Question 1: [17%, Work-out question, Learning Objectives 1, 2, 4, and 5]

1. [1%] What does the acronym "AM-DSB" stand for?

Amplitude Modulation - Double Side Band.

Prof. Wang wanted to transmit an AM-SSB signal. To that end, he wrote the following MATLAB code.

```
% Initialialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read two different .wav files
[x1, f_sample, N]=wavread('x1');
x1=x1';
[x2, f_sample, N]=wavread('x2');
x2=x2';

% Step 0: Initialize several parameters
W_1=pi*4000;
W_2=pi*7000;
W_3=pi*11000;
W_4=pi*11000;
W_5=pi*7000;
W_6=????;
W_7=????;


% Step 1: Make the signals band-limited.
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);

% Step 2: Multiply x1_new and x2_new with a sinusoidal wave.
x1_h=x1_new.*cos(W_2*t+pi/3);
x2_h=x2_new.*cos(W_3*t+pi/8);

% Step 3: Keep one of the two side bands
h_one=1/(pi*t).*(sin(W_4*t))-1/(pi*t).*(sin(W_5*t));
h_two=1/(pi*t).*(sin(W_6*t))-1/(pi*t).*(sin(W_7*t));
```

```
x1_sb=ece301conv(x1_h, h_one);
x2_sb=ece301conv(x2_h, h_two);

% Step 4: Create the transmitted signal
y=x1_sb+x2_sb;
wavwrite(y', f_sample, N, 'y.wav');
```

2. [1.5%] What is the bandwidth (Hz) of the signal x1_new? 2000 Hz

3. [2.5%] Is this AM-SSB transmitting an upper-side-band signal or a lower-side-band signal?

   upper side band.

4. [3%] What should the values of W_6 and W_7 be in the MATLAB code?

   $\pi \cdot 15000$ $\quad$ $\pi \cdot 11000$

Knowing that Prof. Wang used the above code to generate the "y.wav" file, a student tried to demodulate the output waveform "y.wav" by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[y, f_sample, N]=wavread('y');
y=y';

% Initialize several parameters
W_8=????;
W_9=????;
W_10=????;
W_11=????;

% Create the low-pass filter.
h_M=1/(pi*t).*(sin(pi*4000*t));

% Create the band-pass filters.
h_BPF1=1/(pi*t).*(sin(pi*11000*t)-sin(pi*7000*t));
h_BPF2=1/(pi*t).*(sin(W_8*t)-sin(W_9*t));


% demodulate signal 1
y1_BPF=ece301conv(y,h_BPF1);
y1=4*y1_BPF.*cos(pi*W_10*t);
x1_hat=ece301conv(y1,h_M);

sound(x1_hat,f_sample)

% demodulate signal 2
y2_BPF=ece301conv(y,h_BPF2);
y2=4*y2_BPF.*cos(W_11*t);
x2_hat=ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```

5. [4%] Continue from the previous question. What should the values of W_8 to W_11 in the MATLAB code?

$W_8 : \pi \cdot 15000$ $\quad$ $W_{10} : \pi \cdot 7000$

$W_9 - \pi \cdot 11000.$ $\quad$ $W_{11}\ \pi \cdot 11000.$

6. [5%] It turns out that using the above MATLAB code, the demodulated signals x1_hat and x2_hat do not sound the same as the original signals x1_new and x2_new. Please answer the following questions:

   (i) Describe what is the "difference" when playing x1_hat versus x1_new.

   (ii) Which one (radio station 1 versus radio station 2) has a more severe problem?

   (iii) If you can only change the demodulation MATLAB codes, describe how we can fix the code so that we can hear both x1_hat and x2_hat properly.

   Hint: If you do not know the answers of Q1.2 to Q1.6, please simply draw the modulation and demodulation diagrams of AMSSB with upper side band. You need to carefully mark all the parameter values in your diagram. You will receive 10 points for Q1.2 to Q1.6.

(i) The volume is much weaker.

(ii) radio station 1.

(iii) y1 = 4 * y1_BPF .* cos(Pi * 7000 * t + PI/3).

y2 = 4 * y2_BPF .* cos(Pi * 11000 * t + pi/8).

Question 2: [14%, Work-out question, Learning Objectives 1, 4, and 5]

Consider a discrete-time signal

$$x[n] = \begin{cases} \frac{\pi}{4} & \text{if } n = 0 \\ \frac{\sin(\frac{\pi}{4}n)}{\pi n} & \text{otherwise} \end{cases} \quad (1)$$

1. [2%] Plot $x[n]$ for the range of $-5 \leq n \leq 5$.
2. [2%] Plot $X(e^{j\omega})$, the DTFT of $x[n]$, for the range of $-2\pi \leq \omega \leq 2\pi$. Hint: Table 5.2 may be useful.

Consider another signal

$$p[n] = \begin{cases} 1 & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases} \quad (2)$$

3. [1%] Plot $p[n]$ for the range of $-5 \leq n \leq 5$.
4. [2%] Plot $P(e^{j\omega})$, the DTFT of $p[n]$, for the range of $-2\pi \leq \omega \leq 2\pi$. Hint: Table 5.2 may be useful.

