Final Exam of ECE301, Section 2, Prof. Wang's section
10:30am–12:30pm, Wednesday, May 6, 2015, STEW 130.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**
2. **The first few questions may be harder and you may want to try the last few questions first.**
3. This is a closed book exam.
4. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
5. Use the back of each page for rough work.
6. Neither calculators nor help sheets are allowed.

Name:

Student ID:

I certify that I have neither given nor received unauthorized aid on this exam.

Signature: Date:

Question 1: [17%, Work-out question]

1. [1%] What does the acronym "AM-DSB" stand for?

Prof. Wang wanted to transmit an AM-SSB signal. To that end, he wrote the following MATLAB code.

```
% Initialialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read two different .wav files
[x1, f_sample, N]=wavread('x1');
x1=x1';
[x2, f_sample, N]=wavread('x2');
x2=x2';

% Step 0: Initialize several parameters
W_1=pi*4000;
W_2=pi*7000;
W_3=pi*11000;
W_4=pi*11000;
W_5=pi*7000;
W_6=????;
W_7=????;


% Step 1: Make the signals band-limited.
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);

% Step 2: Multiply x1_new and x2_new with a sinusoidal wave.
x1_h=x1_new.*cos(W_2*t+pi/3);
x2_h=x2_new.*cos(W_3*t+pi/8);

% Step 3: Keep one of the two side bands
h_one=1/(pi*t).*(sin(W_4*t))-1/(pi*t).*(sin(W_5*t));
h_two=1/(pi*t).*(sin(W_6*t))-1/(pi*t).*(sin(W_7*t));
```

```
x1_sb=ece301conv(x1_h, h_one);
x2_sb=ece301conv(x2_h, h_two);

% Step 4: Create the transmitted signal
y=x1_sb+x2_sb;
wavwrite(y', f_sample, N, 'y.wav');
```

2. [1.5%] What is the bandwidth (Hz) of the signal x1_new?
3. [2.5%] Is this AM-SSB transmitting an upper-side-band signal or a lower-side-band signal?
4. [3%] What should the values of W_6 and W_7 be in the MATLAB code?

Knowing that Prof. Wang used the above code to generate the "y.wav" file, a student tried to demodulate the output waveform "y.wav" by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[y, f_sample, N]=wavread('y');
y=y';

% Initialize several parameters
W_8=????;
W_9=????;
W_10=????;
W_11=????;

% Create the low-pass filter.
h_M=1/(pi*t).*(sin(pi*4000*t));

% Create the band-pass filters.
h_BPF1=1/(pi*t).*(sin(pi*11000*t)-sin(pi*7000*t));
h_BPF2=1/(pi*t).*(sin(W_8*t)-sin(W_9*t));


% demodulate signal 1
y1_BPF=ece301conv(y,h_BPF1);
y1=4*y1_BPF.*cos(W_10*t);
x1_hat=ece301conv(y1,h_M);

sound(x1_hat,f_sample)

% demodulate signal 2
y2_BPF=ece301conv(y,h_BPF2);
y2=4*y2_BPF.*cos(W_11*t);
x2_hat=ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```

5. [4%] Continue from the previous question. What should the values of W_8 to W_11 in the MATLAB code?

6. [5%] It turns out that using the above MATLAB code, the demodulated signals

x1_hat and x2_hat do not sound the same as the original signals x1_new and x2_new. Please answer the following questions:

(i) Describe what is the “difference” when playing x1_hat versus x1_new.

(ii) Which one (radio station 1 versus radio station 2) has a more severe problem?

(iii) If you can only change the demodulation MATLAB codes, describe how we can fix the code so that we can hear both x1_hat and x2_hat properly.

Hint: If you do not know the answers of Q1.2 to Q1.6, please simply draw the modulation and demodulation diagrams of AMSSB. You need to carefully mark all the parameter values in your diagram. You will receive 10 points for Q1.2 to Q1.6.

