Final Exam of ECE301, Section 1 (Prof. Chih-Chun Wang)
1–3pm, Tuesday, December 13, 2016, EE 129.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name: Solution

Student ID:

I certify that I have neither given nor received unauthorized aid on this exam.

Signature: Date:

Question 1: [20%, Work-out question]

1. [1%] What is the difference between "AM synchronous demodulation" and "AM asynchronous demodulation"?

   Hint 1: The former requires a very specific component in its demodulator while the latter, also known as *envelope detector*, does not need that component. If you can write down the name of that component, you will receive the full credit.

   Hint 2: The said component is usually generated by a block called "PLL."

Ans: Whether they use a synchronous carrier.

Prof. Wang wanted to transmit an AM-SSB signal. To that end, he wrote the following MATLAB code.

```
% Initialialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read two different .wav files
[x1, f_sample, N]=audioread('x1.wav');
x1=x1';
[x2, f_sample, N]=audioread('x2.wav');
x2=x2';

% Step 0: Initialize several parameters
W_1=pi*2000;
W_2=pi*4000;
W_3=pi*8000;
W_4=pi*3000;
W_5=pi*7000;
W_6=????;



% Step 1: Make the signals band-limited.
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);
```

Prof. Wang's Exam

```
% Step 2: Multiply x1_new and x2_new with a ~~cosine~~ sine wave.
x1_h=x1_new.*sin(W_2*t);
x2_h=x2_new.*sin(W_3*t);

% Step 3: Keep one of the two side bands
h_one=1/(pi*t).*(2*sin(W_4*t)).*(cos(W_5*t));
h_two=1/(pi*t).*(2*sin(W_4*t)).*(cos(W_6*t));
x1_sb=ece301conv(x1_h, h_one);
x2_sb=ece301conv(x2_h, h_two);

% Step 4: Create the transmitted signal
y=x1_sb+x2_sb;
audiowrite('y.wav', y, f_sample);
```

2. [1.5%] What is the bandwidth (Hz) of the signal x1_new?
3. [1.5%] For the first signal x1_new, is this AM-SSB transmitting an upper-side-band signal or a lower-side-band signal?
4. [1.5%] What should the value of W_6 be in the MATLAB code, if we decide to use a lower-side-band transmission for the second signal x2_new?
5. [1.5%] What should the value of W_6 be in the MATLAB code, if we decide to use an upper-side-band transmission for the second signal x2_new?

2. 1000 Hz

3. Upper Side Band

4. 5000π

5. 11000π

Knowing that Prof. Wang used the above code to generate the "y.wav" file, a student tried to demodulate the output waveform "y.wav" by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[y, f_sample, N]=audioread('y.wav');
y=y';

% Initialize several parameters
W_8=????;
W_9=????;
W_10=????;
W_11=????;
W_12=????;


% Create the low-pass filter.
h_M=1/(pi*t).*(sin(W_8*t));

% demodulate signal 1
h_BPF1=1/(pi*t).*(sin(W_9*t))-1/(pi*t).*(sin(W_10*t));
y1_BPF=ece301conv(y,h_BPF1);
y1=4*y1_BPF.*sin(pi*4000*t);
x1_hat=ece301conv(y1,h_M);

sound(x1_hat,f_sample)

% demodulate signal 2
h_BPF2=1/(pi*t).*(sin(W_11*t))-1/(pi*t).*(sin(W_12*t));
y2_BPF=ece301conv(y,h_BPF2);
y2=4*y2_BPF.*sin(pi*W_3*t);
x2_hat=ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```

6. [7.5%] Continue from the previous questions. What should the values of W_8 to W_12 be in the MATLAB code? When answering this question, please assume that the second radio x2_new is transmitted using the *upper side-band*.

7. [2.5%] Suppose that Prof. Wang decides to use the *upper-side-band* transmission for x2_new but at the same time wants to make the "frequency bands" of x1_new and x2_new as close to each other as possible. Question: What is the smallest value of W_3 that Prof. Wang can still use while not affecting the sound quality of either x1_new or x2_new?

