ECE 301 Exam 3 Solutions

Question 1:

1. $x[0] = \sum_{k=0}^{19} a_k e^{jk\omega_0 \cdot 0} = \sum_{k=0}^{19} a_k = (3-3)\times 5 + (-1)\times 6 + 0\times 10 = -6$

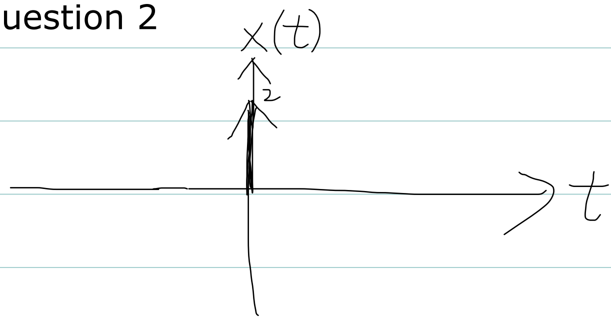
2. Parseval's: $\sum_{n=-3}^{16} |x[n]|^2 = 20 \sum_{k=0}^{20} |a_k|^2 = 20(10\cdot 3^2 + 6\cdot 1^2) = 1920$

3. $-1 = e^{-j\cdot 10\cdot(2\pi/20)}$

$\therefore \sum_{n=0}^{19} (-1)^n x[n] = \sum_{n=0}^{19} x[n] e^{-j\cdot 10\cdot(2\pi/20)n} = 20\cdot a_{10} = -20$

Question 2

1.



2. No.

3. CTFS $\because T=2\pi \quad \therefore \omega_0=1$

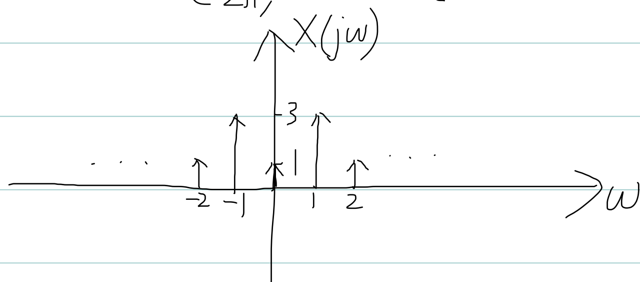
$$a_k=\frac{1}{2\pi}\int_{t=-1}^{2\pi-1} x(t)e^{-jkt}dt=\frac{1}{2\pi}\int_{t=-1}^{2\pi-1}\left(2\delta(t)-\delta(t-\pi)\right)e^{-jkt}dt$$

$$=\frac{1}{2\pi}\left(e^{-jk0}\cdot 2-e^{-jk\pi}\right)$$

$$=\frac{1}{\pi}-\frac{1}{2\pi}\cdot(-1)^k$$

$$\therefore a_k=\begin{cases}\frac{1}{2\pi}, & k \text{ is even}\\ \frac{3}{2\pi}, & k \text{ is odd}\end{cases}$$

Use first property in Table 4.2:



If used hint 2 instead:

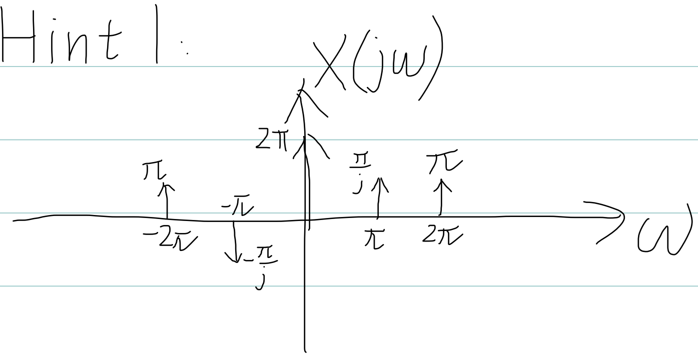
$$X(j\omega)=\pi\left[\delta(\omega-1)+\delta(\omega+1)\right]+\frac{\pi}{j}\left[\delta(\omega-\sqrt{3})-\delta(\omega+\sqrt{3})\right]$$

Question 3

$$x(t) = 1 + \frac{1}{2j}\left(e^{j\pi t} - e^{-j\pi t}\right) + \frac{1}{2}\left(e^{j2\pi t} + e^{-j2\pi t}\right)$$

$$\therefore y(t) = 1 \cdot H(j0) + \frac{1}{2j}\left(e^{j\pi t}H(j\pi) - e^{-j\pi t}H(-j\pi)\right)$$
$$+ \frac{1}{2}\left(e^{j2\pi t}H(j2\pi) + e^{-j2\pi t}H(-j2\pi)\right)$$
$$= 0 + \frac{1}{2j}\left(j\pi e^{j\pi t} + j\pi e^{-j\pi t}\right) + 0$$
$$= \pi \cos(\pi t)$$

