Midterm #1 of ECE301, Section 3 (CRN 17101-003)
8–9pm, Wednesday, September 13, 2017, EE 170.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

I certify that I have neither given nor received unauthorized aid on this exam.

Signature: Date:

Question 1: [19%, Work-out question, Learning Objectives 1 and 2]

1. [6%] Consider a DT signal

$$x[n] = \begin{cases} 2n + 3j & \text{if } 0 \leq n \leq 2 \\ 1 & \text{if } 6 \leq n \leq 100 \\ 0 & \text{otherwise} \end{cases}. \tag{1}$$

   Compute the total energy of this signal.

2. [3%] Consider a DT signal

$$y[n] = \begin{cases} 1 & \text{if } n = 0 \\ 2 & \text{if } 3 \leq n \leq 7 \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

   Plot $y[n]$ for the range of $-2 \leq n \leq 10$.

3. [10%] Suppose that we can express the above $y[n]$ signal by

$$y[n] = \sum_{k=-\infty}^{\infty} \beta_k \mathcal{U}[n-k] \tag{3}$$

   where $\mathcal{U}[n]$ is the unit step signal. Find out the values of β_k for the range of $0 \leq k \leq 9$.

   Hint: If you do not know how to solve this question, you may solve the following question instead. You will receive 5 points if your answer is correct.

   An alternative question: Suppose that we can express the above $y[n]$ signal by

$$y[n] = \sum_{k=-\infty}^{\infty} \alpha_k \delta[n-k] \tag{4}$$

   where $\delta[n]$ is the unit impulse signal. Find out the values of α_k for the range of $0 \leq k \leq 9$.

Q1| 1) Total Energy $= \sum_{n=-\infty}^{\infty} |x[n]|^2$

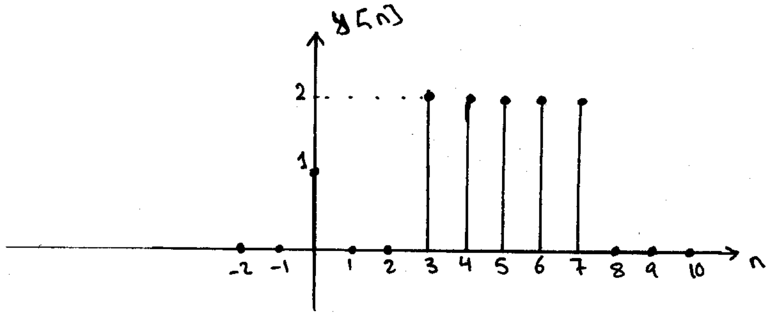
$$E = \sum_{n=0}^{2} |2n+3j|^2 + \sum_{n=6}^{100} |1|^2$$

$$= \sum_{n=0}^{2} \left(\sqrt{(2n)^2+(3)^2}\right)^2 + \sum_{n=6}^{100} 1$$

$$= \sum_{n=0}^{2} 4n^2+9 + \sum_{n=6}^{100} 1$$

$$= (0+9)+(4+9)+(16+9)+95 = 142$$

2)



3) $y[n] = \sum_{k=-\infty}^{\infty} \beta_k u[n-k] \qquad 0 \leq k \leq 9$

$$= u[n] - u[n-1] + 2u[n-3] - 2u[n-8]$$

$\therefore \beta_0 = 1$

$\beta_1 = -1$

$\beta_3 = 2$

$\beta_8 = -2$

$\beta_2 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_9 = 0$

Question 2: [16%, Work-out question, Learning Objectives 1, 2, and 3]

Define two discrete-time signals:

$$x[n] = \begin{cases} e^{jn} & \text{if } 1 \leq n \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

and

$$y[n] = \begin{cases} 3^{-n} & \text{if } 1 \leq n \\ 0 & \text{otherwise} \end{cases}, \tag{6}$$

respectively. Compute the expression of the following sum

$$z[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k]. \tag{7}$$

Hint 1: You may need the following formulas:

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r} \quad \text{if } |r| < 1 \tag{8}$$

$$\sum_{k=1}^{K} ar^{k-1} = \frac{a(1-r^K)}{1-r} \quad \text{if } r \neq 1 \tag{9}$$

$$\sum_{k=1}^{\infty} akr^{k-1} = \frac{a}{(1-r)^2} \quad \text{if } |r| < 1. \tag{10}$$

Hint 2: You can leave your answer to be of the form like $\frac{e^{3j}-2^{-12}}{3+10j}$. There is no need to further simplify the expression.