8. [3%] Continue from the previous sub-question. In addition to changing the value of W_3, the values of W_6, W_11, and W_12 also need to be changed. Question: What are their new values?

   Hint: If you do not know the answers of Q1.2 to Q1.8, please simply draw the AMSSB modulation and demodulation diagrams and mark carefully all the parameter values. You will receive 10 points for Q1.2 to Q1.8.

6. $W_8 = 2000\pi$

   $W_9 = 6000\pi$

   $W_{10} = 4000\pi$

   $W_{11} = 10000\pi$

   $W_{12} = 8000\pi$

7. $W_3 = 6000\pi$

8. $W_{11} = 8000\pi$

   $W_{12} = 6000\pi$

   $W_6 = 9000\pi$

or

$W_3 = 2000\pi$

$W_{11} = 4000\pi$

$W_{12} = 2000\pi$

$W_6 = 5000\pi$

Question 2: [12%, Work-out question]

1. [4%] Consider the following discrete time signal

$$x_1[n] = \sin\left(\frac{4\pi}{3}n\right). \tag{1}$$

Let $X_1(e^{j\omega})$ denote the corresponding DTFT. Find the expression of $X_1(e^{j\omega})$ and plot $X_1(e^{j\omega})$ for the range of $-2\pi < \omega < 2\pi$.

2. [4%] Consider a discrete time signal $x_2[n]$ with the corresponding DTFT being

$$X_2(e^{j\omega}) = \sin\left(5\omega + \frac{\pi}{4}\right). \tag{2}$$

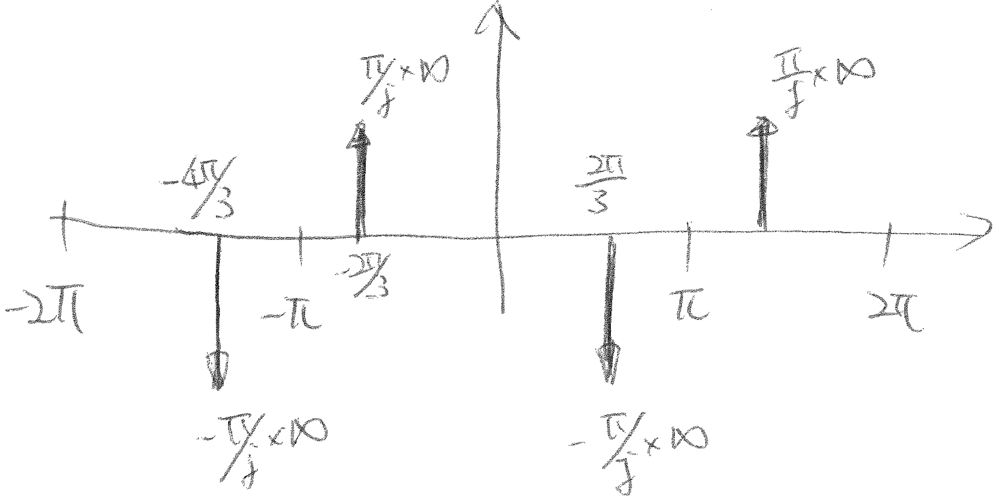
Find the expression of $x_2[n]$.

3. [4%] Consider the following discrete time signal

$$x_3[n] = \begin{cases} n & \text{if } -100 \le n \le 100 \\ 0 & \text{otherwise} \end{cases}. \tag{3}$$

Let $X_3(e^{j\omega})$ denote the corresponding DTFT. Find the value of $\int_{\omega=100\pi}^{102\pi} X_3(e^{j\omega})d\omega$.

$$X_1(e^{j\omega}) = \begin{cases} \frac{\pi}{j}\delta\left(\omega + \frac{2\pi}{3}\right) - \frac{\pi}{j}\delta\left(\omega - \frac{2\pi}{3}\right) & \text{if } -\pi < \omega \le \pi \\ \text{period } 2\pi. \end{cases}$$



