Final Exam of ECE301, Section 3 (CRN 17101-003)
8–10am, Wednesday, December 13, 2017, Hiler Thtr.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have two hours to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name: Solution

Student ID:

I certify that I have neither given nor received unauthorized aid on this exam.

Signature: Date:

Question 1: [21.5%, Work-out question] Amplitude Modulation

1. [1%] What does the acronym AM-SSB stand for? Single-Side Band
2. [1%] What does the acronym FDM stand for when referring to the technique of simultaneously broadcasting multiple AM signals from the same antenna tower?

Frequency Division Multiplexing

Prof. Wang wanted to transmit an AM-SSB signal. To that end, he wrote the following MATLAB code.

```
% Initialialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read two different .wav files
[x1, f_sample, N]=audioread('x1.wav');
x1=x1';
[x2, f_sample, N]=audioread('x2.wav');
x2=x2';

% Step 0: Initialize several parameters
W_1=????;
W_2=pi*5000;
W_3=pi*8000;
W_4=pi*2000;
W_5=pi*3000;
W_6=????;


% Step 1: Make the signals band-limited.
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);

% Step 2: Multiply x1_new and x2_new with a sinusoidal wave.
x1_h=x1_new.*cos(W_2*t);
x2_h=x2_new.*sin(W_3*t);

% Step 3: Keep one of the two side bands
h_one=1/(pi*t).*(2*sin(W_4*t)).*(cos(W_5*t));
h_two=1/(pi*t).*(2*sin(W_4*t)).*(cos(W_6*t));
x1_sb=ece301conv(x1_h, h_one);
x2_sb=ece301conv(x2_h, h_two);
```

```
% Step 4: Create the transmitted signal
y=x1_sb+x2_sb;
audiowrite('y.wav', y, f_sample);
```

3. [1.5%] What is the carrier frequency (Hz) of the signal x2_new? 4000 Hz

4. [1.5%] For the first signal x1_new, is this AM-SSB transmitting an upper-side-band signal or a lower-side-band signal? lower side band

5. [1.5%] What should the value of W_6 be in the MATLAB code, if we decide to use an upper-side-band transmission for the second signal x2_new? 10000π

6. [1.5%] What should the value of W_6 be in the MATLAB code, if we decide to use a lower-side-band transmission for the second signal x2_new? 6000π

7. [2%] Continue from the previous sub-question. Suppose lower-side-band transmission is used for the second signal x2_new. To ensure that the receiver side can have the best possible quality, it is important for the transmitter to choose the largest W_1 value when possible. What is the largest W_1 value that can be used without significantly degrading the quality of any of the two transmitted signals?

$W_1 = 3000\pi$

Knowing that Prof. Wang decided to use a lower-side-band transmission for the second signal x2_new and choose the W_1 value to be $W_1 = 2000 \times \pi$. He then used the above code to generate the "y.wav" file, a student tried to demodulate the output waveform "y.wav" by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[y, f_sample, N]=audioread('y.wav');
y=y';

% Initialize several parameters
W_8=????;
W_9=????;
W_10=????;
W_11=????;
W_12=????;


% Create the low-pass filter.
h_M=1/(pi*t).*(sin(W_8*t));

% demodulate signal 1
h_BPF1=1/(pi*t).*(sin(W_9*t));
y1_BPF=ece301conv(y,h_BPF1);
y1=2*y1_BPF.*cos(pi*5000*t);
x1_hat=ece301conv(y1,h_M);

sound(x1_hat,f_sample)

% demodulate signal 2
h_BPF2=1/(pi*t).*(sin(W_10*t))-1/(pi*t).*(sin(W_11*t));
y2_BPF=ece301conv(y,h_BPF2);
y2=2*y2_BPF.*cos(pi*W_12*t);
x2_hat=ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```

8. [7.5%] Continue from the previous questions. What should the values of W_8 to W_12 be in the MATLAB code? When answering this question, please assume that

the second radio x2_new is transmitted using the *lower side-band* and $W_1 = 2000 \times \pi$.

9. [2%] It turns out that the above MATLAB code is not written correctly and neither signal x1_new nor signal x2_new can be correctly demodulated. Please use 2 to 3 sentences to (i) what kind of problem does x1_new have, i.e., how does the problem impact the sound quality of "sound(x1_hat,f_sample)"? (ii) how can the MATLAB code be corrected so that the playback/demodulation can performed successfully?

10. [2%] Please use 2 to 3 sentences to (i) what kind of problem does x2_new have, i.e., how does the problem impact the sound quality of "sound(x2_hat,f_sample)"? (ii) how can the MATLAB code be corrected so that the playback/demodulation can performed successfully?