Let $y[n] = x[n] \cdot p[n]$.

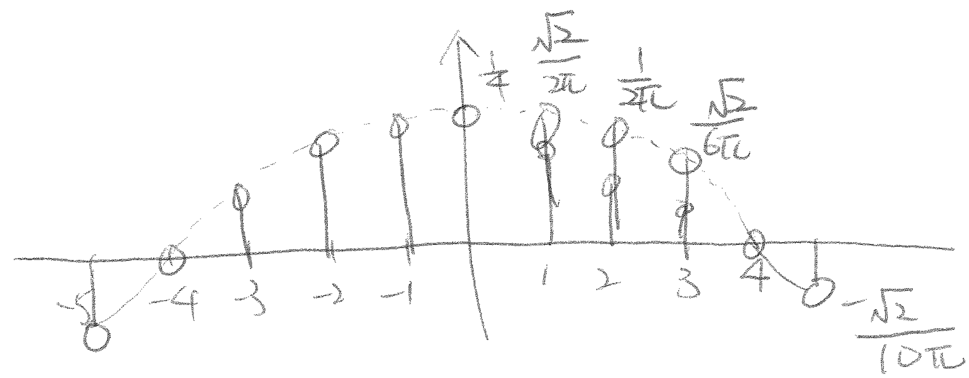
5. [2%] Plot $y[n]$ for the range of $-5 \leq n \leq 5$.
6. [5%] Plot $Y(e^{j\omega})$, the DTFT of $y[n]$, for the range of $-2\pi \leq \omega \leq 2\pi$. Hint: If you do not know how to solve Q2.6, you can solve the following question instead and you will get 3.5 points if your answer is correct.

   Suppose

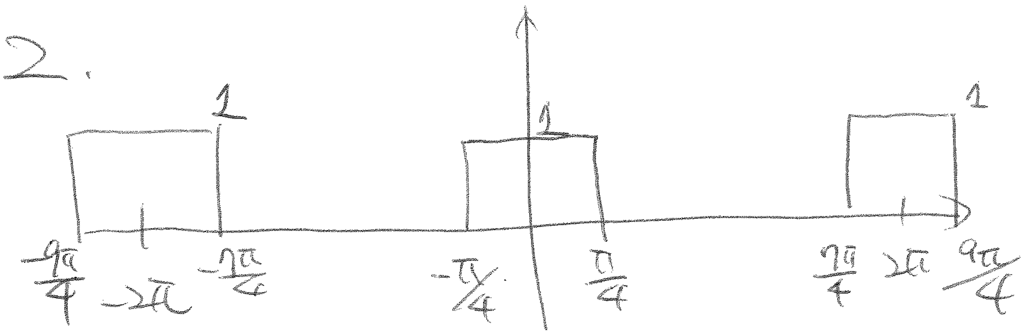
$$Z(j\omega) = \begin{cases} \omega + \frac{\pi}{3} & \text{if } -\frac{\pi}{3} \leq \omega \leq 0 \\ -\omega + \frac{\pi}{3} & \text{if } 0 \leq \omega \leq \frac{\pi}{3} \\ 0 & \text{otherwise} \end{cases} . \quad (3)$$

   Define $W(j\omega) = Z(j\omega) * \sum_{h=-\infty}^{\infty} \delta(\omega - h\pi)$. Plot $W(j\omega)$ for the range of $-2\pi \leq \omega \leq 2\pi$.

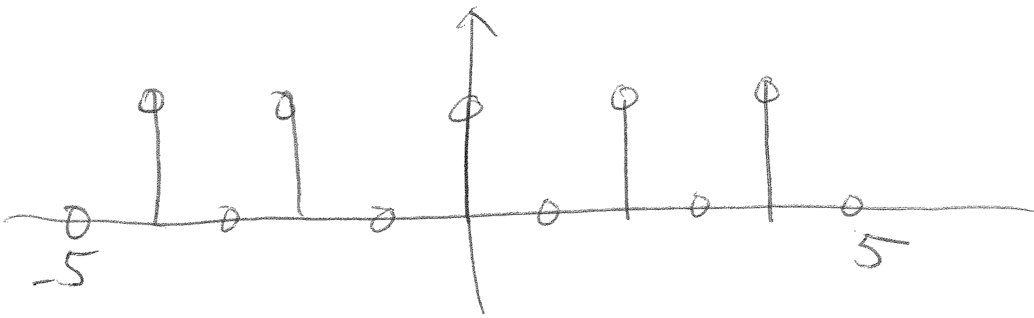
1.