2. $X_2(e^{j\omega}) = \frac{1}{2j} e^{j5\omega} \cdot e^{j\frac{\pi}{4}} - \frac{1}{2j} e^{-j5\omega} \cdot e^{-j\frac{\pi}{4}}$

$$\Rightarrow x_2[n] = \begin{cases} \frac{e^{j\frac{\pi}{4}}}{2j} & \text{if } n = -5 \\ -\frac{e^{-j\frac{\pi}{4}}}{2j} & \text{if } n = 5 \\ 0 & \text{otherwise.} \end{cases}$$

3. 0

Question 3: [17%, Work-out question]

Consider a continuous time signal $x(t) = \cos(15\pi t) + \sin(10\pi t)$ and we use a digital voice recorder to convert the continuous time signal $x(t)$ to its discrete time counterpart $x[n]$ with sampling frequency 10Hz. Answer the following questions.

1. [3%] Write the explicit expression of $x[n]$ in this question. Namely, your answer should be something like $x[n] = \cos \ldots$.

2. [3%] Plot $x[n]$ for the range of $-4 \leq n \leq 4$.

3. [3%] Continue from the previous question. We use "zero-order hold" to reconstruct the original signal. We denote the output by $\hat{x}_{\mathrm{ZOH}}(t)$. Plot $\hat{x}_{\mathrm{ZOH}}(t)$ for the range of $-0.5 < t < 0.5$.

   [Alternative question:] If you do not know the answer to this question, you can assume that the sampling period is $T = \frac{1}{5}$ and the sampled values are $x[n] = \sin(0.5\pi n)$. Plot the Zero-Order-Hold output $\hat{x}_{\mathrm{ZOH}}(t)$ for the range of $t = -1$ to 1. If your answer is correct, you will receive 2.5 points.

4. [4%] Let $x_p(t)$ denote the *impulse train-sampled signal* of $x(t)$. Plot $x_p(t)$ for the range of $-0.5 < t < 0.5$.

5. [4%] Consider an LTI system with impulse response

$$h_2(t) = \frac{\sin(10\pi t))}{10\pi t} \tag{4}$$

   Let $y_2(t)$ denote the convolution $y_2(t) = x_p(t) * h_2(t)$. Find the expression of $y_2(t)$ and plot $y_2(t)$ for the range of $-0.5 < t < 0.5$.

   Hint 1: This is a relatively harder question. You should not spend too much time on this question if you do not know how to approach it.

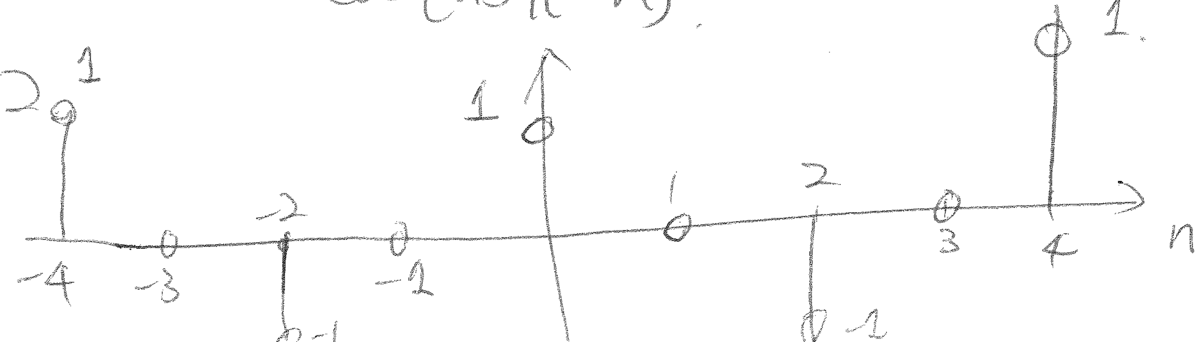
   Hint 2: You should first plot $y_2(t)$ and try to "guess" what is the expression of $y_2(t)$ and then write two to four sentences to justify your guess.