Hint: If you do not know the answers of Q1.3 to Q1.10, please simply draw the AMSSB modulation and demodulation diagrams and mark carefully all the parameter values. You will receive 12 points for Q1.3 to Q1.10.

$W_8 = 2000\pi$

$W_9 = 5000\pi$

$W_{10} = 8000\pi$

$W_{11} = 6000\pi$

$W_{12} = \cancel{8000\pi}$ 8000

9. Too weak.

$y1 = 4 \cdot y_{1,BPF} \cdot \cos(pi * 5000 \cdot t)$

10. Silent.

$y_2 = 4 \cdot y_{2,BPF} \cdot \sin(pi\ W_{12} \cdot t)$

Question 2: [11.5%, Work-out question]

1. [1.5%] Consider a continuous time signal $x(t)$

$$x(t) = \frac{\sin(2\pi t)}{\pi t}. \tag{1}$$

Plot the CTFT $X(j\omega)$ of $x(t)$ for the range of $-4\pi \leq \omega \leq 4\pi$.

2. [2%] We then construct $y(t)$ by

$$y(t) = 2\cos(100\pi t). \tag{2}$$

Plot the CTFT $Y(j\omega)$ of $y(t)$ for the range of $-104\pi \leq \omega \leq 104\pi$.

3. [3%] Finally we construct $z(t)$ by

$$z(t) = y(t) * \frac{\sin(101\pi t)}{\pi t} \tag{3}$$

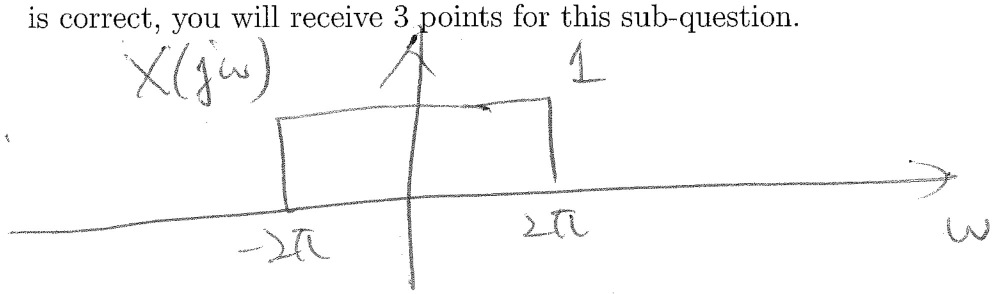
Plot the CTFT $Z(j\omega)$ of $z(t)$ for the range of $-104\pi \leq \omega \leq 104\pi$.

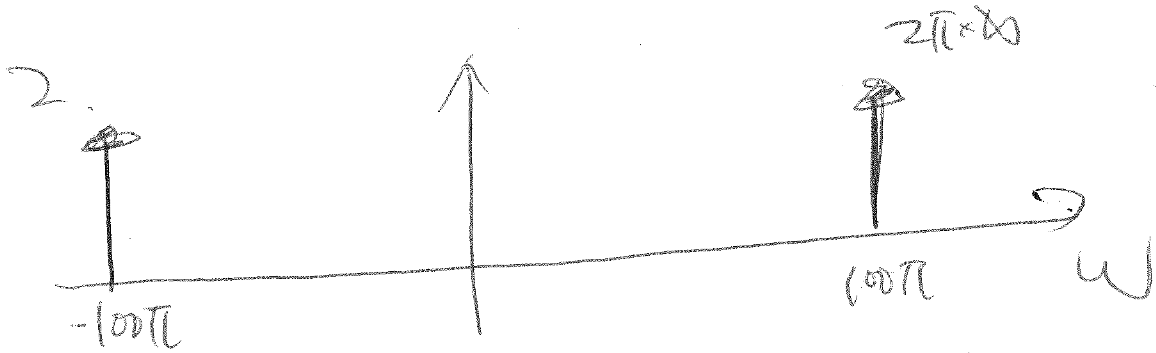
Hint: If you do not know how to solve this question, you can solve the following alternative question instead. You will receive 2 points if your answer is correct. Suppose $h(t) = \frac{\sin(100t)}{t} * \frac{\sin(20t)}{\pi t}$. Find the CTFT $H(j\omega)$.

4. [5%] Continue from the previous sub-question. Plot $z(t)$ for the range of $-5 < t < 5$.

Hint: If you do not know how to solve this question, please write down what is the definition of *AM asynchronous demodulation* and give a detailed example how to use AM asynchronous demodulation to demodulate an AM signal. If your answer is correct, you will receive 3 points for this sub-question.