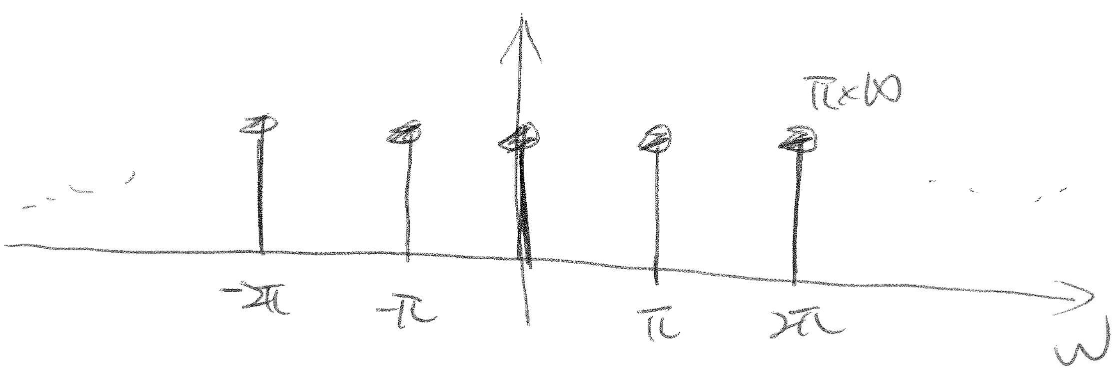
2.



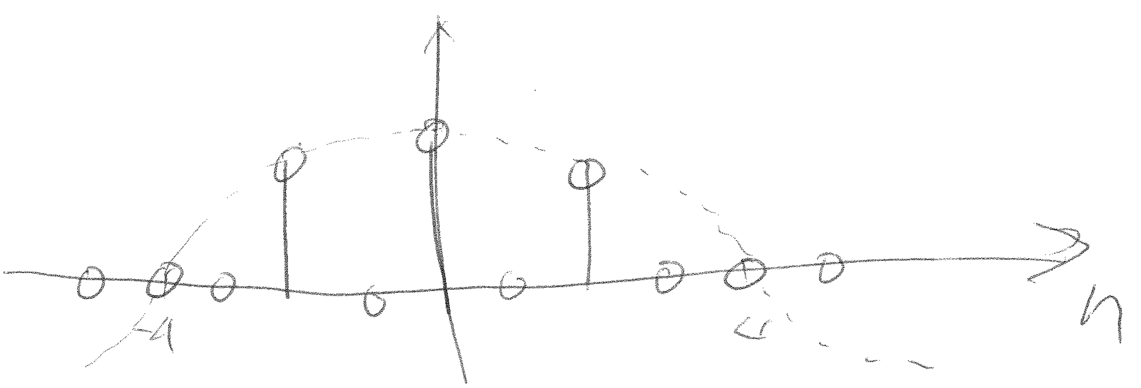
3.



4.

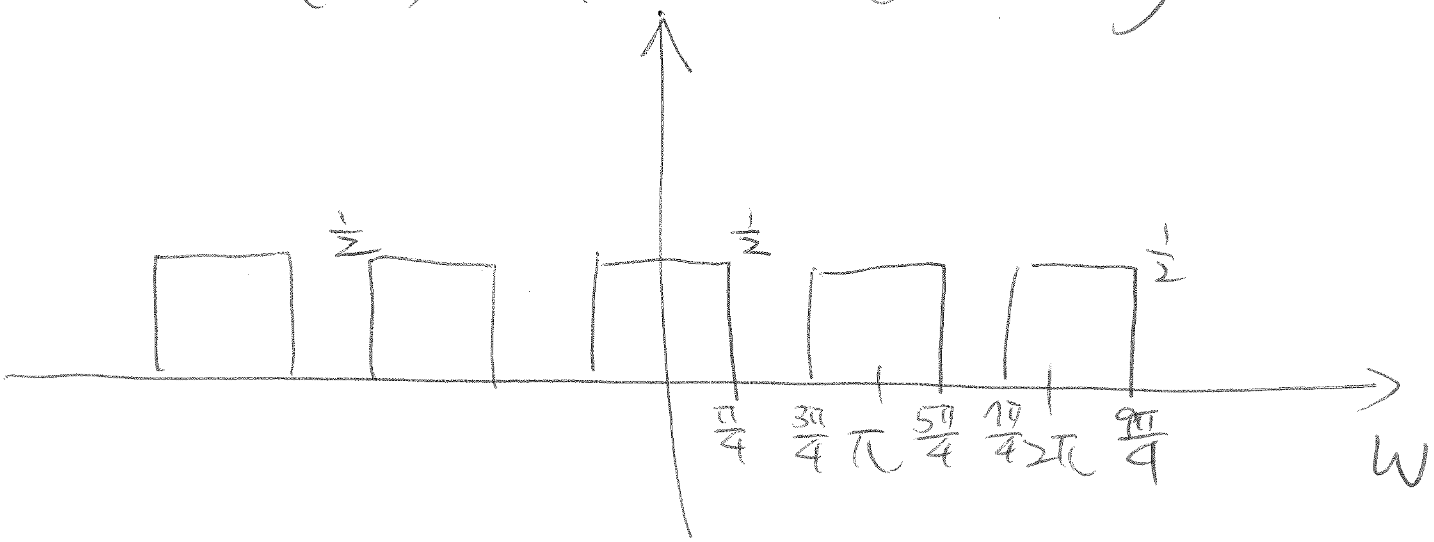


5.