   Hint 3: If you do not know the answer to this sub-question, you can plot $h_2(t)$ for the range of $-0.5 < t < 0.5$. and then plot $y_2(t)$ for the range of $-0.5 < t < 0.5$ without finding the expression of $y_2(t)$. You will still receive 3 points if your answers are correct.

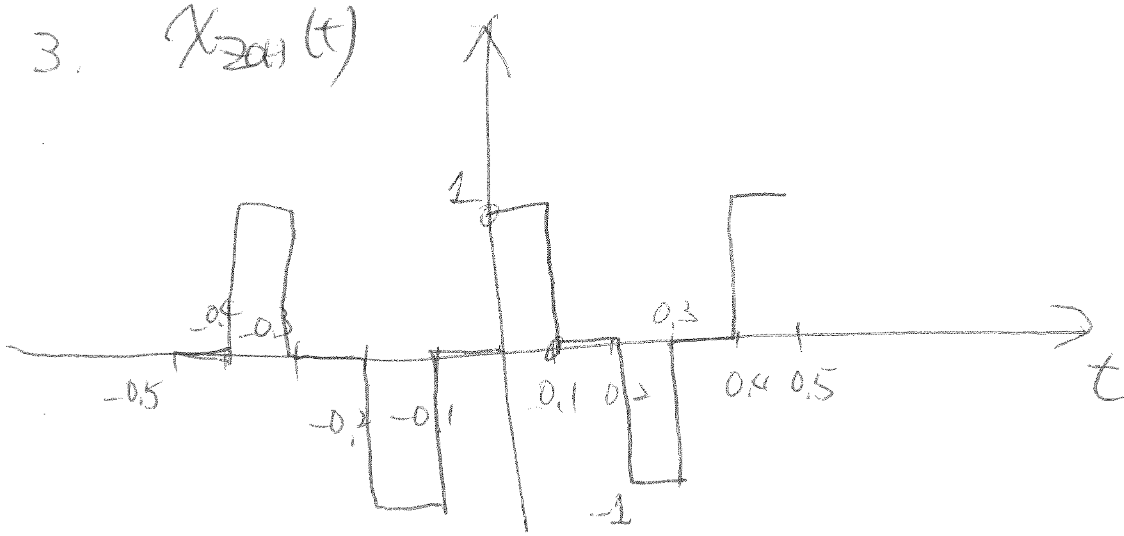
1. $X[n] = \cos(1.5\pi \cdot n) + \sin(\pi \cdot n)$

$= \cos(1.5\pi \cdot n)$

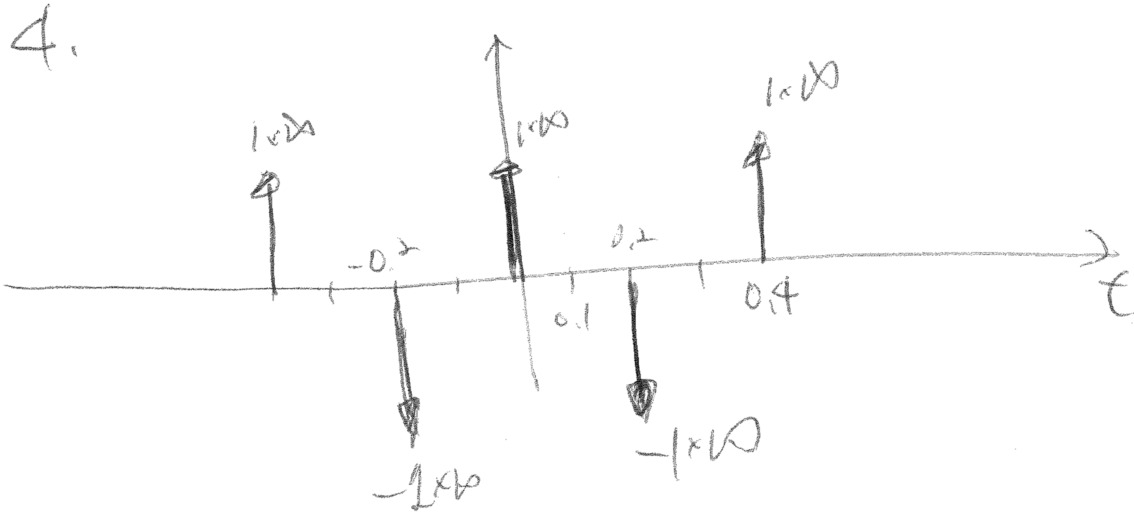
2.