1.





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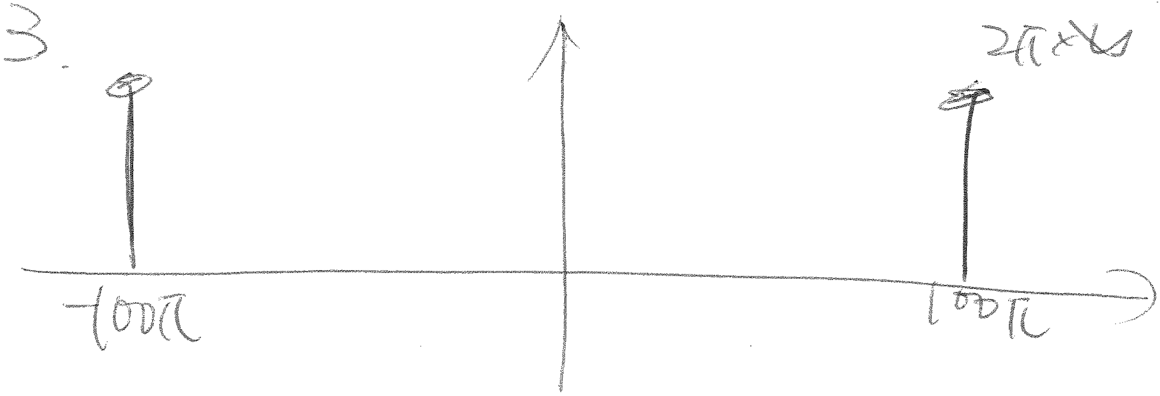
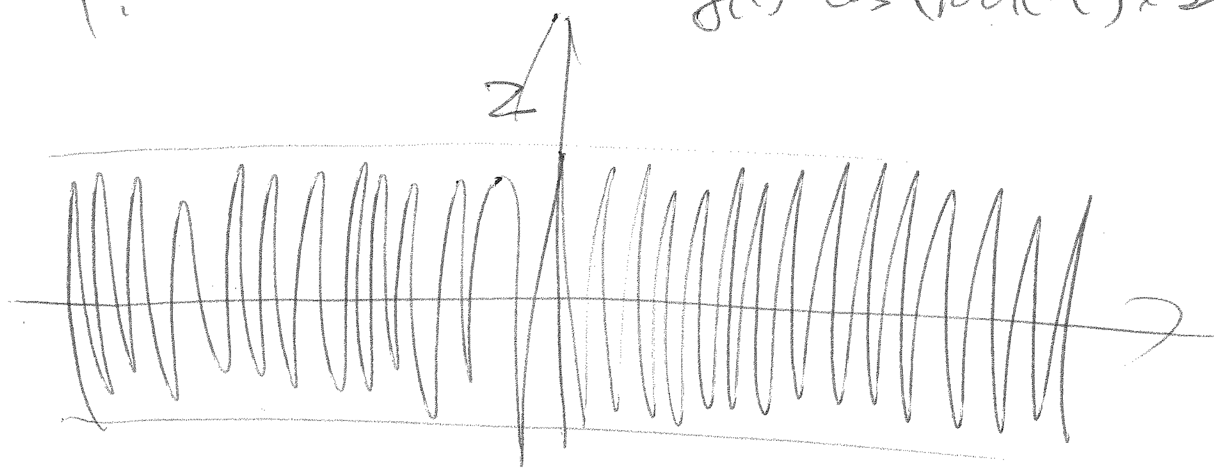


4.

$g(t) = \cos(100\pi \cdot t) \times 2$



Question 3: [13%, Work-out question]

1. [2%] Consider a continuous time signal

$$x(t) = \begin{cases} 2t & \text{if } 0.001 < t < 1.001 \\ 2 & \text{if } 1.001 < t < 2.001 \\ \text{periodic with period 2} & \end{cases} \tag{4}$$

   Plot $x(t)$ for the range of $-4 < t < 4$.

2. [3%] We sample $x(t)$ with the sampling frequency 1.5Hz and denote the sampled values by $x[n]$. Plot $x[n]$ for the range of $-5 \le n \le 5$.

