1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**

2. This is a closed book exam.

3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.

4. Use the back of each page for rough work.

5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

I certify that I have neither given nor received unauthorized aid on this exam.

Signature: Date:

Question 1: [22%, Work-out question, Learning Objectives 4, 5] Consider the following discrete-time periodic signal $x[n]$:

$$x[n] = \begin{cases} |n| & \text{if } -3 \leq n \leq 4 \\ \text{periodic with period 8} \end{cases} \tag{1}$$

Let a_k denote the Discrete-Time Fourier Series coefficients of $x[n]$.

1. [6%] Find the value of a_0.

2. [7%] Find the value of $\sum_{k=0}^{7} j^k a_k$. Hint: The Fourier series synthesis formula will be useful.

   Also define $y[n] = (1 - e^{j\frac{3\pi}{4}n})$ and let $z[n] = y[n] \cdot x[n]$.

3. [9%] Denote the Discrete-Time Fourier Series coefficients of $z[n]$ by c_k. Find the expression of c_2 in terms of a_k. Namely, you can assume a_k is known completely and write down your expression of c_7 in terms of a_k.

Question 2: [18%, Work-out question, Learning Objective 4] Consider the following differential CT signal:

$$y(t + 1.5) = \frac{d}{dt}x(t) + 0.5\frac{d^2}{dt^2}y(t). \tag{2}$$

1. [6%] Find the *frequency response* $H(j\omega)$ of the above system.

2. [3%] Suppose the input signal is $x(t) = \cos(2t)$. Find the Fourier series representation of $x(t)$.

3. [9%] Suppose the input signal is $x(t) = \cos(2t)$. Find the Fourier series representation of the output $y(t)$.

   Hint: if you do not know the $H(j\omega)$ in the previous question, you can assume $H(j\omega) = \frac{e^{-j\omega/2}\omega^4}{1+3\omega^2}$. You will receive full credit if your answer is correct.

Question 3: [15%, Work-out question, Learning Objectives 1, 2, 3, 4, 5, 6] Consider a continuous time signal $x(t)$:

$$x(t) \begin{cases} 3 & \text{if } 0 \leq t < 2 \\ 0 & \text{if } 2 \leq t < 5 \\ \text{periodic with period 5} \end{cases} \tag{3}$$

There is a typo. It should be "x(t)=".

The equality sign "=" is missing.

1. [3%] Plot $x(t)$ for the range of $-8 < t < 8$.

2. [12%] Find the Fourier transform $X(j\omega)$. Hint: If you do not know how to solve this question, you can find the CTFS of $x(t)$ instead. You will receive 8 points if your answer is correct.

Question 4: [13%, Work-out question, Learning Objectives 4, 5] Consider the following signal $x(t)$

$$x(t) = \frac{(\sin(4t))^2}{3t^2} \cos(4t) \qquad (4)$$

Find the corresponding Fourier transform $X(j\omega)$ and plot it for the range of $-12 < \omega <$ 12.

Question 5: [12%, Work-out question, Learning Objectives 4, 5, and 6] Consider a discrete time signal

$$x[n] = 2^{-|n|}e^{j3n}$$

Find the corresponding Fourier transform $X(e^{j\omega})$.

There is a typo. The formula works if |r|<1. That is, the "not-equal" sign should be replaced by "<".

Hint: You may need the following formula: If $|r| \neq 1$, then

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}. \tag{6}$$

Question 6: [20%, Learning Objectives 2, 3, 4, and 5] Consider the following AM transmission system. The input signal is $x(t) = \cos(5.5t)$. We first multiply $x(t)$ by $\cos(3t)$. That is,

$$y(t) = x(t) \cdot \cos(3t).$$

The transmitter then transmits signal $y(t)$ through the antenna.

At the receiver side, we first multiply $y(t)$ by $2\cos(3t)$. That is $z(t) = y(t) \cdot 2\cos(3t)$ and then pass $z(t)$ through a low pass filter with cutoff frequency $W = 1.5$ rad/sec. Denote the final output by $w(t) = z(t) * h_{\mathrm{LPF}}(t)$.

Find the expression of $w(t)$.

Hint: If you do not know how to solve this question, you can assume $x(t) = \cos(0.001t)$ instead, and solve the following alternative question: Let $v(t)$ denote the output of the asynchronous AM demodulation. Plot $v(t)$ for the range $-5000\pi < t < 5000\pi$. If your answer is correct, you will receive 14 points.

Discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \tag{1}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n} \tag{2}$$

Continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \tag{3}$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt \tag{4}$$

Continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \tag{5}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{6}$$

Discrete-time Fourier transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(j\omega) e^{j\omega n} d\omega \tag{7}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \tag{8}$$

Laplace transform

$$x(t) = \frac{1}{2\pi} e^{\sigma t} \int_{-\infty}^{\infty} X(\sigma + j\omega) e^{j\omega t} d\omega \tag{9}$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \tag{10}$$

Z transform

$$x[n] = r^n \mathcal{F}^{-1}(X(re^{j\omega})) \tag{11}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{12}$$

TABLE 3.1   PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{array}{l}x(t)\\y(t)\end{array}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t}x(t) = e^{jM(2\pi/T)t}x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\displaystyle\int_T x(\tau)y(t - \tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\displaystyle\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\dfrac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\dfrac{2\pi}{T}a_k$
Integration		$\displaystyle\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\dfrac{1}{jk\omega_0}\right)a_k = \left(\dfrac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases}a_k = a^*_{-k}\\ \Re e\{a_k\} = \Re e\{a_{-k}\}\\ \Im m\{a_k\} = -\Im m\{a_{-k}\}\\ \|a_k\| = \|a_{-k}\|\\ \sphericalangle a_k = -\sphericalangle a_{-k}\end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases}x_e(t) = \mathcal{E}v\{x(t)\} & [x(t)\text{ real}]\\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t)\text{ real}]\end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

three examples, we illustrate this. The last example in this section then demonstrates how properties of a signal can be used to characterize the signal in great detail.

Example 3.6

Consider the signal $g(t)$ with a fundamental period of 4, shown in Figure 3.10. We could determine the Fourier series representation of $g(t)$ directly from the analysis equation (3.39). Instead, we will use the relationship of $g(t)$ to the symmetric periodic square wave $x(t)$ in Example 3.5. Referring to that example, we see that, with $T = 4$ and $T_1 = 1$,

$$g(t) = x(t - 1) - 1/2.$$

Thus, in general, *none* of the finite partial sums in eq. (3.52) yield the exact values of $x(t)$, and convergence issues, such as those considered in Section 3.4, arise as we consider the problem of evaluating the limit as the number of terms approaches infinity.

3.7 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

There are strong similarities between the properties of discrete-time and continuous-time Fourier series. This can be readily seen by comparing the discrete-time Fourier series properties summarized in Table 3.2 with their continuous-time counterparts in Table 3.1.

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$x[n]$ ⎫ Periodic with period N and $y[n]$ ⎭ fundamental frequency $\omega_0 = 2\pi/N$	a_k ⎫ Periodic with b_k ⎭ period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n} x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a_{-k}^*
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\dfrac{1}{m} a_k \left(\begin{array}{l}\text{viewed as periodic} \\ \text{with period } mN\end{array}\right)$
Periodic Convolution	$\displaystyle\sum_{r=\langle N\rangle} x[r]y[n - r]$	$N a_k b_k$
Multiplication	$x[n]y[n]$	$\displaystyle\sum_{l=\langle N\rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n - 1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\displaystyle\sum_{k=-\infty}^{n} x[k] \left(\begin{array}{l}\text{finite valued and periodic only}\\ \text{if } a_0 = 0\end{array}\right)$	$\left(\dfrac{1}{(1 - e^{-jk(2\pi/N)})}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a_{-k}^* \\ \mathfrak{Re}\{a_k\} = \mathfrak{Re}\{a_{-k}\} \\ \mathfrak{Im}\{a_k\} = -\mathfrak{Im}\{a_{-k}\} \\ \|a_k\| = \|a_{-k}\| \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}v\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{O}d\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\mathfrak{Re}\{a_k\}$ $j\mathfrak{Im}\{a_k\}$
Parseval's Relation for Periodic Signals		
$\dfrac{1}{N}\displaystyle\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$		

4.6 TABLES OF FOURIER PROPERTIES AND OF BASIC FOURIER TRANSFORM PAIRS

In the preceding sections and in the problems at the end of the chapter, we have considered some of the important properties of the Fourier transform. These are summarized in Table 4.1, in which we have also indicated the section of this chapter in which each property has been discussed.