6.

$$Y(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) \circledast P(e^{j\omega})$$



Question 3: [14%, Work-out question, Learning Objectives 1, 2, 3, 4, and 5] Suppose a continuous-time signal $x(t)$ is sampled with sampling frequency 2Hz, and the sampled array values are

$$x[n] = \begin{cases} 2 & \text{if } n = 0 \\ 1 & \text{if } n = 4 \\ 0 & \text{otherwise} \end{cases}. \tag{4}$$

1. [2%] Let $x_{\text{lin}}(t)$ denote the reconstructed signal using *linear interpolation.* Plot $x_{\text{lin}}(t)$ for the range of $-4 \leq t \leq 4$.

2. [2%] Let $x_{\text{zoh}}(t)$ denote the reconstructed signal using *Zero-Order Hold.* Plot $x_{\text{zoh}}(t)$ for the range of $-4 \leq t \leq 4$.

3. [3%] Let $x_{\text{opt}}(t)$ denote the reconstructed signal using the optimal *band-limited interpolation.* Plot $x_{\text{opt}}(t)$ for the range of $-4 \leq t \leq 4$.

Prof. Wang downloaded a *high-quality* audio file, which is sampled at 176.4kHz and stored in the mono .wav format. Namely, when read by MATLAB, the audio file of Prof. Wang can be represented by an array $w[n]$.

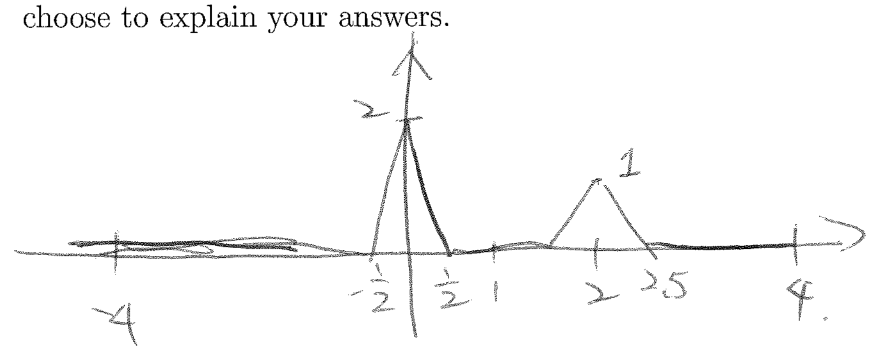
After downloading the file, Prof. Wang realized that his music player can only play at the sampling rate 44.1kHz. Because of this match (176.4kHz versus 44.1kHz), Prof. Wang cannot listen to the audio file properly.

4. [3%] If we use the 44.1kHz music player to play this high-quality audio file (176.4kHz), how will it sound?

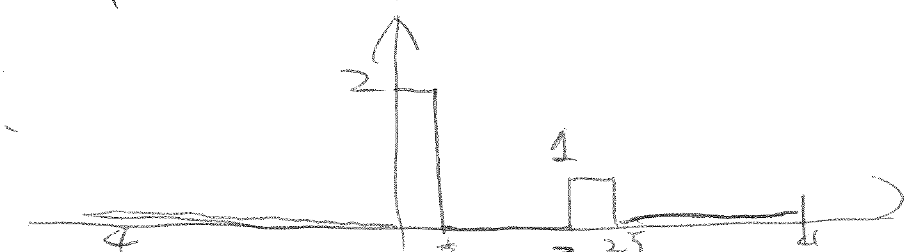
5. [4%] How would you use MATLAB to "convert" the file $w[n]$ so that the new file can be played by Prof. Wang's music player?

   Hint: You can either just describe carefully, in plain English, what is your main idea; or you can choose to use a pseudo code to answer this question; or you can actually write down the MATLAB code directly. Your answer will be graded based on whether the concepts are correct. It will not be graded based on which way you choose to explain your answers.

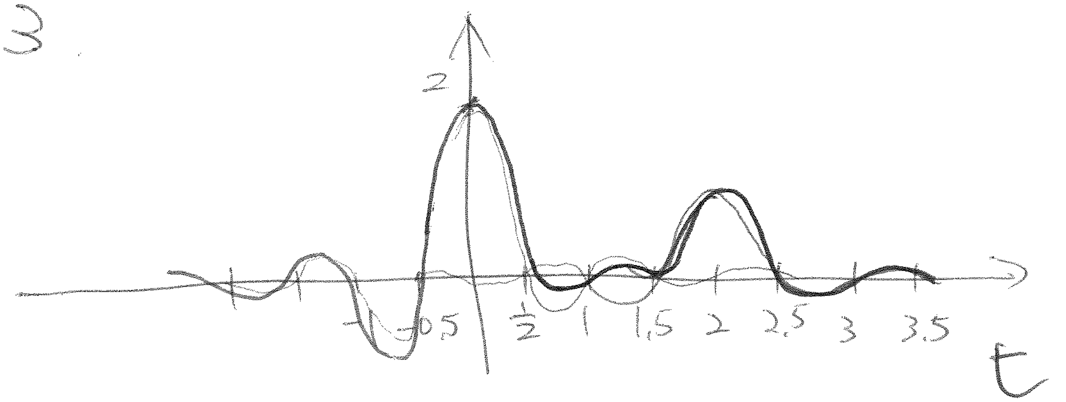
1.