3. $\hat{x}_{ZOH}(t)$



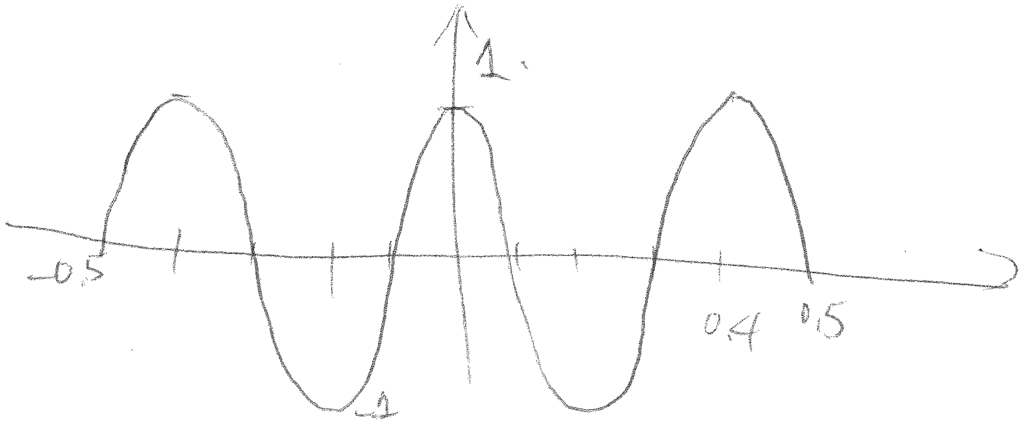
4.



5.

$$y_2(t) = \cos\left(\frac{2\pi}{0.4}t\right)$$

$$= \cos(5\pi \cdot t)$$



Question 4: [10%, Work-out question]

Consider a digital voice recorder that converts a continuous time waveform $x(t)$ into a discrete time array $x[n]$ with sampling frequency 44.1kHz. The $x[n]$ is stored in a .wav file. Answer the following questions.

1. [4%] Suppose I pass $x[n]$ through the following discrete-time signal processing equation:

$$y[n] = x[n] * h[n] \tag{5}$$

where

$$h[n] = \frac{\sin(0.5\pi n)}{\pi n}. \tag{6}$$

After processing $x[n]$ to generate $y[n]$ and storing it into a new .wav file, when I play the new wav file (containing $y[n]$), how will the new music file sound?

Hint 1: A couple of sentences describing the qualitative behavior will be enough. There is no need to have very exact expression.

Hint 2: If you do not know the answer, you can assume $h[n] = \frac{1}{5}(\sum_{k=-2}^{2} \delta[n-k])$ and answer the following alternative question instead: Is this signal processing block a low-pass filter or a high-pass filter? You will receive 3 points if your answer is correct.

2. [4%] Suppose my very old MP3 player can only play audio files that are sampled at 22.05kHz. Also suppose that I did not know the limitation of my MP3 player and incorrectly fed my recorded array $x[n]$ to the old MP3 player. How will the music file sound in the old MP3 player when the "sampling frequency 44.1kHz" is now twice as large as the "playback frequency 22.05kHz"? A quick sentence or two that explain how the sound quality will change is sufficient.

