ECE 301 Exam 2 Solutions

Question 1

1. Impulse response is the output of a system when the input is $\delta(t) / \delta[n]$

2. 1 is not causal. 2 is not causal. 3 is causal

3. 2 is stable. 3 is stable.

4. $h_{1\to 2}(t) = h_1 * h_2 = \delta(t+2.5) * h_2(t)$

$= h_2(t+2.5)$

$= e^{-|t+0.5|}$

5. $h_{2\to 3}(t) = h_2(t) * h_3(t)$

$= \int_{-\infty}^{+\infty} e^{-|\tau-2|} e^{-t+\tau} u(2(t-\tau)-\pi) d\tau$

$u(2(t-\tau)-\pi) = 1 \iff \tau \leq t - \frac{\pi}{2}$

$\therefore$ When $2 \geq t - \frac{\pi}{2}$ $(t \leq 2 + \frac{\pi}{2})$:

$h_{2\to 3}(t) = \int_{-\infty}^{t-\frac{\pi}{2}} e^{\tau-2} e^{-t+\tau} d\tau = e^{-t-2} \cdot \int_{-\infty}^{t-\frac{\pi}{2}} e^{2\tau} d\tau = \frac{1}{2} e^{t-\pi-2}$

When $t > 2 + \frac{\pi}{2}$

$h_{2\to 3}(t) = \int_{-\infty}^{2} e^{\tau-2} e^{-t+\tau} d\tau + \int_{2}^{t-\frac{\pi}{2}} e^{2-\tau} e^{-t+\tau} d\tau$

$= \frac{1}{2} e^{2-t} + (t - \frac{\pi}{2} - 2) \cdot e^{2-t}$

$= (t - \frac{\pi}{2} - 1.5) e^{2-t}$

Question 2.

$$H(j\omega) = \int_{-\infty}^{+\infty} h(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} [e^{(1-j\omega)}]^{t} dt$$

$$= \frac{1}{1-j\omega}$$

$$x(t) = e^{jt} + \frac{1}{2}\left(e^{j(\sqrt{3}t-1)} + e^{-j(\sqrt{3}t-1)}\right)$$

$\uparrow \omega=1$ $\quad \uparrow \omega=\sqrt{3}$ $\quad \uparrow \omega=-\sqrt{3}$

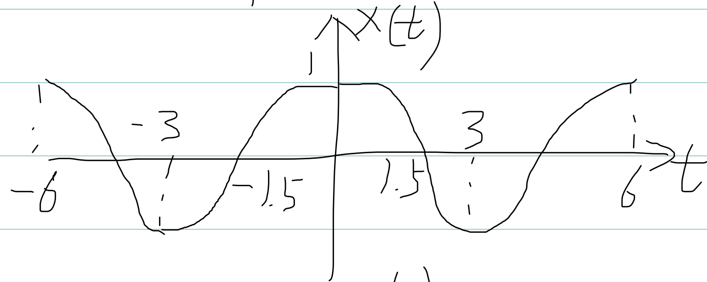
$$\therefore y(t) = e^{jt} H(j) + \frac{1}{2} e^{j(\sqrt{3}t-1)} H(\sqrt{3}j) + \frac{1}{2} e^{-j(\sqrt{3}t-1)} H(-\sqrt{3}j)$$

$$= \frac{e^{jt}}{1-j} + \frac{1}{2} e^{j(\sqrt{3}t-1)} \cdot \frac{1}{1-\sqrt{3}j} + \frac{1}{2} e^{-j(\sqrt{3}t-1)} \cdot \frac{1}{1+\sqrt{3}j}$$

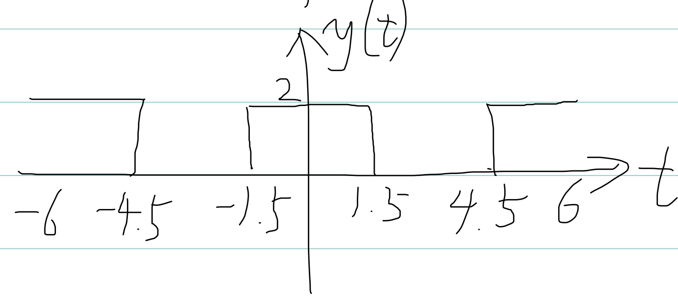
Question 3

1. $\omega_0 = \frac{\pi}{3} \quad x(t) = \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$

$\therefore a_1 = a_{-1} = \frac{1}{2} \quad a_k = 0$ for all other k



2.



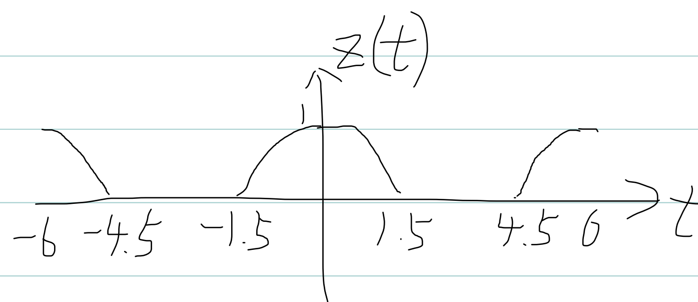
This is a rectangular waveform with period 6, amplitude 2, and duty cycle $\frac{1}{2}$ $(T_1 = 1.5, T = 6)$

$\therefore b_0 = 2 \cdot \frac{1}{2} = 1$

$b_k = 2 \cdot \frac{\sin(1.5k\frac{2\pi}{6})}{k\pi} = \frac{2\sin(\frac{\pi}{2}k)}{k\pi} \quad (k \neq 0)$

$\therefore b_9 = \frac{2}{9\pi} \quad b_{-4} = 0$

3.



$z(t) = x(t) \cdot \frac{1}{2} y(t)$

$\therefore C_k = \sum_{l=-\infty}^{+\infty} a_l \cdot \frac{1}{2} b_{k-l}$

$= \frac{1}{2}(a_1 \cdot b_{k-1} + a_{-1} \cdot b_{k+1}) = \frac{1}{4}(b_{k-1} + b_{k+1})$

Question 4

Parseval's relation:

$$\sum_{k=0}^{199} |a_k|^2 = \frac{1}{200} \sum_{n=0}^{199} |x[n]|^2$$

$\because |e^{jn}| = 1$, $|-1+j|^2 = 2$

$\therefore \frac{1}{200} \sum_{n=0}^{199} |x[n]|^2$

$$= \frac{1}{200}(50 \cdot 1 + 50 \cdot 2 + 100 \cdot 0)$$

$$= \frac{3}{4}$$

Question 5

	System 1	System 2
1.	No	No
2.	Yes	No
3.	Yes	Yes
4.	No	No
5.	Yes	No