Q2 $x[k] = \begin{cases} e^{jk}, & k \geq 1 \\ 0, & \text{otherwise} \end{cases}$ $y[n-k] = \begin{cases} 3^{-(n-k)}, & k \leq n-1 \\ 0, & \text{otherwise} \end{cases}$

$$z[n] = \sum_{k=-\infty}^{\infty} x[k]\, y[n-k]$$

2 Cases:

$\boxed{1}$ $n-1 < 1 \rightarrow n < 2$

$$z[n] = \sum (0) \cdot y[n-k] = 0$$

$\boxed{2}$ $n-1 \geq 1 \rightarrow n \geq 2$

$$z[n] = \sum_{k=1}^{n-1} (e^{jk})(3^{-(n-k)})$$

$$= \left(\frac{1}{3}\right)^n \sum_{k=1}^{n-1} (3e^{j})^k = \left(\frac{1}{3}\right)^n \sum_{k=1}^{n-1} (3e^{j})^{k-1} (3e^{j})$$

$$= \left(\frac{1}{3}\right)^n \frac{3e^{j}\,(1 - 3e^{j(n-1)})}{1 - 3e^{j}} =$$

Question 3: [18%, Work-out question, Learning Objectives 1, 3, and 5] Consider the following signal

$$x(t) = e^{j\frac{\pi t}{3}}. \tag{11}$$

We also know another function $h(t)$ being

$$h(t) = \begin{cases} 3 & \text{if } 0 \leq t \leq 3 \\ 0 & \text{otherwise} \end{cases}. \tag{12}$$

Define

$$y(t) = \int_{-\infty}^{\infty} h(s)x(t-s)ds \tag{13}$$

1. [12%] Find the expression of $y(t)$.
2. [6%] Find the angle (also known as phase) of $y(6)$ and find the amplitude of $y(6)$.

   Hint 1: If you do not know the answer to the last sub-question, you can find the phase and amplitude of $z = e^{3+j\pi} \cdot \left(\frac{1-j}{1+j}\right)$. You will receive full points (6 points) if your answer is correct.

   Hint 2: You can leave your answer to be of the form like $\frac{e^{3j}-2^{-12}}{3+10j}$. There is no need to further simplify the expression.

Q3) 1) $y(t) = \int_{-\infty}^{\infty} h(s)\, x(t-s)\, ds$

$$x(t-s) = e^{j\frac{\pi(t-s)}{3}} = e^{j\frac{\pi t}{3}} \; e^{-j\frac{\pi s}{3}}$$

$$y(t) = \int_{-\infty}^{\infty} h(s)\, e^{j\frac{\pi t}{3}} \; e^{-j\frac{\pi s}{3}} \, ds$$

$$= 3\, e^{j\frac{\pi t}{3}} \int_{0}^{3} e^{-j\frac{\pi s}{3}} \, ds$$

$$= 3\, e^{j\frac{\pi t}{3}} \left[\frac{e^{-j\frac{\pi s}{3}}}{-j\frac{\pi}{3}} \right]_0^3$$

$$= 3\, e^{j\frac{\pi t}{3}} \, j\frac{3}{\pi} \left(e^{-j\pi} - e^{-j0} \right)$$

$$e^{-j\pi} = \cos\pi - j\sin\pi$$
$$= -1$$
$$-j = e^{-j\frac{\pi}{2}}$$

$$= \frac{9}{\pi}\, j\, e^{j\frac{\pi t}{3}} (-2)$$

$$= \frac{18}{\pi}\, e^{-j\frac{\pi}{2}} \, e^{j\frac{\pi}{3}t}$$

$$= \frac{18}{\pi}\, e^{j\left(\frac{\pi}{3}t - \frac{\pi}{2}\right)}$$

2) $y(6) = \frac{18}{\pi} e^{j\left(2\pi - \frac{\pi}{2}\right)}$

$\Rightarrow$ phase $= -\frac{\pi}{2}$

$\Rightarrow$ amp $= \left|\frac{18}{\pi}\right| = \frac{18}{\pi}$

Question 4: [12%, Work-out question, Learning Objectives 1 and 6]
Consider the following signal.

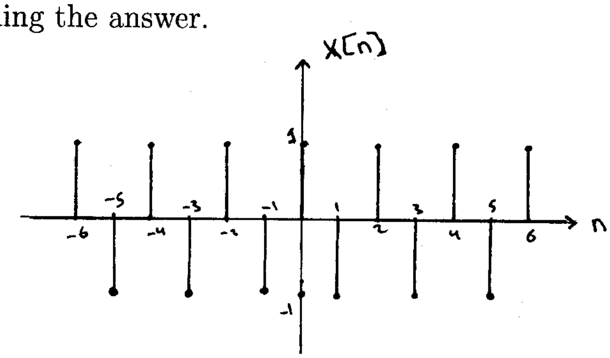
$$x[n] = \cos(\pi n) \tag{14}$$

We generate another signal

$$y[n] = \begin{cases} x[(n+1)/2] & \text{if } n \text{ is odd} \\ x[n/2] & \text{if } n \text{ is even} \end{cases} \tag{15}$$

Plot $y[n]$ for the range of $-6 \le n \le 6$.
Hint: Plotting $x[n]$ may be helpful for finding the answer.