3. [2%] What kind of processing do we need to apply to $x[n]$ first (before playing back using the old MP3 player) so that the new $x_{\text{new}}[n]$ can be properly played by the old MP3 player?

1. The high-freq component has been removed. Specifically, any freq higher than $\frac{44.1k}{4} = 11.025$ kHz has been removed.

2. Sound 2-time slower.

3. Shorten the array by removing entries with odd time indices.

Question 5: [9%, Work-out question]

Consider the following discrete-time signals

$$x[n] = 0.5^n \mathcal{U}[n - 100] \tag{7}$$

1. [1%] What does "ROC" stand for?
2. [3.5%] Denote the Z-transform of $x[n]$ by $X(z)$. Find out the value of $X(1+\sqrt{3}j)$. Namely evaluate $X(z)$ when $z = 1 + \sqrt{3}j$.
3. [4.5%] Find the Z-transform $X(z)$ for general z values and carefully specify and draw its ROC in the corresponding z-plane.

   Hint: To decide the ROC, you need to carefully check whether you can apply the following formula that solves a geometric series.

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r} \quad \text{if } |r| < 1. \tag{8}$$

1. Region of Convergence.

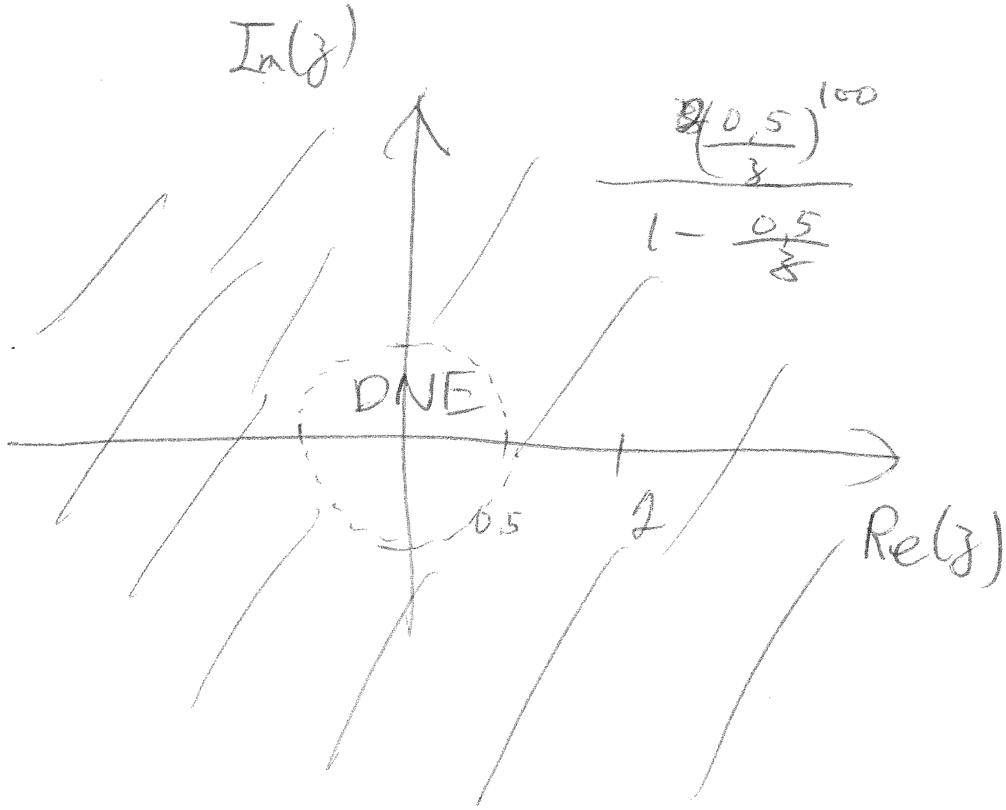
2. $X(1+\sqrt{3}j) = \sum_{n=100}^{\infty} 0.5^n (1+\sqrt{3}j)^{-n}$

$$= \frac{\left(\frac{0.5}{1+\sqrt{3}j}\right)^{100}}{1 - \frac{0.5}{1+\sqrt{3}j}}$$

