Midterm #1 of ECE301, Section 1
8–9pm, Thursday, September 15, 2016, ME 1061.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

I certify that I have neither given nor received unauthorized aid on this exam.

Signature: Date:

Question 1: [21%, Work-out question, Learning Objectives 2 and 3]

1. [6%] Consider a signal

$$h(t) = \begin{cases} 2\cos(t) & \text{if } 0 \le t \le 2\pi \\ 0 & \text{otherwise} \end{cases}. \tag{1}$$

Plot the signal $w(t) = \delta(t-\pi) \cdot h(t)$ for the range of $-2\pi \le t \le 2\pi$.

Hint 1: If you do not know how to compute/plot $w(t)$, please simply plot $h(t)$. You will receive 2 points if your answer is correct.

2. [15%] Continue from the previous question. Consider another signal

$$x(t) = \begin{cases} e^{2t} & \text{if } t < 2\pi \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

Define

$$y(t) = \int_{s=-\infty}^{\infty} x(s)h(t-s)ds. \tag{3}$$

Compute the general expression of $y(t)$ and plot $y(t)$ for the range of $-5\pi \le t \le 5\pi$.

Hint 2: You may need to use the following formulas:

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j} \tag{4}$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}. \tag{5}$$

Hint 3: You can leave your answer in the form like $e^{j\sqrt{t}+2.5\pi} - e^{-j2\sqrt{t}-0.4}$. There is no need to further simplify this type of expression.

Question 2: [10%, Work-out question, Learning Objectives 4 and 5]

Given a discrete-time signal

$$x(t) = \begin{cases} 2^{-t} & \text{if } 0 \leq t < 3 \\ 0 & \text{otherwise} \end{cases}. \tag{6}$$

Define $y(t) = x(t-1)$. Compute the expression of the following integral

$$f(\omega) = \int_{t=-\infty}^{\infty} y(s) e^{-j\omega(t-s)} ds. \tag{7}$$

Hint: You can leave your answer in the form like $e^{j\sqrt{t}+2.5\pi} - e^{-j2\sqrt{t}-0.4}$. There is no need to further simplify this type of expression.

Question 3: [20%, Work-out question, Learning Objectives 1 and 4]

We assume ω is a constant satisfying $\omega \neq 0$. Consider the following continuous-time signals:

$$x_2(t) = e^{j2\omega t} \tag{8}$$

$$x_{-3}(t) = e^{j\cdot(-3)\omega t}. \tag{9}$$

1. [3%] Find the total energy of $x_2(t)$.
2. [5%] Find the overall average power of $x_{-3}(t)$.
3. [6%] Compute the expression of $\int_{-\frac{2\pi}{\omega}}^{\frac{2\pi}{\omega}} x_2(t)x_{-3}(-t)dt$.
4. [6%] Compute the expression of $\int_{-\frac{2\pi}{\omega}}^{\frac{2\pi}{\omega}} x_2(t)x_2(-t)dt$.

Hint: You may need the formula that $e^{j\theta} = \cos(\theta) + j\sin(\theta)$.

Question 4: [10%, Work-out question, Learning Objectives 1 and 2]
Consider the following signal.

$$x(t) = (t+2) \cdot \mathcal{U}(t+1) - (t-2) \cdot \mathcal{U}(t-2) \tag{10}$$

1. [3%] Plot $x(t)$ for the range of $t = -4$ to 4.
2. [7%] Plot the even part of $x(t)$ for the range of $t = -4$ to 4.

Question 5: [19%, Work-out question, Learning Objective 1]

Consider the following system that takes a discrete-time signal $x(t)$ as input and outputs a continuous-time signal $y(t)$:

$$y(t) = \int_{0.5t-2}^{0.5t+1} x(2s)ds. \tag{11}$$

1. [14%] Is the above system is time-invariant or not? Carefully explain the steps how you decide whether the system is time-invariant or not.

2. [5%] Prove that the above system is not invertible. Carefully explain the steps how you prove that the system is not invertible.

Question 6: [20%, Multiple Choices, Learning Objectives 1 and 6]

The following questions are multiple-choice questions and there is no need to justify your answers. Consider two continuous-time signals:

$$x_1(t) = \sum_{k=4}^{9} 2^{-k}(e^{j(\sqrt{2})kt} - e^{-j(\sqrt{2})kt}) \tag{12}$$

$$x_2(t) = e^{j(t-1)}\cos(1.5t + \pi) \tag{13}$$

and two discrete-time signals:

$$x_3[n] = \sin(8\pi n^2) + \cos(\frac{5\pi}{7}n) \tag{14}$$

$$x_4[n] = \cos(\pi n^3) + \sum_{k=-\infty}^{\infty} \delta[n - (3k-1)]. \tag{15}$$

1. [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not, *respectively.* If it is periodic, write down the fundamental period. Please state explicitly which signal is periodic and which is not.

2. [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them, respectively. Please state explicitly which signal is even, which is odd, and which is neither.