Question 1: [18%, Work-out question]

1. [1%] What does the acronym "AM-DSB" stand for?

Amplitude-Modulation Double-Side-Band.

Prof. Wang wanted to transmit an AM-SSB signal. To that end, he wrote the following MATLAB code.

```
% Initialialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read two different .wav files
[x1, f_sample, N]=wavread('x1');
x1=x1';
[x2, f_sample, N]=wavread('x2');
x2=x2';

% Step 0: Initialize several parameters
W_1=pi*3000;
W_2=pi*5000;
W_3=pi*15000;
W_4=pi*8000;
W_5=????;
W_6=????;




% Step 1: Make the signals band-limited.
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);

% Step 2: Multiply x1_new and x2_new with a cosine wave.
x1_h=x1_new.*cos(W_2*t);
x2_h=x2_new.*cos(W_3*t);

% Step 3: Keep one of the two side bands
h_one=h.*(cos(W_4*t));
```

```
h_two=1/(pi*t).*(sin(W_5*t))-1/(pi*t).*(sin(W_6*t));
x1_sb=ece301conv(x1_h, h_one);
x2_sb=ece301conv(x2_h, h_two);

% Step 4: Create the transmitted signal
y=x1_sb+x2_sb;
wavwrite(y', f_sample, N, 'y.wav');
```

2. [1.5%] What is the bandwidth (Hz) of the signal x1_new?

3. [1.5%] For the first signal x1_new, is this AM-SSB transmitting an upper-side-band signal or a lower-side-band signal?

4. [3%] What should the values of W_5 and W_6 be in the MATLAB code, if we decide to use a lower-side-band transmission for the second signal x2_new ?

2. 1560 Hz

3. Upper side band

4. $W_5 = 15000\pi$.

$W_6 = 12000\pi$.

Knowing that Prof. Wang used the above code to generate the "y.wav" file, a student tried to demodulate the output waveform "y.wav" by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[y, f_sample, N]=wavread('y');
y=y';

% Initialize several parameters
W_8=????;
W_9=????;
W_10=????;
W_11=????;
W_12=????;


% Create the low-pass filter.
h_M=1/(pi*t).*(sin(W_8*t));

% demodulate signal 1
h_BPF1=1/(pi*t).*(sin(W_9*t))-1/(pi*t).*(sin(W_10*t));
y1_BPF=ece301conv(y,h_BPF1);
y1=4*y1_BPF.*cos(pi*5000*t);
x1_hat=ece301conv(y1,h_M);

sound(x1_hat,f_sample)

% demodulate signal 2
h_BPF2=1/(pi*t).*(sin(W_11*t))-1/(pi*t).*(sin(W_12*t));
y2_BPF=ece301conv(y,h_BPF2);
y2=4*y2_BPF.*cos(pi*15000*t);
x2_hat=ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```

5. [7.5%] Continue from the previous questions. What should the values of W_8 to W_12 be in the MATLAB code?

6. [4.5%] It turns out that Prof. Wang's MATLAB code gives one of the two stations

an "unfair disadvantage." Please (i) indicate which station is given unfair disadvantage; (ii) explain/specify what is the disadvantage; and (iii) Describe how to fix the demodulation codes of yours to rectify the problem.

Hint: If you do not know the answers of Q1.2 to Q1.6, please simply draw the AMSSB modulation and demodulation diagrams and mark carefully all the parameter values. You will receive 10 points for Q1.2 to Q1.6.

5. $W_8 = 3000\pi$

$W_9 = 8000\pi$

$W_{10} = 5000\pi$.

$W_{11} = 15000\pi$

$W_{12} = 12000\pi$.

6. Station 1.

Volume is weaker.

```
y1 = 8* y1_BPF * cos(pi*5000*t);
```

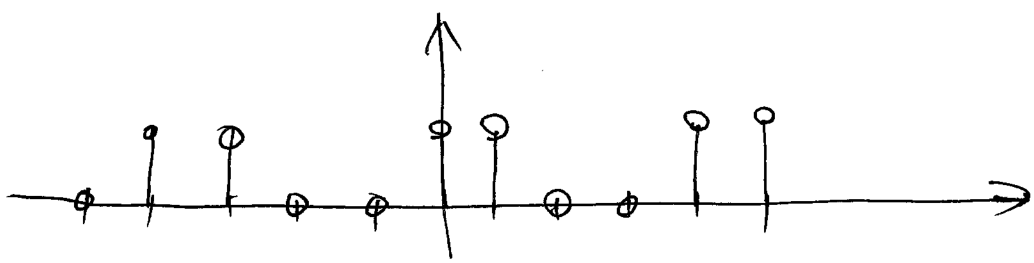
Question 2: [9%, Work-out question] Consider a discrete time signal

$$x[n] = \begin{cases} 1 & \text{if } n = 0 \text{ or } n = 1 \\ 0 & \text{if } n = 2 \text{ or } n = 3 \\ \text{periodic with period 4} & \end{cases} \quad (1)$$