2.

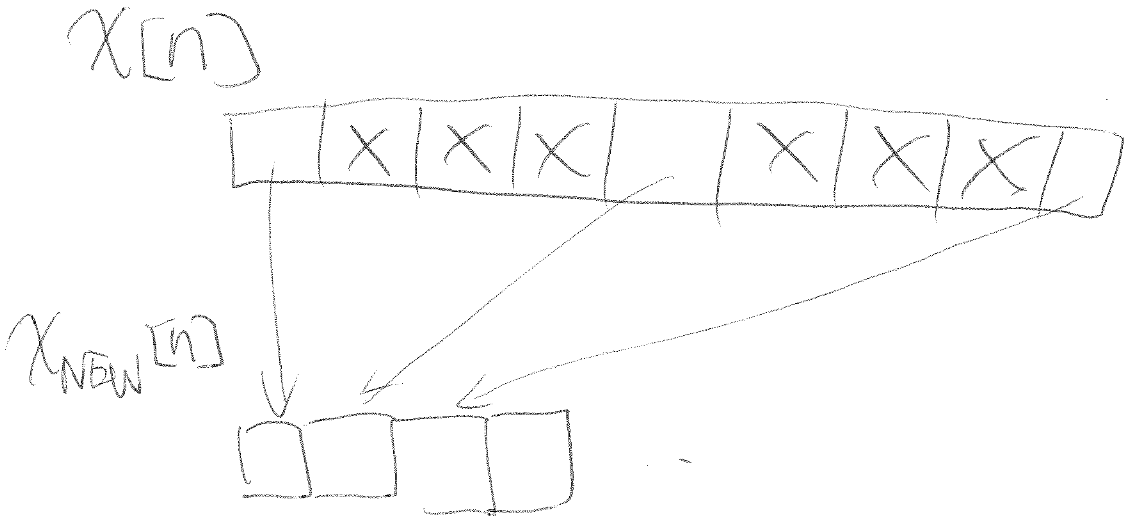


3.



4. Sound very "slow".

5. Remove 3 entries out of every 4 values.



Question 4: [15%, Work-out question, Learning Objectives 3, 4, and 5] For any continuous time signal $x(t)$, let $x_p(t)$ denote the corresponding impulse-train-sampled signal with sampling frequency 2Hz.

1. [3%] Suppose $x(t) = \sin(3\pi t)$. Plot the corresponding CTFT $X(j\omega)$ for the range of $-4\pi \leq \omega \leq 4\pi$.

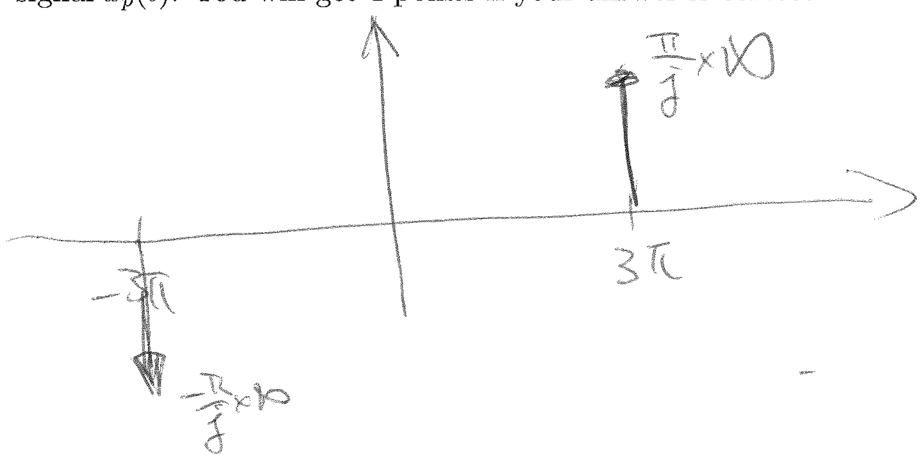
2. [4%] Continue from the above question. Plot the CTFT $X_p(j\omega)$ of the impulse-train-sampled signal $x_p(t)$ for the range of $-4\pi \leq \omega \leq 4\pi$.

3. [4%] Continue from the above questions. Assuming the optimal band-limited reconstruction is used to generate the reconstructed signal $\hat{x}(t)$ from $x_p(t)$. Plot the CTFT $\hat{X}(j\omega)$ of the reconstructed signal $\hat{x}(t)$ for the range of $-4\pi \leq \omega \leq 4\pi$.