Question 2: [14%, Work-out question, Learning Objective 6]

Consider a discrete-time signal

$$x[n] = \begin{cases} \frac{1}{4} & \text{if } n = 0 \\ \frac{\sin(\frac{\pi}{4}n)}{\pi n} & \text{otherwise} \end{cases} \tag{1}$$

1. [2%] Plot $x[n]$ for the range of $-5 \leq n \leq 5$.
2. [2%] Plot $X(e^{j\omega})$, the DTFT of $x[n]$, for the range of $-2\pi \leq \omega \leq 2\pi$. Hint: Table 5.2 may be useful.

Consider another signal

$$p[n] = \begin{cases} 1 & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases} \tag{2}$$

3. [1%] Plot $p[n]$ for the range of $-5 \leq n \leq 5$.
4. [2%] Plot $P(e^{j\omega})$, the DTFT of $p[n]$, for the range of $-2\pi \leq \omega \leq 2\pi$. Hint: Table 5.2 may be useful.

Let $y[n] = x[n] \cdot p[n]$.

5. [2%] Plot $y[n]$ for the range of $-5 \leq n \leq 5$.
6. [5%] Plot $Y(e^{j\omega})$, the DTFT of $y[n]$, for the range of $-2\pi \leq \omega \leq 2\pi$. Hint: If you do not know how to solve Q2.6, you can solve the following question instead and you will get 3.5 points if your answer is correct.

   Suppose

$$Z(j\omega) = \begin{cases} \omega + \frac{\pi}{3} & \text{if } -\frac{\pi}{3} \leq \omega \leq 0 \\ -\omega + \frac{\pi}{3} & \text{if } 0 \leq \omega \leq \frac{\pi}{3} \\ 0 & \text{otherwise} \end{cases} . \tag{3}$$

   Define $W(j\omega) = Z(j\omega) * \sum_{h=-\infty}^{\infty} \delta(\omega - h\pi)$. Plot $W(j\omega)$ for the range of $-2\pi \leq \omega \leq 2\pi$.

Question 3: [14%, Work-out question, Learning Objective 6] Suppose a continuous-time signal $x(t)$ is sampled with sampling frequency 2Hz, and the sampled array values are

$$x[n] = \begin{cases} 2 & \text{if } n = 0 \\ 1 & \text{if } n = 4 \\ 0 & \text{otherwise} \end{cases}. \tag{4}$$

1. [3%] Let $x_{\text{zoh}}(t)$ denote the reconstructed signal using *Zero-Order Hold*. Plot $x_{\text{zoh}}(t)$ for the range of $-4 \leq t \leq 4$.

2. [4%] Let $x_{\text{opt}}(t)$ denote the reconstructed signal using the optimal *band-limited interpolation*. Plot $x_{\text{opt}}(t)$ for the range of $-4 \leq t \leq 4$.

Prof. Wang downloaded a *high-quality* audio file, which is sampled at 176.4kHz and stored in the mono .wav format. Namely, when read by MATLAB, the audio file of Prof. Wang can be represented by an array $w[n]$.

After downloading the file, Prof. Wang realized that his music player can only play at the sampling rate 44.1kHz. Because of this match (176.4kHz versus 44.1kHz), Prof. Wang cannot listen to the audio file properly.

3. [4%] If we use the 44.1kHz music player to play this high-quality audio file (176.4kHz), how will it sound?

4. [3%] How would you use MATLAB to "convert" the file $w[n]$ so that the new file can be played by Prof. Wang's music player?

   Hint: You can either just describe carefully, in plain English, what is your main idea; or you can choose to use a pseudo code to answer this question; or you can actually write down the MATLAB code directly. Your answer will be graded based on whether the concepts are correct. It will not be graded based on which way you choose to explain your answers.

Question 4: [15%, Work-out question, Learning Objective 6] For any continuous time signal $x(t)$, let $x_p(t)$ denote the corresponding impulse-train-sampled signal with sampling frequency 2Hz.

1. [3%] Suppose $x(t) = \sin(3\pi t)$. Plot the corresponding CTFT $X(j\omega)$ for the range of $-4\pi \leq \omega \leq 4\pi$.