1. [1.5%] Plot $x[n]$ for the range of $n = -5$ to 5.
2. [1.5%] Is $X(e^{j\omega})$ periodic? What is its period?
3. [6%] Find the expression of the DTFT $X(e^{j\omega})$ of $x[n]$ and plot $X(e^{j\omega})$ for the range of $-2\pi < \omega < 2\pi$.

   Hint: If you do not know the answer to this subquestion, you can solve the following question instead and you will get 4.5 points if your answer is correct. Assume $y[n] = \cos(\frac{\pi}{3}n + \frac{\pi}{4})$. Find the expression of the DTFT $Y(e^{j\omega})$ of $y[n]$ and plot $Y(e^{j\omega})$ for the range of $-2\pi < \omega < 2\pi$.

1.

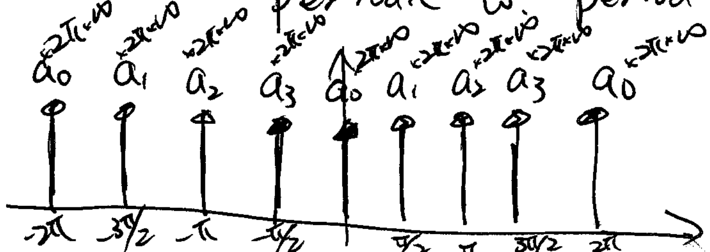


2. Periodic, period 2π.

3. $$a_k = \frac{1}{4}\sum_{n=0}^{3} x[n]\, e^{-jk\frac{2\pi}{4}n}$$

$$= \frac{1}{4}\left(1 + e^{-jk\frac{\pi}{2}}\right)$$

$$\Rightarrow X(e^{j\omega}) = \begin{cases} \sum_{k=0}^{3} \left(\frac{1}{4} + \frac{e^{-jk\frac{\pi}{2}}}{4}\right)\cdot 2\pi \cdot \delta(\omega - k\frac{\pi}{2}) & \text{if } 0 \le \omega < 2\pi \\ \text{periodic w. period } 2\pi & \end{cases}$$



Question 3: [17%, Work-out question]

Consider a continuous time signal $x(t) = \sin(3\pi t)$ and we use a digital voice recorder to convert the continuous time signal $x(t)$ to its discrete time counter part $x[n]$ with sampling frequency 2Hz. Answer the following questions.

1. [1%] What is the relationship between $x(t)$ and $x[n]$? A short equation would suffice.
2. [2%] Plot $x[n]$ for the range of $-4 \leq n \leq 4$.
3. [2.5%] Continue from the previous question. We use "zero-order hold" to reconstruct the original signal. We denote the output by $\hat{x}_{\text{ZOH}}(t)$. Plot $\hat{x}_{\text{ZOH}}(t)$ for the range of $t = -4$ to 4.

   [Alternative question:] If you do not know the answer to this question, you can assume that the sampling period is $T = \frac{1}{4}$ and the sampled values are $x[n] = \cos(0.5\pi n)$. Plot the Zero-Order-Hold output $\hat{x}_{\text{ZOH}}(t)$ for the range of $t = -1$ to 1. If your answer is correct, you will receive 2.5 points.
4. [1.5%] Let $x_p(t)$ denote the *impulse train-sampled signal* of $x(t)$. What is the relationship between $x(t)$ and $x_p(t)$? A short equation would suffice.
5. [3%] Let $X_p(j\omega)$ be the CTFT of $x_p(t)$. Find the expression of $X_p(j\omega)$ and plot $X_p(j\omega)$ for the range of $-4\pi < \omega < 4\pi$.
6. [5%] Continue from Question 3.3 (NOT the alternative question). Suppose we use the optimal band-limited reconstruction and denote the output by $\hat{x}_{\text{sinc}}(t)$. Find the expression of $\hat{x}_{\text{sinc}}(t)$ and plot $\hat{x}_{\text{sinc}}(t)$ for the range of $t = -4$ to 4.