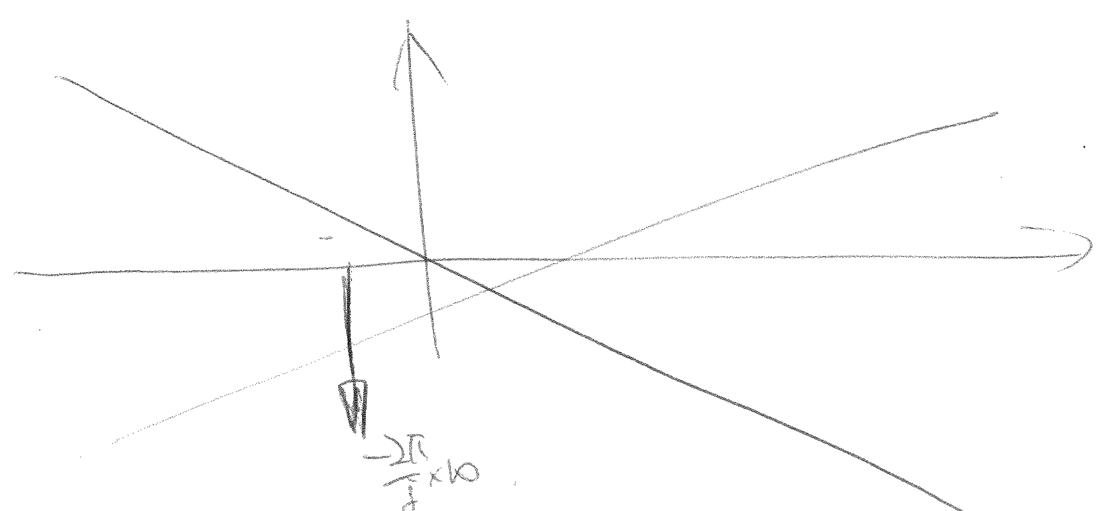
4. [4%] Continue from the above questions. Find the expression of the reconstructed signal $\hat{x}(t)$. There is no need to plot $\hat{x}(t)$.

   Hint: If you do not know the answers of Q4.3 and Q4.4, you can simply draw the diagram how to reconstruct the original signal $\hat{x}(t)$ from the impulse-train-sampled signal $x_p(t)$. You will get 4 points if your answer is correct.

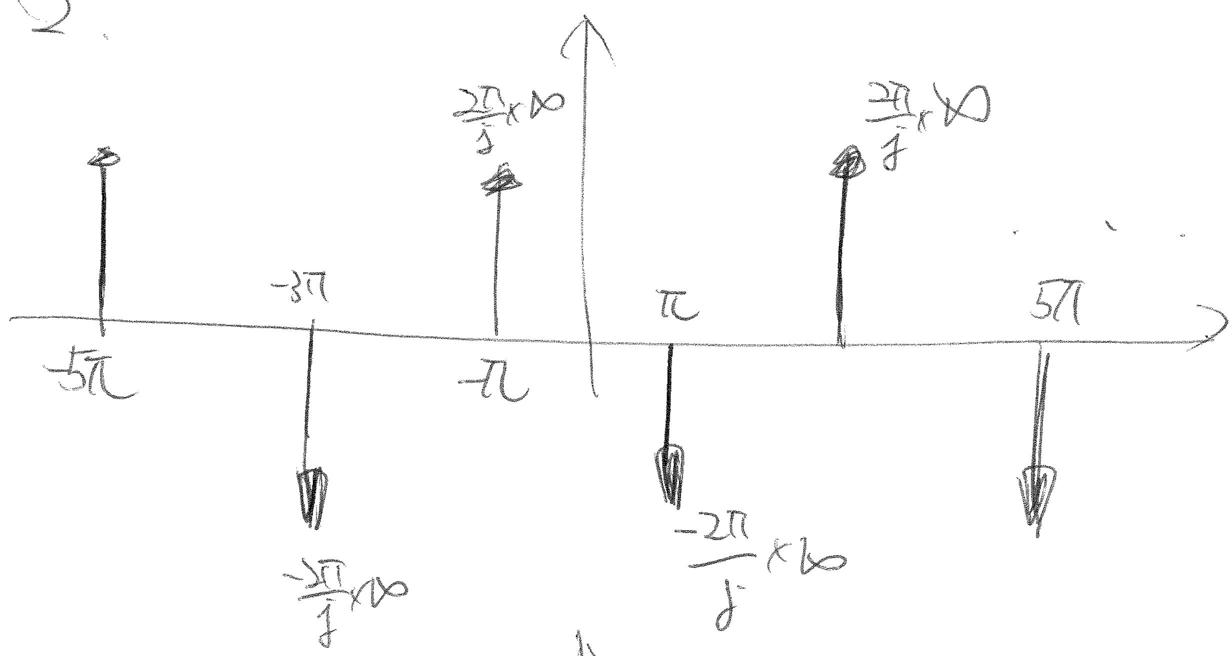
1.