$x[n] = \cos(\pi n) = (-1)^n$



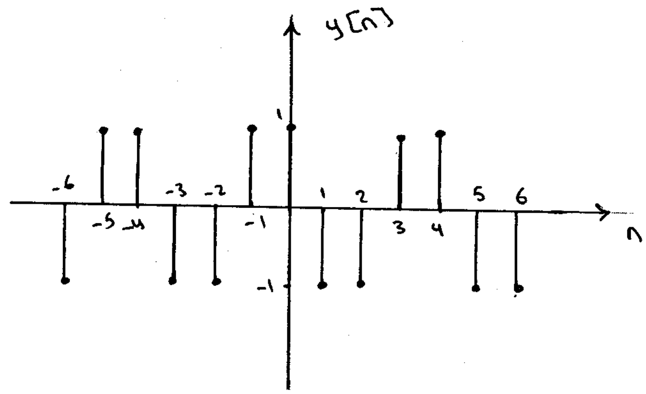
if n is odd

$y[n] = x[(n+1)/2]$

$y[1] = -1$ $y[-1] = 1$

$y[3] = 1$ $y[-3] = -1$

$y[5] = -1$ $y[-5] = 1$



if n is even

$y[n] = x[n/2]$

$y[0] = 1$

$y[2] = -1$ $y[-2] = -1$

$y[4] = 1$ $y[-4] = 1$

$y[6] = -1$ $y[-6] = -1$

Question 5: [15%, Work-out question, Learning Objective 1]

Consider the following system that takes a continuous-time signal $x(t)$ as input and outputs a continuous-time signal $y(t)$:

$$y(t) = \int_{0.5t-2}^{0.5t+1} x(2s)ds. \quad (16)$$

Is the above system is linear or not? Carefully explain the steps how you prove that the system is linear or is not linear.

$$x_1(t) \longrightarrow \boxed{\text{system}} \longrightarrow y_1(t) = \int_{\frac{t}{2}-2}^{\frac{t}{2}+1} x_1(2s)\,ds$$

$$x_2(t) \longrightarrow \boxed{\text{system}} \longrightarrow y_2(t) = \int_{\frac{t}{2}-2}^{\frac{t}{2}+1} x_2(2s)\,ds$$

$$x_3(t) = a_1 x_1(t) + a_2 x_2(t)$$

$$x_3(t) \longrightarrow \boxed{\text{system}} \longrightarrow y_3(t) = \int_{\frac{t}{2}-2}^{\frac{t}{2}+1} x_3(2s)\,ds$$

$$= \int_{\frac{t}{2}-2}^{\frac{t}{2}+1} \left(a_1 x_1(2s) + a_2 x_2(2s)\right) ds$$

$$= \int_{\frac{t}{2}-2}^{\frac{t}{2}+1} a_1 x_1(2s)\,ds + \int_{\frac{t}{2}-2}^{\frac{t}{2}+1} a_2 x_2(2s)\,ds$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

$\Rightarrow$ system is Linear

Question 6: [20%, Multiple Choices, Learning Objectives 1 and 6]

The following questions are multiple-choice questions and there is no need to justify your answers. Consider two continuous-time signals:

$$x_1(t) = \sum_{k=1}^{\infty} 2^{-k} \cos((\pi + k)t) \quad (17)$$

$$x_2(t) = (e^{-t^3} + e^{t^3}) \cos(1.5t) \quad (18)$$

and two discrete-time signals:

$$x_3[n] = e^{j\frac{7\pi}{5}n} + \cos(3\pi n^3) \quad (19)$$

$$x_4[n] = \cos(\frac{\pi}{2}|n|) + \sin(\frac{\pi}{2}|n|). \quad (20)$$

1. [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not, *respectively.* If it is periodic, write down the fundamental period. Please state explicitly which signal is periodic and which is not.
2. [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them, respectively. Please state explicitly which signal is even, which is odd, and which is neither.

1) $X_1(t) \to$ aperiodic

$X_2(t) \to$ aperiodic

$X_3[n] \to$ periodic , $N=10$

$X_4[n] \to$ aperiodic

2) $X_1(t) \to$ even

$X_2(t) \to$ even

$X_3[n] \to$ neither

$X_4[n] \to$ even