2. [4%] Continue from the above question. Plot the CTFT $X_p(j\omega)$ of the impulse-train-sampled signal $x_p(t)$ for the range of $-4\pi \leq \omega \leq 4\pi$.

3. [4%] Continue from the above questions. Assuming the optimal band-limited reconstruction is used to generate the reconstructed signal $\hat{x}(t)$ from $x_p(t)$. Plot the CTFT $\hat{X}(j\omega)$ of the reconstructed signal $\hat{x}(t)$ for the range of $-4\pi \leq \omega \leq 4\pi$.

4. [4%] Continue from the above questions. Find the expression of the reconstructed signal $\hat{x}(t)$. There is no need to plot $\hat{x}(t)$.

   Hint: If you do not know the answers of Q4.3 and Q4.4, you can simply draw the diagram how to reconstruct the original signal $\hat{x}(t)$ from the impulse-train-sampled signal $x_p(t)$. You will get 4 points if your answer is correct.

Question 5: [8%, Work-out question]
Define $x(t) = e^{-t}\mathcal{U}(t)$ and

$$h(t) = \begin{cases} e^{(1+j)t} & \text{if } t \leq 0 \\ 0 & \text{otherwise} \end{cases}. \tag{5}$$

Define $y(t) = x(t) * h(t)$. Find the expression of $y(t)$. Hint: You may want to try direct computation.

Question 6: [7%, Work-out question]
Consider a discrete-time periodic signal $x[n]$

$$x[n] = \begin{cases} 2^{-n} & \text{if } 0 \le n \le 19 \\ \text{periodic with period } N = 20 & \end{cases}. \tag{6}$$

Find the discrete-time Fourier-series (DTFS) representation of $x[n]$. Hint: You may need to use the formula:

$$\sum_{k=1}^{K} ar^{k-1} = \frac{a(1-r^K)}{1-r} \quad \text{if } r \neq 1. \tag{7}$$

Question 7: [10%, Work-out question]
Consider a continuous-time signal $x(t) = \cos(\pi t)$. We perform Amplitude Modulation on $x(t)$ with the carrier frequency 1000Hz. Let $y(t)$ denote the modulated signal.

1. [7%] Write down the expression of $y(t)$ in terms of $x(t)$, and plot $y(t)$ for the range of $-2 \leq t \leq 2$.

2. [3%] Suppose we use *asynchronous demodulation* and generate $\hat{x}_{\text{asyn}}(t)$ from the received signal $y(t)$. Plot $\hat{x}_{\text{asyn}}(t)$ for the range of $-2 \leq t \leq 2$.

   Hint: If you do not know the answer of Q7.2, please write down a couple of sentences on how to perform asynchronous demodulation. You will get 2 points if your answers are correct.

Question 8: [15%, Multiple-choice question] Consider two signals

$$h_1(t) = e^{(\sin(2t))^2} \cos(t) \tag{8}$$

and

$$h_2[n] = \begin{cases} \sin(99\pi n) & \text{if } n \leq 99 \\ e^n & \text{if } 100 \leq n \leq 200 \\ e^{400-n} & \text{if } 201 \leq n \end{cases} \tag{9}$$

1. [1.25%] Is $h_1(t)$ periodic?
2. [1.25%] Is $h_2[n]$ periodic?
3. [1.25%] Is $h_1(t)$ even or odd or neither?
4. [1.25%] Is $h_2[n]$ even or odd or neither?
5. [1.25%] Is $h_1(t)$ of finite energy?
6. [1.25%] Is $h_2[n]$ of finite energy?

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless?
2. [1.25%] Is System 2 memoryless?
3. [1.25%] Is System 1 causal?
4. [1.25%] Is System 2 causal?
5. [1.25%] Is System 1 stable?
6. [1.25%] Is System 2 stable?

Discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \tag{1}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n} \tag{2}$$

Continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \tag{3}$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt \tag{4}$$

Continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \tag{5}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{6}$$

Discrete-time Fourier transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(j\omega) e^{j\omega n} d\omega \tag{7}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \tag{8}$$

Laplace transform

$$x(t) = \frac{1}{2\pi} e^{\sigma t} \int_{-\infty}^{\infty} X(\sigma + j\omega) e^{j\omega t} d\omega \tag{9}$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \tag{10}$$

Z transform

$$x[n] = r^n \mathcal{F}^{-1}(X(re^{j\omega})) \tag{11}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{12}$$

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{array}{l} x(t) \\ y(t) \end{array}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} x(t) = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T} a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right) a_k = \left(\frac{1}{jk(2\pi/T)}\right) a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

three examples, we illustrate this. The last example in this section then demonstrates how properties of a signal can be used to characterize the signal in great detail.

Example 3.6

Consider the signal $g(t)$ with a fundamental period of 4, shown in Figure 3.10. We could determine the Fourier series representation of $g(t)$ directly from the analysis equation (3.39). Instead, we will use the relationship of $g(t)$ to the symmetric periodic square wave $x(t)$ in Example 3.5. Referring to that example, we see that, with $T = 4$ and $T_1 = 1$,

$$g(t) = x(t-1) - 1/2. \quad (3.[illegible])$$

Thus, in general, *none* of the finite partial sums in eq. (3.52) yield the exact values of $x(t)$, and convergence issues, such as those considered in Section 3.4, arise as we consider the problem of evaluating the limit as the number of terms approaches infinity.

3.7 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

There are strong similarities between the properties of discrete-time and continuous-time Fourier series. This can be readily seen by comparing the discrete-time Fourier series properties summarized in Table 3.2 with their continuous-time counterparts in Table 3.1.

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{matrix} a_k \\ b_k \end{matrix}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n}x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m}a_k \begin{pmatrix}\text{viewed as periodic} \\ \text{with period } mN\end{pmatrix}$
Periodic Convolution	$\sum_{r=\langle N\rangle} x[r]y[n-r]$	Na_kb_k
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N\rangle} a_lb_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k] \begin{pmatrix}\text{finite valued and periodic only} \\ \text{if } a_0 = 0\end{pmatrix}$	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a^*_{-k} \\ \mathcal{R}e\{a_k\} = \mathcal{R}e\{a_{-k}\} \\ \mathcal{I}m\{a_k\} = -\mathcal{I}m\{a_{-k}\} \\ \lvert a_k\rvert = \lvert a_{-k}\rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}v\{x[n]\} & [\text{x[n] real}] \\ x_o[n] = \mathcal{O}d\{x[n]\} & [\text{x[n] real}] \end{cases}$	$\mathcal{R}e\{a_k\}$ $j\mathcal{I}m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$$

4.6 TABLES OF FOURIER PROPERTIES AND OF BASIC FOURIER TRANSFORM PAIRS

In the preceding sections and in the problems at the end of the chapter, we have considered some of the important properties of the Fourier transform. These are summarized in Table 4.1, in which we have also indicated the section of this chapter in which each property has been discussed.

In Table 4.2, we have assembled a list of many of the basic and important Fourier transform pairs. We will encounter many of these repeatedly as we apply the tools of

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$
4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\theta)Y(j(\omega-\theta))d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^{t}x(t)dt$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \mathcal{R}e\{X(j\omega)\} = \mathcal{R}e\{X(-j\omega)\} \\ \mathcal{I}m\{X(j\omega)\} = -\mathcal{I}m\{X(-j\omega)\} \\ \lvert X(j\omega)\rvert = \lvert X(-j\omega)\rvert \\ \sphericalangle X(j\omega) = -\sphericalangle X(-j\omega) \end{cases}$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\mathcal{R}e\{X(j\omega)\}$ $j\mathcal{I}m\{X(j\omega)\}$

4.3.7 Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty}|x(t)|^2dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\omega)|^2d\omega$$

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$, otherwise
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, otherwise
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, otherwise
$x(t) = 1$	$2\pi\delta(\omega)$	$a_0 = 1,\quad a_k = 0,\ k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & T_1 < \lvert t\rvert \leq \frac{T}{2} \end{cases}$ and $x(t + T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2\sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k
$x(t) \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & \lvert t\rvert > T_1 \end{cases}$	$\frac{2\sin \omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < W \\ 0, & \lvert\omega\rvert > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at}u(t),\ \mathcal{R}e\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$te^{-at}u(t),\ \mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t),$ $\mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—