3. ~~[2%] We use $x_{\text{lin}}(t)$ to represent the reconstructed signal using "linear interpolation". Plot $x_{\text{lin}}(t)$ for the range of $-4 < t < 4$.~~

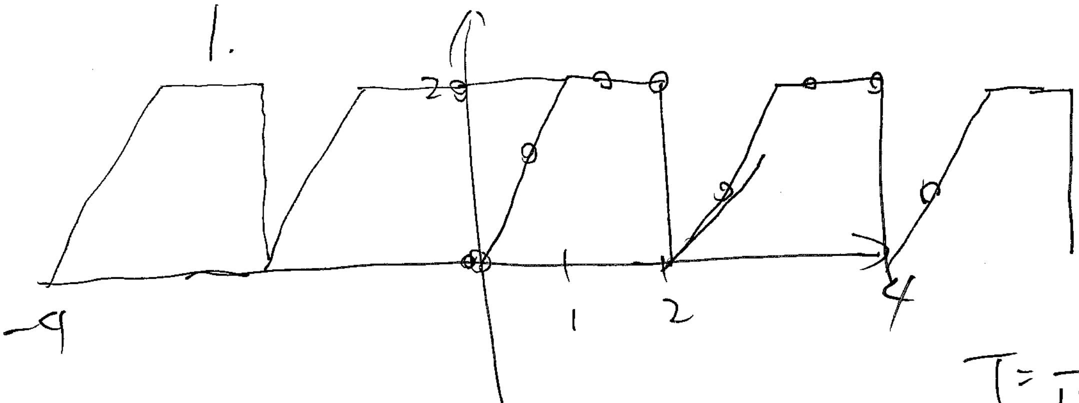
   ~~Hint: if you do not know the answer of $x[n]$, you can assume that $x[n] = 2(-1)^n + 1$ and the sampling frequency is 1.5Hz. You will receive full points if your answer is correct.~~

4. [2%] We use $x_{\text{ZOH}}(t)$ to represent the reconstructed signal using "zero-order hold". Plot $x_{\text{ZOH}}(t)$ for the range of $-4 < t < 4$.

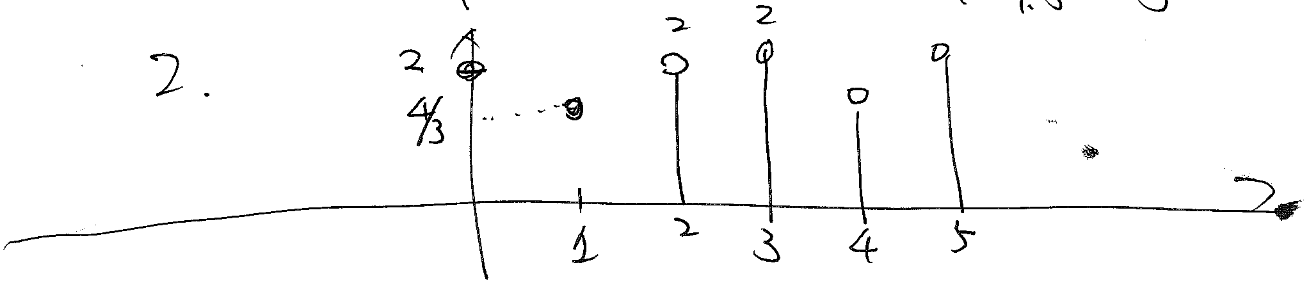
   Hint: if you do not know the answer of $x[n]$, you can assume that $x[n] = 2(-1)^n + 1$ and the sampling frequency is 1.5Hz. You will receive full points if your answer is correct.

5. [4%] Consider another continuous time signal $y(t)$ and again we sample $y(t)$ with sampling frequency 1.5Hz. Suppose the resulting $y[n] = \delta[n - 5]$. Let $y_{\text{opt}}(t)$ denote the optimal band-limited reconstruction of $y(t)$. Plot $y(t)$ for the range of $-6 < t < 6$.

1.