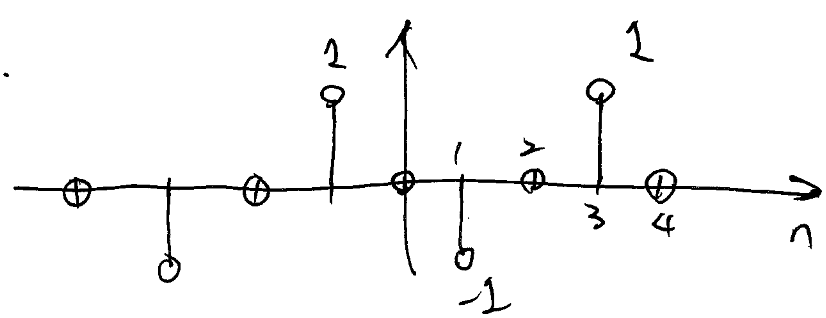
   Hint 1: You may need to use the answer of Q3.5 to help you find the expression of $\hat{x}_{\text{sinc}}(t)$.

   Hint 2: Even if you do not know the expression of $\hat{x}_{\text{sinc}}(t)$, you should till try to plot $\hat{x}_{\text{sinc}}(t)$ directly. You will get 3.5 points if your plot is correct.

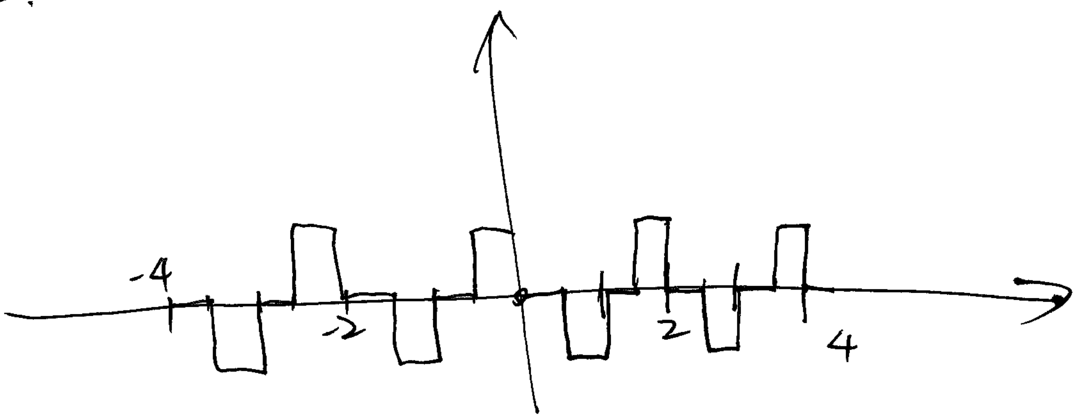
   [Alternative question:] If you do not know the answer to this question, you can assume that the sampling period is $T = 1/4$ and the sampled values are $x[n] = \delta[n] - \delta[n-4]$. Plot the optimal band-limited reconstruction output $\hat{x}_{\text{sinc}}(t)$for the range of $t = -4$ to 4. If your answer is correct, you will receive 3.5 points.
7. [2%] What is the value of the "Nyquist frequency" of $x(t)$? Your answer should be something like 44.1 kHz. There is no need to explain how you find the Nyquist frequency.

1. $x[n] = x(0.5n)$

2.