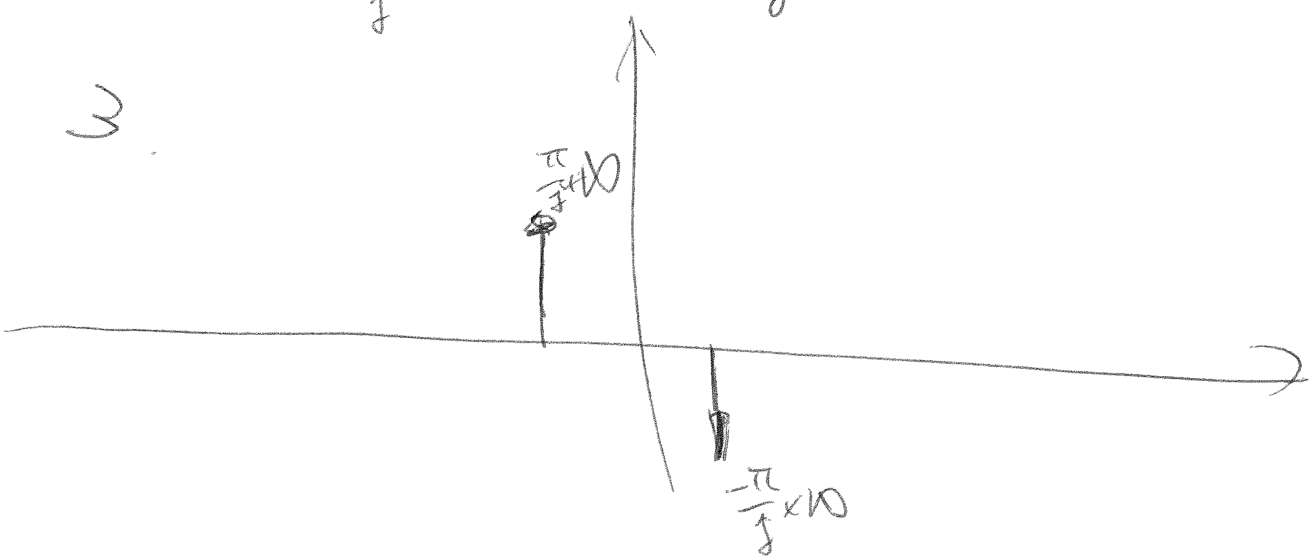
2. $W_s = 2\pi \cdot 2 = 4\pi$ rad/s $\quad \frac{1}{T} = 2$



2.


$\frac{2\pi}{j} \times \infty$
$\frac{2\pi}{j} \times \infty$
-3π
π
5π
-5π
$-\pi$
$\frac{-2\pi}{j} \times \infty$
$\frac{-2\pi}{j} \times \infty$

3.


$\frac{\pi}{j} \times \infty$
$\frac{-\pi}{j} \times \infty$

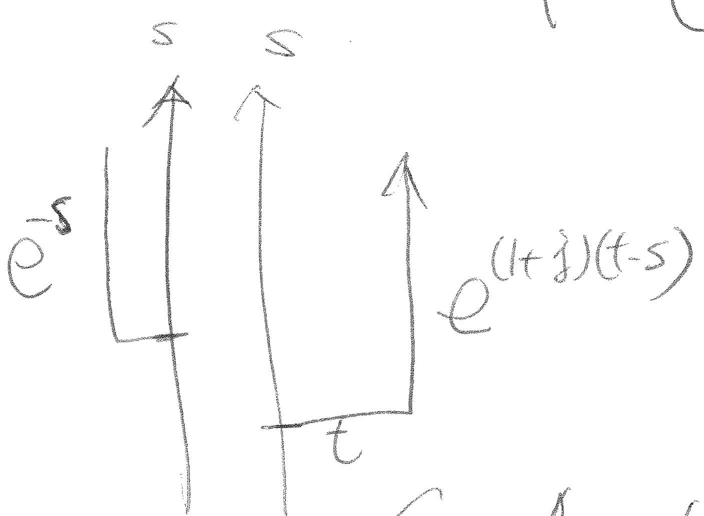
4. $-\sin(\pi t)$

Question 5: [8%, Work-out question, Learning Objectives 3, 4, and 5]
Define $x(t) = e^{-t}\mathcal{U}(t)$ and

$$h(t) = \begin{cases} e^{(1+j)t} & \text{if } t \leq 0 \\ 0 & \text{otherwise} \end{cases}. \tag{5}$$

Define $y(t) = x(t) * h(t)$. Find the expression of $y(t)$.

$$h(t-s) = \begin{cases} e^{+(1+j)(t-s)} & \text{if } t-s \leq 0 \quad s \geq t. \\ 0 & \text{o.w.} \end{cases}$$



Case 1: $t \leq 0$.

$$\Rightarrow y(t) = \int_0^{\infty} e^{-s} e^{(1+j)(t-s)} \, ds$$

$$= e^{(1+j)\cdot t} \cdot \int_0^{\infty} e^{-(2+j)s} \, ds$$

$$= \frac{e^{(1+j)\cdot t}}{(2+j)}$$

Case 2: $t \geq 0$.

$$y(t) = \int_{t}^{\infty} e^{-s} e^{(1+j)(t-s)} \, ds$$

$$= e^{(1+j)t} \int_{t}^{\infty} e^{-(2+j)s} \, ds$$

$$= \frac{e^{-t}}{(2+j)}$$

Ans: $y(t) = \begin{cases} \dfrac{e^{(1+j) \cdot t}}{2+j} & \text{if } t \leq 0. \\ \dfrac{e^{-t}}{2+j} & \text{if } t \geq 0. \end{cases}$

