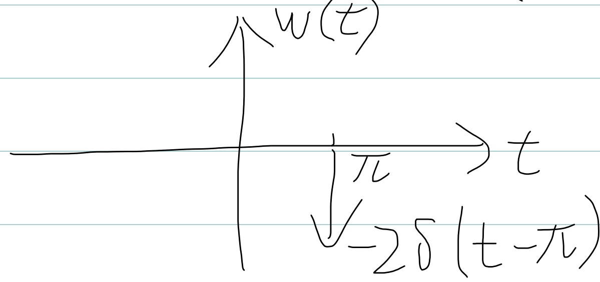
ECE 301 Exam 1 Solutions

Question 1.

1. $w(t)=\delta(t-\pi)\,h(t)=\delta(t-\pi)\,h(\pi)=-2\delta(t-\pi)$



2. $x(s)=0$ if $s\geq 2\pi$

$h(t-s)=0$ if $t-s\notin[0,2\pi]\Rightarrow s\notin[t-2\pi,t]$

$\therefore$ 3 cases:

When $t-2\pi\geq 2\pi$ $(t\geq 4\pi)$:

$x(s)h(t-s)=0$ for all s $\therefore y(t)=0$

When $t-2\pi<2\pi\leq t$: $(2\pi\leq t<4\pi)$

$$y(t)=\int_{t-2\pi}^{2\pi}e^{2s}\cdot 2\cos(t-s)\,ds$$

$$=\int_{t-2\pi}^{2\pi}e^{2s}\left(e^{j(t-s)}+e^{j(s-t)}\right)ds$$

$$=\int_{t-2\pi}^{2\pi}\left(e^{jt}\cdot e^{(2-j)s}+e^{-jt}\cdot e^{(2+j)s}\right)ds$$

$$=\left(\frac{e^{jt}e^{(2-j)s}}{2-j}+\frac{e^{-jt}e^{(2+j)s}}{2+j}\right)\Bigg|_{s=t-2\pi}^{s=2\pi}$$

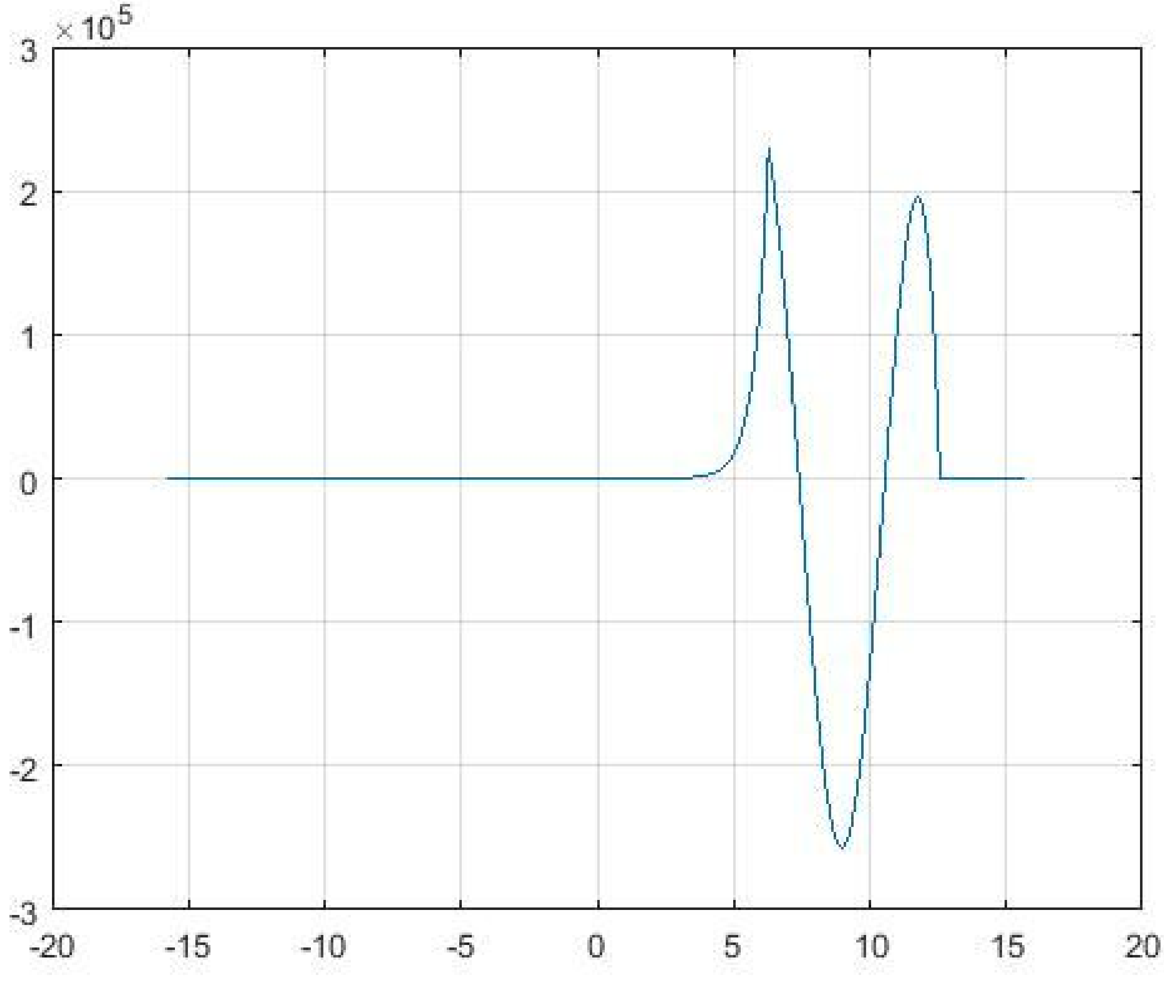
$$=\frac{e^{jt+4\pi}}{2-j}+\frac{e^{-jt+4\pi}}{2+j}-\frac{e^{2t-4\pi}}{2-j}-\frac{e^{2t-4\pi}}{2+j}$$