TABLE 5.1 PROPERTIES OF THE DISCRETE-TIME FOURIER TRANSFORM

Section	Property	Aperiodic Signal	Fourier Transform
		$x[n]$ $y[n]$	$\left.\begin{matrix} X(e^{j\omega}) \\ Y(e^{j\omega}) \end{matrix}\right\}$ periodic with period 2π
5.3.2	Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
5.3.3	Time Shifting	$x[n - n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
5.3.3	Frequency Shifting	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
5.3.4	Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
5.3.6	Time Reversal	$x[-n]$	$X(e^{-j\omega})$
5.3.7	Time Expansion	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
5.4	Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
5.5	Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
5.3.5	Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
5.3.5	Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - e^{-j\omega}}X(e^{j\omega})$ $+\pi X(e^{j0}) \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$
5.3.8	Differentiation in Frequency	$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$
5.3.4	Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \mathcal{R}e\{X(e^{j\omega})\} = \mathcal{R}e\{X(e^{-j\omega})\} \\ \mathcal{I}m\{X(e^{j\omega})\} = -\mathcal{I}m\{X(e^{-j\omega})\} \\ \lvert X(e^{j\omega})\rvert = \lvert X(e^{-j\omega})\rvert \\ \sphericalangle X(e^{j\omega}) = -\sphericalangle X(e^{-j\omega}) \end{cases}$
5.3.4	Symmetry for Real, Even Signals	$x[n]$ real an even	$X(e^{j\omega})$ real and even
5.3.4	Symmetry for Real, Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
5.3.4	Even-odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ $[x[n]$ real$]$ $x_o[n] = \mathcal{O}d\{x[n]\}$ $[x[n]$ real$]$	$\mathcal{R}e\{X(e^{j\omega})\}$ $j\mathcal{I}m\{X(e^{j\omega})\}$
5.3.9	Parseval's Relation for Aperiodic Signals $\sum_{n=-\infty}^{+\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{2\pi} \lvert X(e^{j\omega})\rvert^2 d\omega$		

a duality relationship between the discrete-time Fourier transform and the continuous-time Fourier series. This relation is discussed in Section 5.7.2.

5.7.1 Duality in the Discrete-Time Fourier Series

Since the Fourier series coefficients a_k of a periodic signal $x[n]$ are themselves a periodic sequence, we can expand the sequence a_k in a Fourier series. The duality property for discrete-time Fourier series implies that the Fourier series coefficients for the periodic sequence a_k are the values of $(1/N)x[-n]$ (i.e., are proportional to the values of the original

TABLE 5.2 BASIC DISCRETE-TIME FOURIER TRANSFORM PAIRS

Signal	Fourier Transform	Fourier Series Coefficients (if periodic)
$\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	a_k
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} 1, & k = m, m \pm N, m \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2}, & k = \pm m, \pm m \pm N, \pm m \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$ $a_k = \begin{cases} \frac{1}{2j}, & k = r, r \pm N, r \pm 2N, \ldots \\ -\frac{1}{2j}, & k = -r, -r \pm N, -r \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & N_1 < \lvert n\rvert \le N/2 \end{cases}$ and $x[n+N] = x[n]$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N \sin[2\pi k/2N]}, \; k \ne 0, \pm N, \pm 2N, \ldots$ $a_k = \frac{2N_1 + 1}{N}, \; k = 0, \pm N, \pm 2N, \ldots$
$\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N}$ for all k
$a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{1 - ae^{-j\omega}}$	—
$x[n] = \begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & \lvert n\rvert > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$	—
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} \operatorname{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \le \lvert\omega\rvert \le W \\ 0, & W < \lvert\omega\rvert \le \pi \end{cases}$ $X(\omega)$ periodic with period 2π	—
$\delta[n]$	1	—
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi\delta(\omega - 2\pi k)$	—
$\delta[n - n_0]$	$e^{-j\omega n_0}$	—
$(n+1)a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$	—
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$	—