Question 6: [7%, Work-out question, Learning Objectives 3, 4, and 5]
Consider a discrete-time periodic signal $x[n]$

$$x[n] = \begin{cases} 2^{-n} & \text{if } 0 \leq n \leq 19 \\ \text{periodic with period } N = 20 \end{cases}. \quad (6)$$

Find the discrete-time Fourier-series (DTFS) representation of $x[n]$. Hint: You may need to use the formula:

$$\sum_{k=1}^{K} ar^{k-1} = \frac{a(1-r^K)}{1-r} \quad \text{if } r \neq 1. \quad (7)$$

$N=20$.

$$a_k = \frac{1}{20} \sum_{n=0}^{19} x[n] e^{-jk\frac{2\pi}{20} \times n}$$

$$= \frac{1}{20} \sum_{n=0}^{19} \left(2^{-1} \cdot e^{-jk\frac{2\pi}{20}}\right)^n$$

$$= \frac{1 \cdot \left(1 - \left(2^{-1} e^{-jk\frac{\pi}{10}}\right)^{20}\right)}{20 \cdot \left(1 - 2^{-1} e^{-jk\frac{\pi}{10}}\right)}$$

for all $k=0, \ldots, 19$.

Question 7: [10%, Work-out question, Learning Objectives 3, 4, and 5]
Consider a continuous-time signal $x(t) = \cos(\pi t)$. We perform Amplitude Modulation on $x(t)$ with the carrier frequency $1000Hz$. Let $y(t)$ denote the modulated signal.

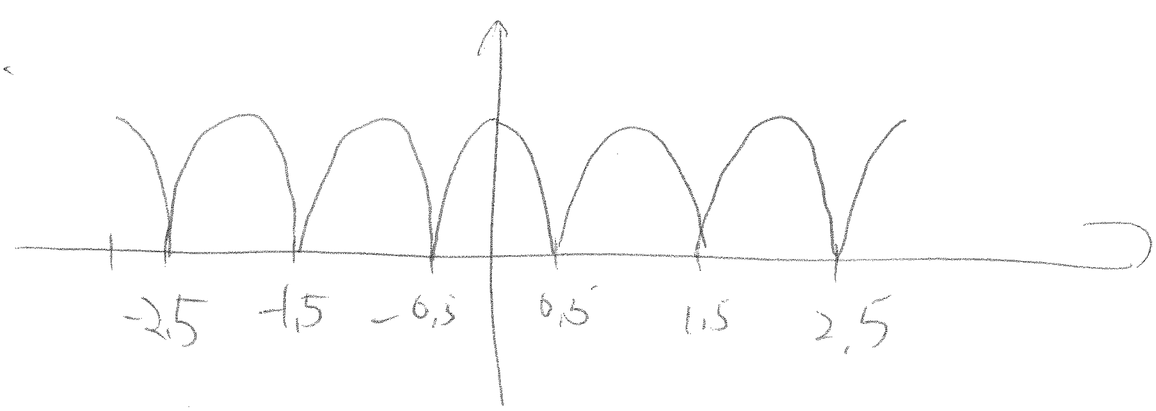
1. [4%] Write down the expression of $y(t)$ in terms of $x(t)$, and plot $y(t)$ for the range of $-3 \leq t \leq t$.

2. [6%] Suppose we use *asynchronous demodulation* and generate $\hat{x}_{\text{asyn}}(t)$ from the received signal $y(t)$. Plot $\hat{x}_{\text{asyn}}(t)$ for the range of $-3 \leq t \leq t$.

   Hint: If you do not know the answer of Q7.2, please write down (i) the most important difference between synchronous demodulation versus asynchronous demodulation, and (ii) a couple of sentences on how to perform asynchronous demodulation. You will get 4 points if your answers are correct.

1. $y(t) = \cos(t\pi) \cdot \cos(2\pi \cdot 1000 \times t)$

2.



Question 8: [15%, Multiple-choice question] Consider two signals

$$h_1(t) = e^{(\sin(2t))^2}\cos(t) \tag{8}$$

and

$$h_2[n] = \begin{cases} \sin(99\pi n) & \text{if } n \leq 99 \\ e^n & \text{if } 100 \leq n \leq 200 \\ e^{400-n} & \text{if } 201 \leq n \end{cases} \tag{9}$$

1. [1.25%] Is $h_1(t)$ periodic? Yes
2. [1.25%] Is $h_2[n]$ periodic? No
3. [1.25%] Is $h_1(t)$ even or odd or neither? even
4. [1.25%] Is $h_2[n]$ even or odd or neither? neither
5. [1.25%] Is $h_1(t)$ of finite energy? No
6. [1.25%] Is $h_2[n]$ of finite energy? Yes.

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless? No
2. [1.25%] Is System 2 memoryless? No.
3. [1.25%] Is System 1 causal? No
4. [1.25%] Is System 2 causal? Yes
5. [1.25%] Is System 1 stable? No
6. [1.25%] Is System 2 stable? Yes