$$=-0.4e^{4\pi}\sin(t)+0.8e^{4\pi}\cos(t)-0.8e^{2t-4\pi}$$

When $t<2\pi$:

$$y(t)=\int_{t-2\pi}^{t}e^{2s}\cdot 2\cos(t-s)\,ds$$

$$=\frac{e^{2t}}{2-j}+\frac{e^{2t}}{2+j}-\frac{e^{2t-4\pi}}{2-j}-\frac{e^{2t-4\pi}}{2+j}=0.8\left(e^{2t}-e^{2t-4\pi}\right)$$



×10^5
3
2
1
0
-1
-2
-3
-20
-15
-10
-5
0
5
10
15
20

Question 2.

$$y(t) = x(t-1) = \begin{cases} 2^{-(t-1)} & \text{if } 0 \le t-1 < 3 \quad (1 \le t < 4) \\ 0 & \text{else} \end{cases}$$

$$\therefore f(w) = \int_1^4 2^{-(s-1)} e^{-jw(t-s)} ds$$

$$= 2e^{-jwt} \int_1^4 \left(\tfrac{1}{2} e^{jw}\right)^s ds$$

$$= 2e^{-jwt} \cdot \frac{2^{-s} e^{jws}}{-\ln 2 + jw} \Bigg|_{s=1}^{4}$$

$$= \frac{e^{-jwt}}{-\ln 2 + jw} \left(\frac{1}{8} e^{j4w} - e^{jw} \right)$$

Question 3.

1. $|x_2(t)| = |e^{j2\omega t}| = 1$ for any t $(\omega \neq 0)$

$\therefore E[x_2(t)] = \int_{-\infty}^{\infty} 1\, dt = \infty$

2. $|x_{-3}(t)| = 1$ for any t

$\therefore P[x_{-3}(t)] = 1$

3. $x_2(t)x_{-3}(-t) = e^{j5\omega t}$ has period $T = \frac{2\pi}{5\omega}$

$\frac{2\pi}{\omega} - \left(-\frac{2\pi}{\omega}\right) = \frac{4\pi}{\omega} = 10T$

$\therefore \int_{-\frac{2\pi}{\omega}}^{\frac{2\pi}{\omega}} e^{j5\omega t} = 0$