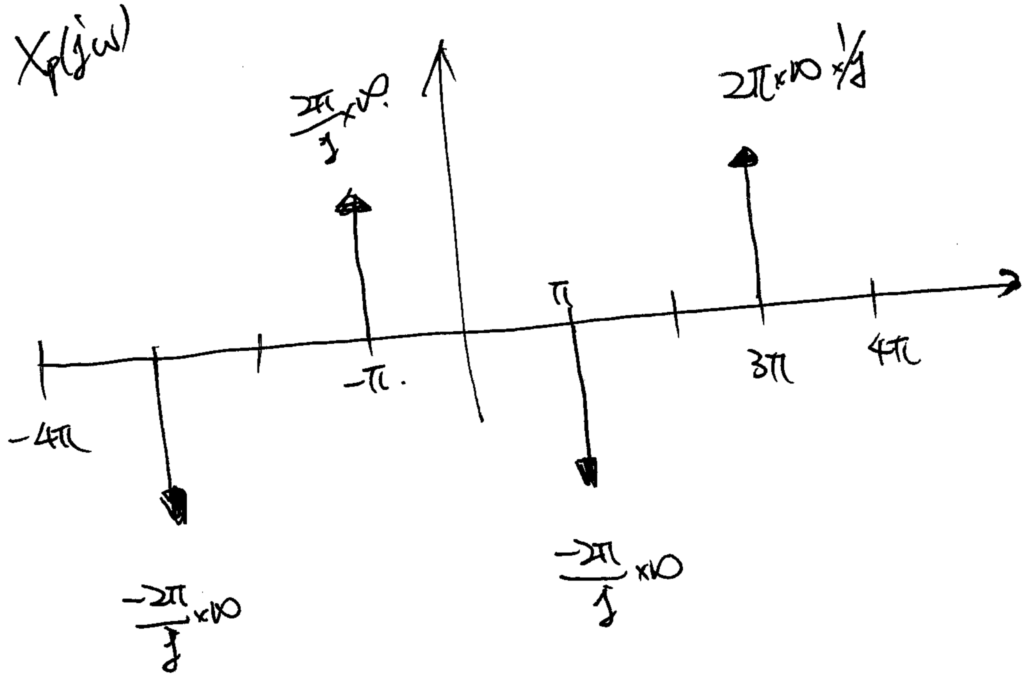


3.

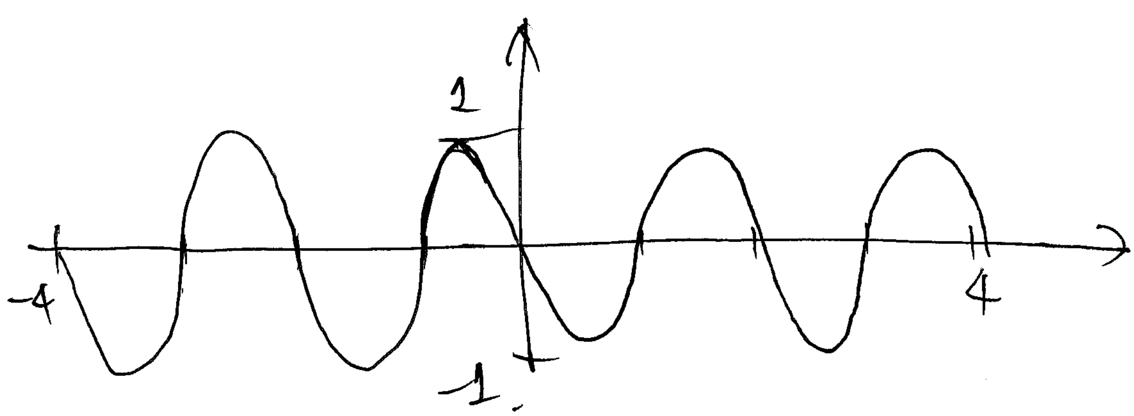


4. $x_p(t) = x(t) \cdot \sum_{k=-10}^{10} \delta(t - 0.5k)$.