$T = \frac{1}{1.5} = \frac{2}{3}$



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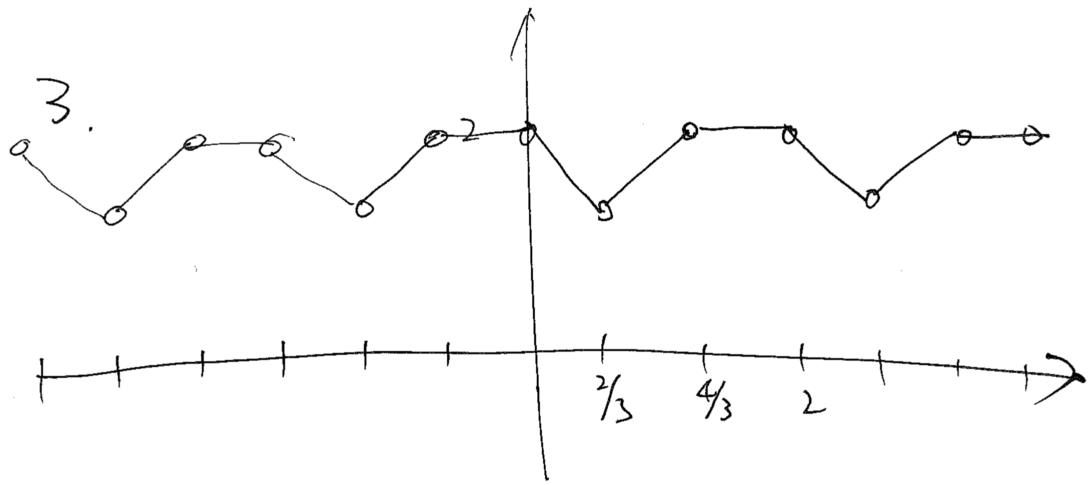

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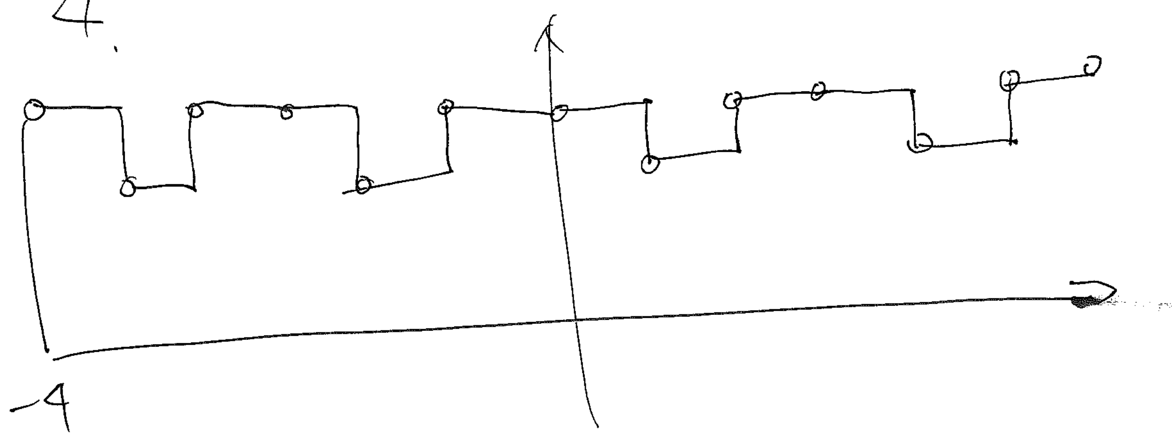

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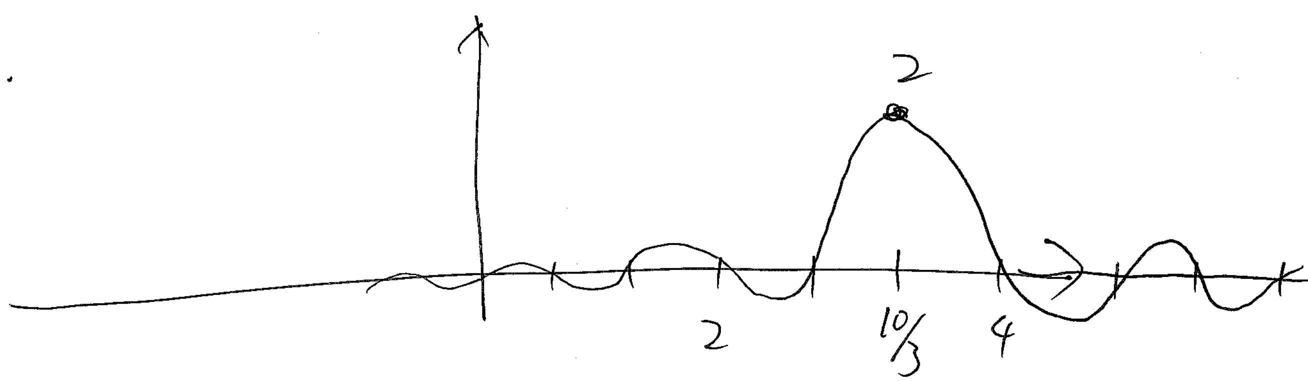


Question 4: [11%, Work-out question]

Consider a signal $x(t) = \sin(1.25\pi t)$. Let $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2k)$. Define $x_p(t) = x(t) \cdot p(t)$, i.e., $x_p(t)$ is the impulse train sampled signal.