4. $x_2(t)x_2(-t) = e^{j2\omega t} \cdot e^{-j2\omega t} = 1$

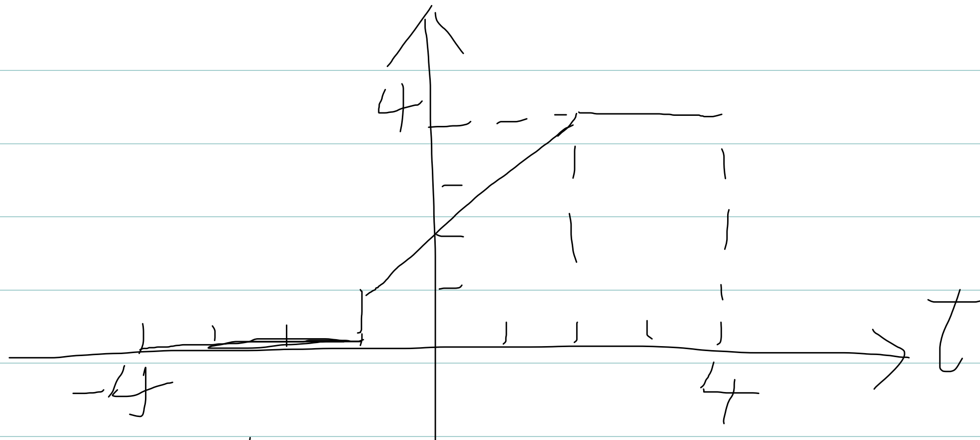
$\therefore \int_{-\frac{2\pi}{\omega}}^{\frac{2\pi}{\omega}} 1\, dt = \frac{4\pi}{\omega}$

Question 4.

1. When $t<-1$: $x(t)=0$

When $-1\leqslant t<2$: $x(t)=t+2$

When $t\geqslant 2$: $x(t)=t+2-(t-2)=4$

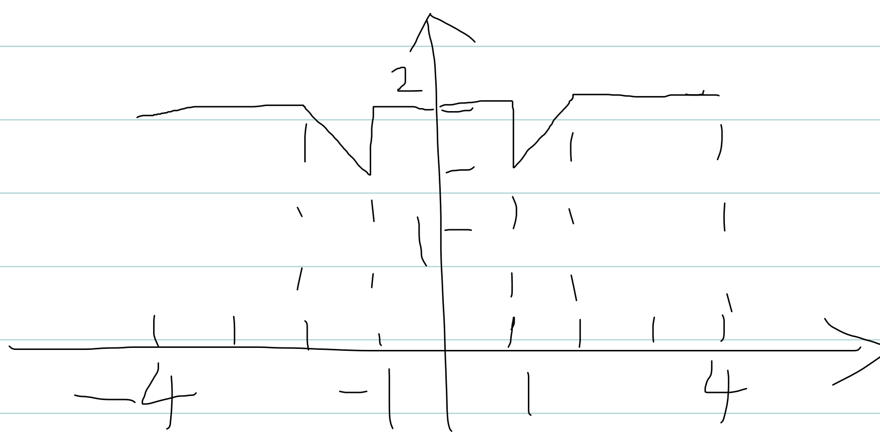


2. $Even(x(t))=\frac{1}{2}(x(t)+x(-t))$

$x(-t)=(2-t)u(1-t)+(t+2)u(-t-2)$

$\therefore Even(x(t))$

$=\frac{1}{2}\{(t+2)[u(-t-2)+u(t+1)]-(t-2)[u(1-t)+u(t-2)]$



Question 5.

1. Given input $x_1(t)$, output $y_1(t)$

If input $x_2(t) = x_1(t+\delta)$

$$y_2(t) = \int_{-0.5t-2}^{0.5t+1} x_1(2s+\delta)\,ds$$

Substitution $k = s + \frac{1}{2}\delta \quad dk = ds$

$$y_2(t) = \int_{0.5t-2+\frac{1}{2}\delta}^{0.5t+1+\frac{1}{2}\delta} x_1(2k)\,dk$$

$$y_1(t+\delta) = \int_{0.5(t+\delta)-2}^{0.5(t+\delta)+1} x_1(2s)\,ds = y_2(t)$$

$\therefore$ Time-invariant

2. Need to find $x_1(t) \neq x_2(t)$ but $y_1(t) = y_2(t)$

Let $x_1(t) = 0$, then $y_1(t) = 0$

Let $x_2(t) = \cos(\frac{\pi}{3}t)$:

$x_2(2t) = \cos(\frac{2\pi}{3}t)$ has period of 3

$$\therefore y_2(t) = \int_{0.5t-2}^{0.5t+1} \cos(\frac{2\pi}{3}s)\,ds = 0 = y_1(t)$$

Q.E.D.

Question 6

1. $x_1(t)$ is periodic, $T_1 = \frac{2\pi}{\sqrt{2}}$
   $x_2(t)$ is periodic, $T_2 = 4\pi$
   $x_3[n]$ is periodic, $T_3 = 14$
   $x_4[n]$ is periodic, $T_4 = 6$

2. $x_1(t)$ odd
   $x_2(t)$ neither
   $x_3[n]$ even
   $x_4[n]$ neither