3. $X(z) = \sum_{n=100}^{\infty} 0.5^n z^{-n}$

$$= \begin{cases} \dfrac{\left(\frac{0.5}{z}\right)^{100}}{1 - \frac{0.5}{z}} & \text{if } \left|\frac{0.5}{z}\right| < 1. \Leftrightarrow |z| > 0.5 \\ \text{does not exist} & \text{if } |z| \leq 0.5 \end{cases}$$

$$\frac{\left(\frac{0.5}{z}\right)^{100}}{1-\frac{0.5}{z}}$$



Question 6: [9%, Work-out question]
Consider a CT LTI system with impulse response

$$h(t) = \frac{\sin(3t)}{\pi t} \tag{9}$$

1. [6%] Find the output $y(t)$ when the input is $x(t) = \cos(2t) + \sin(\pi t)$.
2. [1.5%] Is $x(t)$ periodic? Use 1 sentence to justify your answer.
3. [1.5%] Is $y(t)$ periodic? Use 1 sentence to justify your answer.

   Hint: If you do not know the answer of $y(t)$, you can assume $y(t) = e^{\cos(\pi t)+\sin(3\pi t)^2}$. You will be given full credit if your answer is correct.

1. $y(t) = \cos(2t)$
2. No since LCM$(2, \pi)$ does not exist.
3. Yes. period is $\frac{2\pi}{2} = \pi$.

Question 7: [8%, Work-out question]

Consider the following continuous time system.

$$y(t) = \begin{cases} \int_{s=t-1}^{t} x(s)ds & \text{if } 0 < t \\ \int_{s=t-2}^{t} x(s)ds & \text{if } t \leq 0 \end{cases}. \tag{12}$$

Prove that the above system is not time-invariant. You need to write down the detailed steps in the proof.

$$\longrightarrow \boxed{\text{delay}} \xrightarrow{x(t-\Delta)} \boxed{\text{sys}} \xrightarrow{y(t)}$$

$$y(t) = \begin{cases} \int_{s=t-1}^{t} x(s-\Delta)ds & \text{if } 0<t. \\ \int_{s=t-2}^{t} x(s-\Delta)ds & \text{if } t \leq 0. \end{cases}$$

$$\longrightarrow \boxed{\text{sys}} \longrightarrow \boxed{\text{delay}} \longrightarrow$$

$$y(t) = \begin{cases} \int_{s=t-\Delta-1}^{t-\Delta} x(s)ds & \text{if } 0<t-\Delta. \\ \int_{s=t-\Delta-2}^{t-\Delta} x(s)ds & \text{if } t-\Delta \leq 0. \end{cases}$$

they are NOT equal.

$\Rightarrow$ NOT time-invariant.

Question 8: [15%, Multiple-choice question] Consider two signals

$$h_1(t) = e^{-|t|\cos(t^3)} \tag{11}$$

and

$$h_2[n] = \sin(0.5 \cdot |n+1| \cdot \pi) + \sin(0.5 \cdot (n+1) \cdot \pi) \tag{12}$$

1. [1.25%] Is $h_1(t)$ periodic? No.
2. [1.25%] Is $h_2[n]$ periodic? No
3. [1.25%] Is $h_1(t)$ even or odd or neither? even
4. [1.25%] Is $h_2[n]$ even or odd or neither? neither
5. [1.25%] Is $h_1(t)$ of finite energy? Yes
6. [1.25%] Is $h_2[n]$ of finite energy? No

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless? No
2. [1.25%] Is System 2 memoryless? No
3. [1.25%] Is System 1 causal? No
4. [1.25%] Is System 2 causal? Yes
5. [1.25%] Is System 1 stable? Yes
6. [1.25%] Is System 2 stable? No.