In Table 4.2, we have assembled a list of many of the basic and important Fourier transform pairs. We will encounter many of these repeatedly as we apply the tools of

TABLE 4.1   PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$	$X(j\omega)$
		$y(t)$	$Y(j\omega)$
4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\dfrac{1}{|a|}X\left(\dfrac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{+\infty}X(j\theta)Y(j(\omega - \theta))d\theta$
4.3.4	Differentiation in Time	$\dfrac{d}{dt}x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\displaystyle\int_{-\infty}^{t} x(t)dt$	$\dfrac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j\dfrac{d}{d\omega}X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \mathcal{Re}\{X(j\omega)\} = \mathcal{Re}\{X(-j\omega)\} \\ \mathcal{Im}\{X(j\omega)\} = -\mathcal{Im}\{X(-j\omega)\} \\ |X(j\omega)| = |X(-j\omega)| \\ \sphericalangle X(j\omega) = -\sphericalangle X(-j\omega) \end{cases}$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\mathcal{Re}\{X(j\omega)\}$ $j\mathcal{Im}\{X(j\omega)\}$

4.3.7	Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$

TABLE 4.2   BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\displaystyle\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$\displaystyle 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0,$ otherwise
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0,$ otherwise
$\sin \omega_0 t$	$\dfrac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0,$ otherwise
$x(t) = 1$	$2\pi\,\delta(\omega)$	$a_0 = 1,\quad a_k = 0,\ k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1, & \|t\| < T_1 \\ 0, & T_1 < \|t\| \le \frac{T}{2} \end{cases}$ and $x(t + T) = x(t)$	$\displaystyle\sum_{k=-\infty}^{+\infty} \frac{2\sin k\omega_0 T_1}{k}\,\delta(\omega - k\omega_0)$	$\dfrac{\omega_0 T_1}{\pi}\,\text{sinc}\left(\dfrac{k\omega_0 T_1}{\pi}\right) = \dfrac{\sin k\omega_0 T_1}{k\pi}$
$\displaystyle\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\dfrac{2\pi}{T}\displaystyle\sum_{k=-\infty}^{+\infty} \delta\left(\omega - \dfrac{2\pi k}{T}\right)$	$a_k = \dfrac{1}{T}$ for all k
$x(t)\begin{cases} 1, & \|t\| < T_1 \\ 0, & \|t\| > T_1 \end{cases}$	$\dfrac{2\sin \omega T_1}{\omega}$	—
$\dfrac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \|\omega\| < W \\ 0, & \|\omega\| > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\dfrac{1}{j\omega} + \pi\,\delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at}u(t),\ \mathfrak{Re}\{a\} > 0$	$\dfrac{1}{a + j\omega}$	—
$te^{-at}u(t),\ \mathfrak{Re}\{a\} > 0$	$\dfrac{1}{(a + j\omega)^2}$	—
$\dfrac{t^{n-1}}{(n-1)!}e^{-at}u(t),$ $\mathfrak{Re}\{a\} > 0$	$\dfrac{1}{(a + j\omega)^n}$	—