1. [1%] What is the sampling frequency (Hz) of the impulse train sampled signal $x_p(t)$?
2. [4.5%] What is the CTFT $X_p(j\omega)$ of $x_p(t)$? Plot $X_p(j\omega)$ for the range of $-\pi < \omega < \pi$.

   Hint: If you do not know the answer to this question, you should find CTFT $X(j\omega)$ of $x(t)$ instead. You will receive 2 points if your answer is correct.
3. [5.5%] We reconstruct $x(t)$ by the following operation:

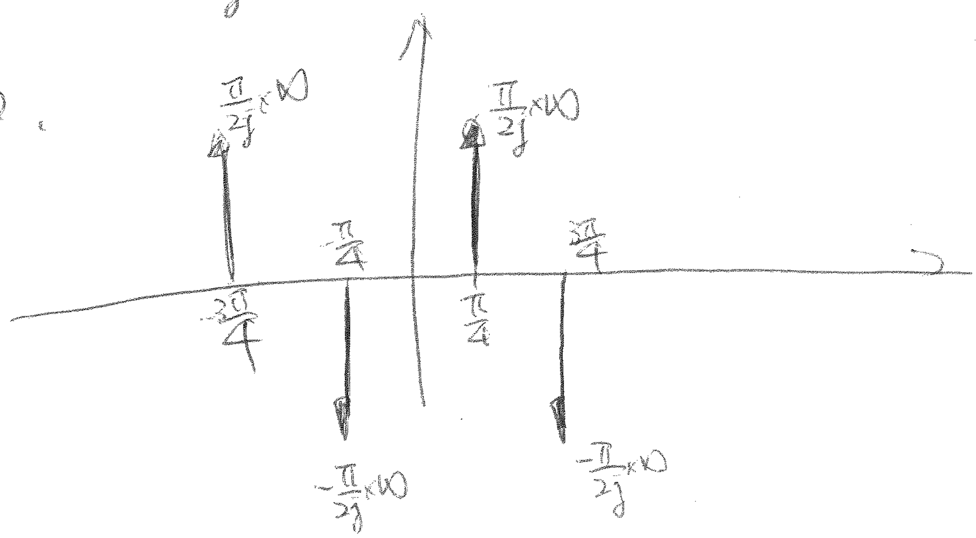
$$\hat{x}(t) = 2x_p(t) * \frac{\sin(0.5\pi t)}{\pi t}. \tag{5}$$

   What is the expression of $\hat{x}(t)$?

   Hint: If you do not know the answer to this question, please write down in details (i) What is the *sampling theorem*? (ii) What is the *Nyquist frequency*? You will receive 3 points if your answers are correct.

1. 0.5 Hz

2.



3. $\hat{x}(t) = \sin\left(\frac{\pi}{4}t\right)$

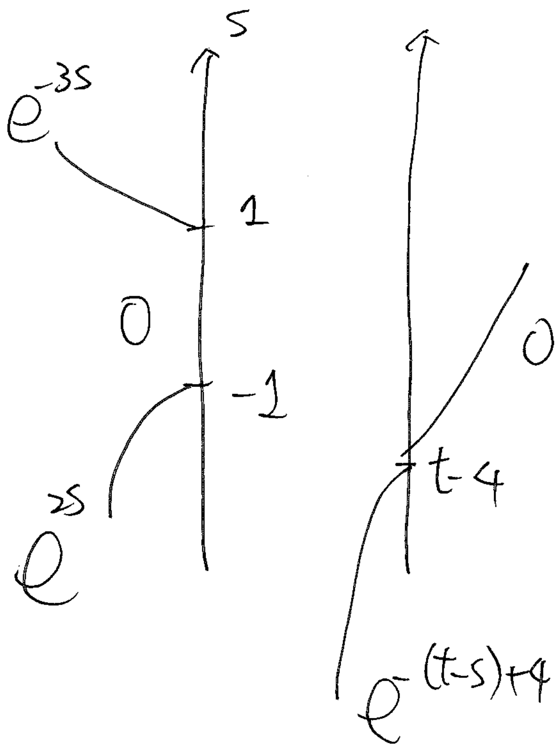
Question 5: [10%, Work-out question]

Consider the following continuous time signals

$$x(t) = \begin{cases} e^{2t} & \text{if } t < -1 \\ 0 & \text{if } -1 \le t < 1 \\ e^{-3t} & \text{if } 1 \le t \end{cases} \tag{6}$$

$$y(t) = e^{-(t-4)}\mathcal{U}(t-4). \tag{7}$$

Find the expression of $z(t) = x(t) * y(t)$ ~~and plot it for the range of $-10 \le t \le 10$~~



$$y(t-s) = \begin{cases} e^{-(t-s-4)} & \text{if } t-s \ge 4 \\ & s \le t-4 \\ 0 & \text{o/w.} \end{cases}$$