Hint 1:

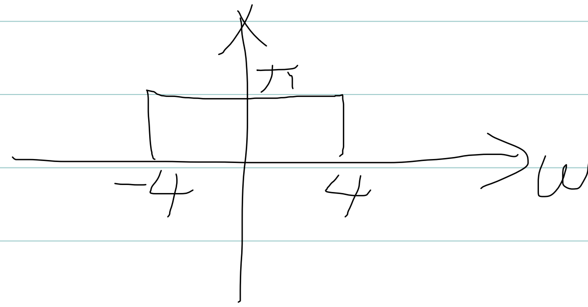


Hint 2:

$$y(t) = 0 \quad \left(\because H(\pm 2\pi j) = H(\pm 3\pi j) = 0\right)$$

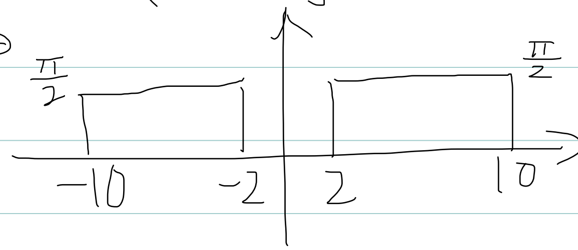
Question 4

$\frac{\sin(4t)}{t} \overset{F}{\longleftrightarrow}$



$\cos(6t) \overset{F}{\longleftrightarrow} \pi[\delta(\omega-6)+\delta(\omega+6)]$

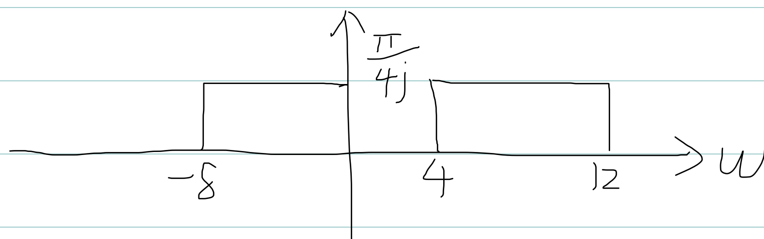
$\therefore \frac{\sin(4t)}{t}\cos(6t) \overset{F}{\longleftrightarrow}$



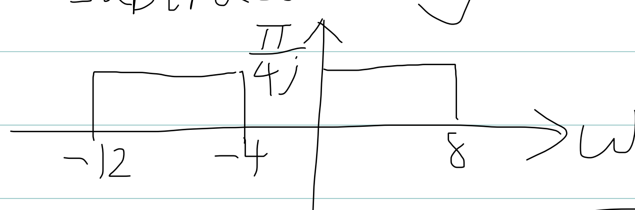
Similarly:

$\sin(2t) \overset{F}{\longleftrightarrow} \frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$

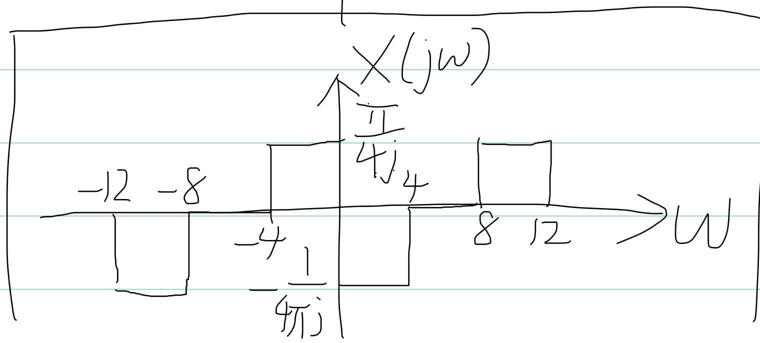
$\therefore$



subtracted by



$\therefore$



Question 5

$$X(e^{jw}) = \sum_{n} e^{n+2} u(100-n) e^{-jwn}$$

$$= \sum_{n=-\infty}^{100} e^{n+2} e^{-jwn}$$

$$= e^{102} e^{-jw100} + (e^{jw-1}) \cdot e^{102} e^{-jw100} + (e^{jw-1})^2 e^{102} e^{-jw100} + \cdots$$

$$= \frac{e^{102-jw100}}{1-e^{jw-1}}$$

Question 6

$$X(j\omega) = \begin{cases} 1, & |\omega| < 1000 \\ 0, & \text{else} \end{cases}$$

$$H(j\omega) = \begin{cases} 1, & |\omega| < 2\pi \\ 0, & \text{else} \end{cases}$$

$\therefore Z(j\omega) = X(j\omega)H(j\omega) = H(j\omega)$

$\therefore z(t) = \frac{\sin(2\pi t)}{\pi t} = h_{LPF}(t)$

$\therefore y(t) = \frac{\sin(2\pi t)}{\pi t} \cos(100\pi t)$