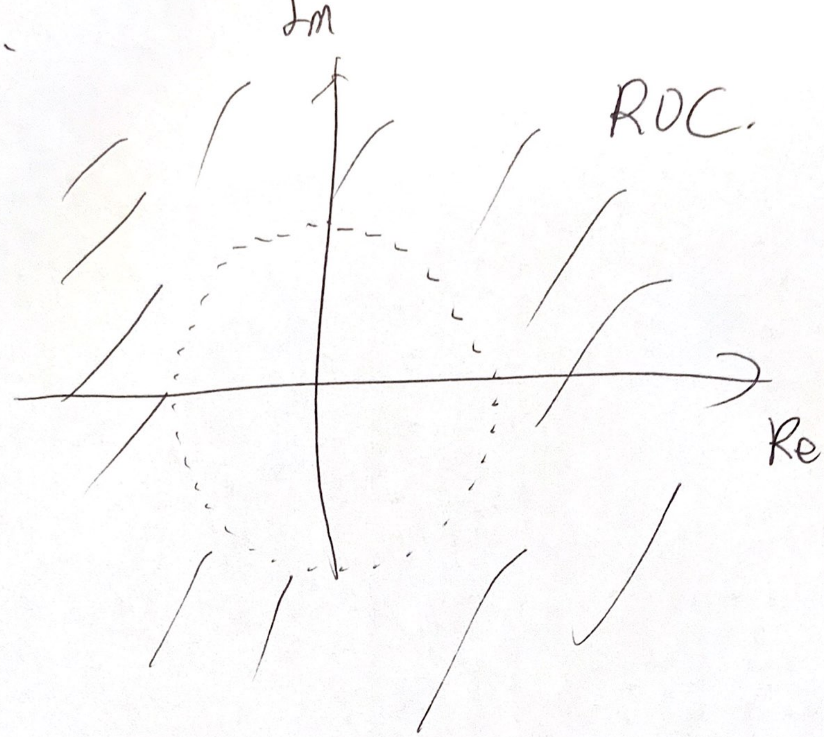
converges if

ROC.

$|z^{-1}| < 1.$

$\Leftrightarrow |z| > 1.$

2.



3. $\sum_{k=0}^{99} a_k (-1)^k = \sum_{k=0}^{99} a_k e^{+jk\frac{2\pi}{100}\times 50}$

$= x[50] = 2$ #

Question 8: [15%, Multiple-choice question] Consider two signals

$$h_1(t) = \begin{cases} \sin(2t) & \text{if } \sin(t) > 0 \\ 0 & \text{if } \sin(t) \leq 0 \end{cases} \tag{11}$$

$\left(\frac{1-j}{2}\right)^n$

and

$$h_2[n] = (1 + j)^{\overset{-n}{n}} \sin(0.75\pi n) + \left(\cancel{\frac{2}{1+j}}\right)^n \sin(0.75\pi|n|) \tag{12}$$

Hint: you should treat $h_1(t)$ as $|e^{x(t)}|$ where $x(t) = e^{jt}$. $h_1(t)$ $h_2[n]$

1. [1.25%] Is $h_1(t)$ periodic? ~~No~~ Yes
2. [1.25%] Is $h_2[n]$ periodic? No
3. [1.25%] Is $h_1(t)$ even or odd or neither? Neither
4. [1.25%] Is $h_2[n]$ even or odd or neither? Neither
5. [1.25%] Is $h_1(t)$ of finite energy? No
6. [1.25%] Is $h_2[n]$ of finite energy? ~~No~~ Yes.

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless? No
2. [1.25%] Is System 2 memoryless? No
3. [1.25%] Is System 1 causal? No
4. [1.25%] Is System 2 causal? Yes
5. [1.25%] Is System 1 stable? No
6. [1.25%] Is System 2 stable? ~~No~~. ~~No~~ Yes