Case 1: $t-4 < -1 \Leftrightarrow t < 3.$

$$z(t) = \int_{-\infty}^{t-4} e^{2s} \cdot e^{s} \cdot e^{4-t}\, ds$$

$$= e^{4-t} \cdot \frac{e^{3(t-4)}}{3} = e^{2(t-4)} \cdot \frac{1}{3}.$$

Case 2: $-1 < t-4 < 1$

$\Leftrightarrow 3 < t < 5$

$$z(t) = \int_{-\infty}^{-1} e^{2s} e^{s} \cdot e^{4-t} \, ds$$

$$= e^{4-t} \cdot \frac{e^{3(-1)}}{3} = e^{1-t} \times \frac{1}{3}$$

Case 3: $1 < t-4 \iff 5 < t$

$$z(t) = \int_{-\infty}^{-1} e^{2s} e^{s} \cdot e^{4-t} \, ds$$

$$+ \int_{1}^{t-4} e^{-3s} e^{s} e^{4-t} \, ds$$

$$= \frac{e^{1-t}}{3} + e^{4-t} \cdot \left(\frac{e^{-2(t-4)} - e^{-2\cdot 1}}{-2} \right)$$

$$= \frac{e^{1-t}}{3} + \frac{e^{2-t}}{2} - \frac{e^{-(t-4)\times 3}}{2}$$

#

Question 6: [11%, Work-out question]

Consider the following discrete-time signals.

$$x[n] = \frac{\sin(0.75\pi t)}{\pi t} \tag{8}$$

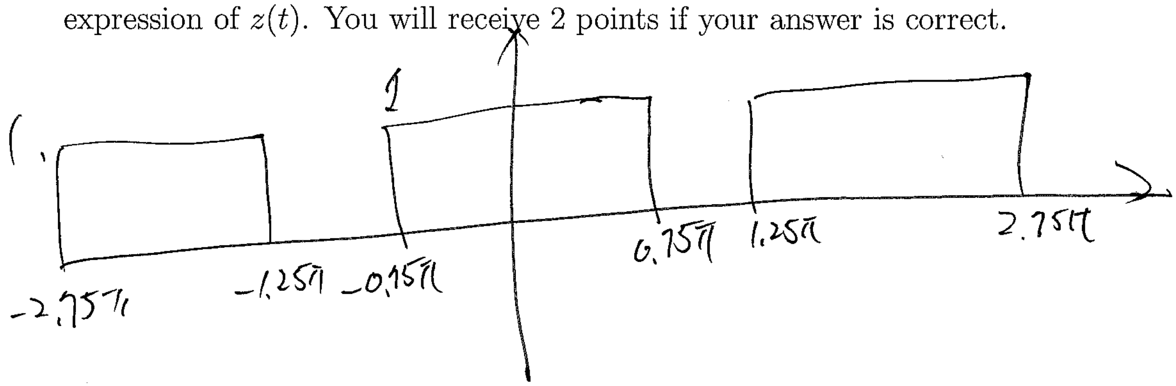
1. [3%] Plot DTFT $X(e^{j\omega})$ for the range of $\overset{-3\pi}{\cancel{-5\pi}} < \omega < \overset{3\pi}{\cancel{5\pi}}$.
2. [3%] Let $y[n] = x[n](-1)^n$. Plot DTFT $Y(e^{j\omega})$ for the range of $\overset{-3\pi}{\cancel{-5\pi}} < \omega < \overset{3\pi}{\cancel{5\pi}}$.

   Hint: if you do not know what is $X(e^{j\omega})$, you can assume $X(e^{j\omega}) = \cos(\omega) + 1$. You will receive full points if your answer is correct.