TABLE 5.1   PROPERTIES OF THE DISCRETE-TIME FOURIER TRANSFORM

Section	Property	Aperiodic Signal	Fourier Transform
		$x[n]$	$X(e^{j\omega})$ } periodic with
		$y[n]$	$Y(e^{j\omega})$ } period 2π
5.3.2	Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
5.3.3	Time Shifting	$x[n - n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
5.3.3	Frequency Shifting	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
5.3.4	Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
5.3.6	Time Reversal	$x[-n]$	$X(e^{-j\omega})$
5.3.7	Time Expansion	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
5.4	Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
5.5	Multiplication	$x[n]y[n]$	$\dfrac{1}{2\pi}\displaystyle\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
5.3.5	Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
5.3.5	Accumulation	$\displaystyle\sum_{k=-\infty}^{n} x[k]$	$\dfrac{1}{1 - e^{-j\omega}}X(e^{j\omega})$ $+\pi X(e^{j0})\displaystyle\sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$
5.3.8	Differentiation in Frequency	$nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
5.3.4	Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \mathcal{R}e\{X(e^{j\omega})\} = \mathcal{R}e\{X(e^{-j\omega})\} \\ \mathcal{I}m\{X(e^{j\omega})\} = -\mathcal{I}m\{X(e^{-j\omega})\} \\ |X(e^{j\omega})| = |X(e^{-j\omega})| \\ \sphericalangle X(e^{j\omega}) = -\sphericalangle X(e^{-j\omega}) \end{cases}$
5.3.4	Symmetry for Real, Even Signals	$x[n]$ real an even	$X(e^{j\omega})$ real and even
5.3.4	Symmetry for Real, Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
5.3.4	Even-odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ $[x[n]$ real$]$ $x_o[n] = \mathcal{O}d\{x[n]\}$ $[x[n]$ real$]$	$\mathcal{R}e\{X(e^{j\omega})\}$ $j\mathcal{I}m\{X(e^{j\omega})\}$
5.3.9	Parseval's Relation for Aperiodic Signals $\displaystyle\sum_{n=-\infty}^{+\infty} |x[n]|^2 = \dfrac{1}{2\pi}\displaystyle\int_{2\pi} |X(e^{j\omega})|^2 d\omega$		

a duality relationship between the discrete-time Fourier transform and the continuous-time Fourier series. This relation is discussed in Section 5.7.2.

5.7.1 Duality in the Discrete-Time Fourier Series

Since the Fourier series coefficients a_k of a periodic signal $x[n]$ are themselves a periodic sequence, we can expand the sequence a_k in a Fourier series. The duality property for discrete-time Fourier series implies that the Fourier series coefficients for the periodic sequence a_k are the values of $(1/N)x[-n]$ (i.e., are proportional to the values of the original

TABLE 5.2 BASIC DISCRETE-TIME FOURIER TRANSFORM PAIRS

Signal	Fourier Transform	Fourier Series Coefficients (if periodic)
$\displaystyle\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$\displaystyle 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	a_k
$e^{j\omega_0 n}$	$\displaystyle 2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi m}{N}$ $\quad a_k = \begin{cases} 1, & k = m, m\pm N, m\pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\cos\omega_0 n$	$\displaystyle \pi \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$ $\quad a_k = \begin{cases} \frac{1}{2}, & k = \pm m, \pm m\pm N, \pm m\pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\sin\omega_0 n$	$\displaystyle \frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$ $\quad a_k = \begin{cases} \frac{1}{2j}, & k = r, r\pm N, r\pm 2N, \ldots \\ -\frac{1}{2j}, & k = -r, -r\pm N, -r\pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$x[n] = 1$	$\displaystyle 2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, & \|n\| \le N_1 \\ 0, & N_1 < \|n\| \le N/2 \end{cases}$ and $x[n + N] = x[n]$	$\displaystyle 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \dfrac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N \sin[2\pi k/2N]}, \; k \ne 0, \pm N, \pm 2N, \ldots$ $a_k = \dfrac{2N_1 + 1}{N}, \; k = 0, \pm N, \pm 2N, \ldots$
$\displaystyle\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\displaystyle \frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \dfrac{1}{N}$ for all k
$a^n u[n], \quad \|a\| < 1$	$\dfrac{1}{1 - ae^{-j\omega}}$	—
$x[n] = \begin{cases} 1, & \|n\| \le N_1 \\ 0, & \|n\| > N_1 \end{cases}$	$\dfrac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$	—
$\dfrac{\sin Wn}{\pi n} = \dfrac{W}{\pi}\,\text{sinc}\left(\dfrac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \le \|\omega\| \le W \\ 0, & W < \|\omega\| \le \pi \end{cases}$ $X(\omega)$ periodic with period 2π	—
$\delta[n]$	1	—
$u[n]$	$\dfrac{1}{1 - e^{-j\omega}} + \displaystyle\sum_{k=-\infty}^{+\infty} \pi\delta(\omega - 2\pi k)$	—
$\delta[n - n_0]$	$e^{-j\omega n_0}$	—
$(n + 1)a^n u[n], \quad \|a\| < 1$	$\dfrac{1}{(1 - ae^{-j\omega})^2}$	—
$\dfrac{(n + r - 1)!}{n!(r - 1)!} a^n u[n], \quad \|a\| < 1$	$\dfrac{1}{(1 - ae^{-j\omega})^r}$	—