Midterm #1 of ECE301, Section 2
8–9pm, Wednesday, September 16, 2015, RHPH 172.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**

2. This is a closed book exam.

3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.

4. Use the back of each page for rough work.

5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

I certify that I have neither given nor received unauthorized aid on this exam.

Signature: Date:

Question 1: [20%, Work-out question, Learning Objectives 2 and 3] Consider two continuous-time signals $x(t)$ and $y(t)$.

$$x(t) = \begin{cases} e^{-t} & \text{if } 2 < t \\ 0 & \text{otherwise} \end{cases}$$

$$y(t) = \begin{cases} \cos(2t)e^{-t} & \text{if } 1 < t \\ 0 & \text{otherwise} \end{cases}$$

Compute the expression of

$$z(t) = \int_{s=-\infty}^{\infty} x(t-s)y(s)ds.$$

Hint 1: You may need to use the following formulas:

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j} \tag{1}$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}. \tag{2}$$

Hint 2: You need to study different cases in your answer. The computation of some cases may be significantly longer and you may want to work on some other questions first before you come back to finish the computation of some of the cases.

Question 2: [15%, Work-out question, Learning Objectives 1, 4, and 5] Given a discrete-time signal

$$x[n] = \begin{cases} 2^{n-10}e^{jn} & \text{if } 15 \leq n \\ 0 & \text{otherwise} \end{cases}$$

Based on $x[n]$, we construct another function

$$y(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}. \tag{3}$$

1. [7.5%] Compute the average power $x[n]$ for the duration of $n = -100$ to 100.

2. [7.5%] Compute the value of $y(3 + 4j)$.

Hint: You may need to use the following two formulas: If $|r| \neq 1$, then we have

$$\sum_{k=1}^{K} ar^{k-1} = \frac{a(1 - r^K)}{1 - r}. \tag{4}$$

If $|r| < 1$, then we have

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1 - r}. \tag{5}$$

If $|r| \geq 1$, then we have

$$\sum_{k=1}^{\infty} ar^{k-1} \text{ does not exist.} \tag{6}$$

Question 3: [16%, Work-out question, Learning Objectives 1, 4, and 6] Consider the following discrete time signal.

$$x[n] = e^{j\frac{5\pi}{2}n} + e^{-j\pi n}.$$

1. [6%] Plot the imaginary part of $x[n]$ for the range of $-4 \le n \le 4$.

2. [6%] Find out the fundamental period of $x[n]$.

3. [4%] Find out the instantaneous power of $x[n]$ when $n = 3$.

Hint: You may need the formula that $e^{j\theta} = \cos(\theta) + j\sin(\theta)$.

Question 4: [15%, Work-out question, Learning Objectives 1, 2, and 6] Consider the following signal.

$$x(t) = \delta(t - 2.5) + \left(\sum_{k=-\infty}^{\infty} \delta(t - k) \right) + U(t + 1.5) \tag{7}$$

1. [7%] Plot $x(t)$ for the range of $t = -3.5$ to 3.5.

2. [8%] Plot the odd part of $x(t)$ for the range of $t = -3.5$ to 3.5.

   Hint: For the second sub-question, if you do not know how to plot $x(t)$ in the first sub-question, you can solve the following alternative question instead. You will still get 5 points if your answer is correct.

   Alternative question: Suppose $y(t) = e^t U(t - 1)$. Plot the odd part of $y(t)$ for the range of $t = -3$ to 3.

Question 5: [14%, Work-out question, Learning Objective 1]

Consider the following system that takes a continuous-time signal $x(t)$ as input and outputs a discrete-time signal $y[n]$:

$$y[n] = x(0.25n^2) - 3x(-n^3) \tag{8}$$

Is the above system is linear or not? Carefully explain the steps how you decide whether the system is linear or not.

Question 6: [20%, Multiple Choices] The following questions are multiple-choice questions and there is no need to justify your answers. Consider two continuous-time signals:

$$x_1(t) = e^{2\sin(t)} \cdot e^{-2\sin(\pi t)}$$
$$x_2(t) = e^{jt^2 - |t|}(1 + j)^{t^4}$$

and two discrete-time signals:

$$x_3[n] = \cos(\pi n^2) + \sin(\frac{\pi}{3}n)$$
$$x_4[n] = \cos(\pi n)U[n - 1] - \cos(3\pi n)U[n - 2] + \delta[n + 1].$$

1. [10%, Learning Objective 1] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not, *respectively.* If it is periodic, write down the fundamental period. Please state explicitly which signal is periodic and which is not.

2. [10%, Learning Objective 1] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them, respectively. Please state explicitly which signal is even, which is odd, and which is neither.