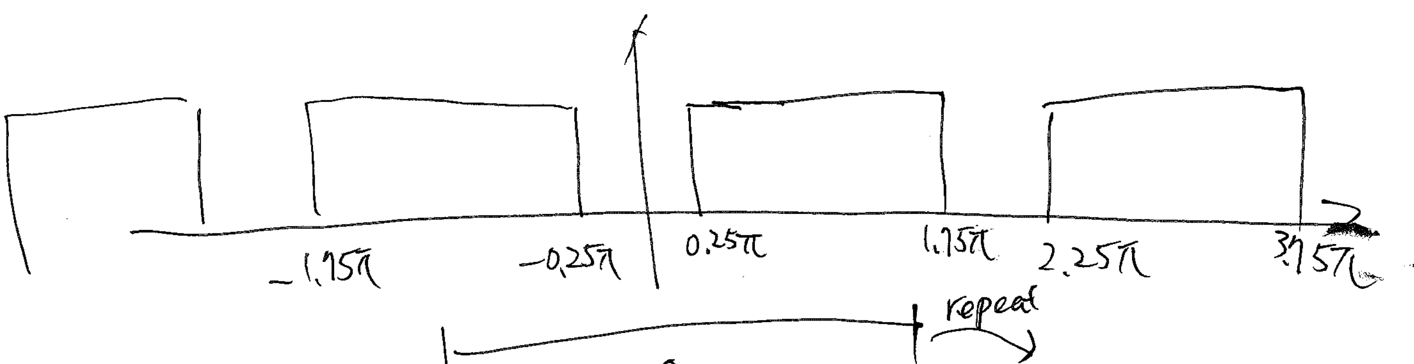
3. [5%] Let $z[n] = x[n] * y[n]$. Find the expression of $z[n]$.

   Hint: if you do not know how to solve this question, you can solve the following question instead.

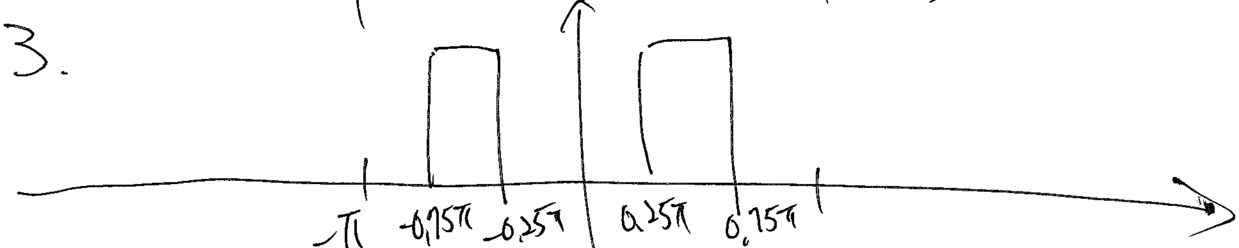
   $x(t) = \cos(t) + \sin(2t)$, $y(t) = \sin(1.5t)/(1.5t)$, and $z(t) = x(t) * y(t)$. Find the expression of $z(t)$. You will receive 2 points if your answer is correct.



$y[n] = x[n]\, e^{+j\pi n}$



3.



$$z[n] = \frac{\sin(0.75\pi n) - \sin(0.25\pi n)}{\pi n}$$

Question 7: [8%, Work-out question]
Consider the following discrete time system.

$$y[n] = \begin{cases} \sum_{k=n}^{n+5} x[k] & \text{if } n \leq -8 \\ \sum_{k=n-5}^{n} x[k] & \text{if } -7 \leq n \end{cases}. \tag{9}$$

1. [5%] Find the expression of the impulse response $h[n]$ of this system.
2. [3%] Is the above system causal? This is not a yes-no question. Please carefully explain your answer in 1 to 3 sentences.

1. $h[n] = \begin{cases} 1 & \text{if } 0 \leq n \leq 5 \\ 0 & \text{otherwise.} \end{cases}$

2. No. $\because$ $y[-10]$ depends on $x[-5]$.

Question 8: [15%, Multiple-choice question] Consider two signals

$$h_1(t) = |e^{e^{jt}}| \quad = \left| e^{\cos t + j \sin t} \right| \tag{10}$$

and

$$= e^{\cos t}$$

$$h_2[n] = \left(\cos(0.25\pi n^3) + e^{j\pi n} - e^{-j\pi n}\right] \cdot u[n-1000] \tag{11}$$

$u[n+999] -$

Hint: you should treat $h_1(t)$ as $|e^{x(t)}|$ where $x(t) = e^{jt}$.

1. [1.25%] Is $h_1(t)$ periodic? Yes.
2. [1.25%] Is $h_2[n]$ periodic? No
3. [1.25%] Is $h_1(t)$ even or odd or neither? even
4. [1.25%] Is $h_2[n]$ even or odd or neither? even
5. [1.25%] Is $h_1(t)$ of finite energy? No
6. [1.25%] Is $h_2[n]$ of finite energy? Yes

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless? No
2. [1.25%] Is System 2 memoryless? No
3. [1.25%] Is System 1 causal? No
4. [1.25%] Is System 2 causal? No
5. [1.25%] Is System 1 stable? No
6. [1.25%] Is System 2 stable? Yes.