5. $X_p(j\omega)$



6. $\hat{x}_{sinc}(t) = -\sin(\pi t)$

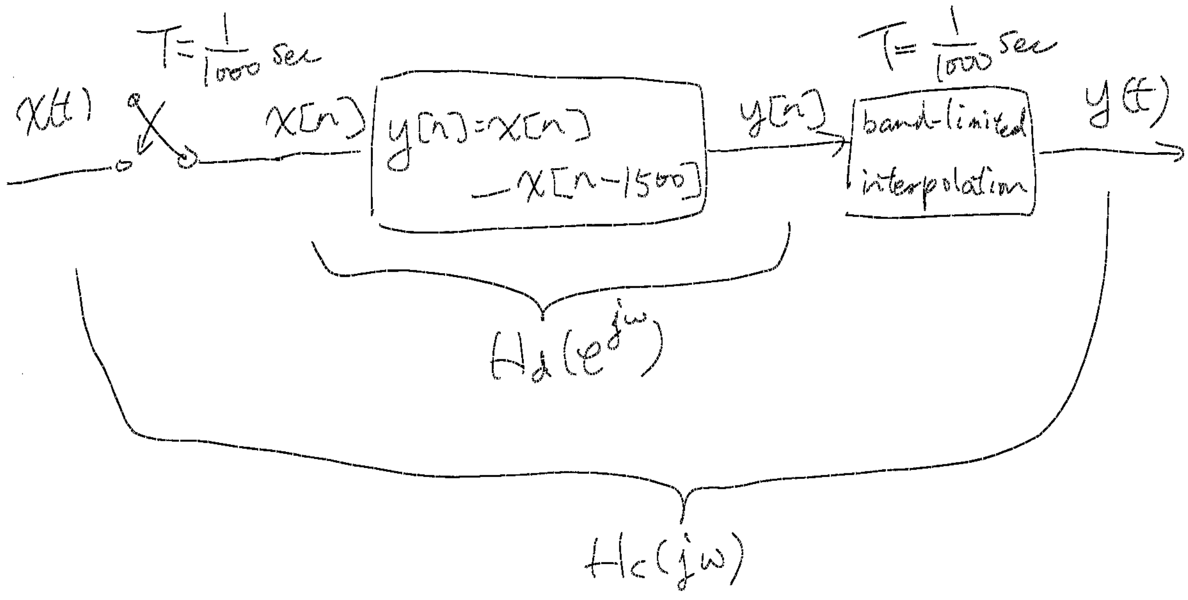


$\cancel{x(t) = 3}$

7. Nyquist freq $= 3\,\text{Hz}$

Question 4: [8%, Work-out question]

Consider the following discrete-time processing system for continuous-time signals.



Find the expression of the continuous-time frequency response $H_c(j\omega)$.

Hint: If you do not know the answer to this question, you can answer the following questions instead: (i) If the input is $x(t) = 5$, what is the output? (ii) What type of filter is the overall system? A low-pass filter? A high-pass filter? Or a band-pass filter? Use a single sentence to justify your answer. If both your answers are correct, you will still get 4 points.

$$H_c(j\omega) = \begin{cases} 1 - e^{-j\omega \cdot 1.5} & \text{if } -1000\pi < \omega < 1000\pi \\ 0 & \text{otherwise.} \end{cases}$$

Reason: $x[n-1500]$ corresponds to delay it by $\frac{1500}{1000}$ sec.

Question 5: [11%, Work-out question]

Consider the following discrete-time signals

$$x_1[n] = \begin{cases} 4^n & \text{if } n \leq 0 \\ 0 & \text{if } n \geq 1 \end{cases} \tag{2}$$

$$x_2[n] = \begin{cases} 0 & \text{if } n \leq 0 \\ 2^n & \text{if } n \geq 1 \end{cases} \tag{3}$$

1. [1%] What does "ROC" stand for?

2. [5%] Find the Z-transform of $x_1[n]$ and carefully specify and draw its ROC.

   Hint: If you do not know the answer to this question, you can answer the following alternative question instead: Whether the DTFT of $x_1[n]$ exists or not. You will receive 1.5 points if your answer is correct.

3. [3%] Find the Z-transform of $x_2[n]$ and carefully specify and draw its ROC.

   Hint: If you do not know the answer to this question, you can answer the following alternative question instead: Whether the DTFT of $x_2[n]$ exists or not. You will receive 1.5 points if your answer is correct.

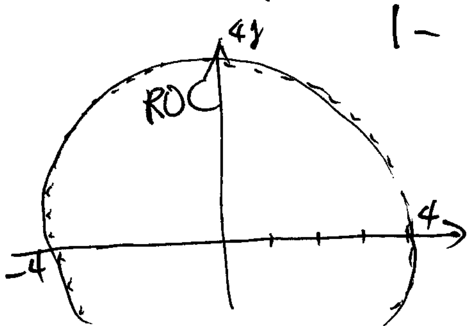
4. [2%] Let $x[n] = x_1[n] + x_2[n]$. Find the Z-transform of $x[n]$ and carefully specify and draw its ROC.

1. Region of convergence.

2. $$X_1(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{0} 4^n z^{-n}$$

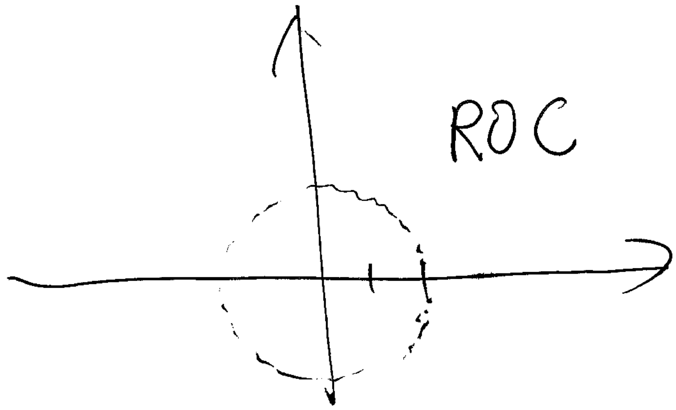
$$= \frac{1}{1 - \frac{z}{4}} \quad \text{if } \left|\frac{z}{4}\right| < 1 \Leftrightarrow |z| < 4.$$



3. $X_2(z) = \sum_{n=1}^{\infty} 2^n z^{-n}$

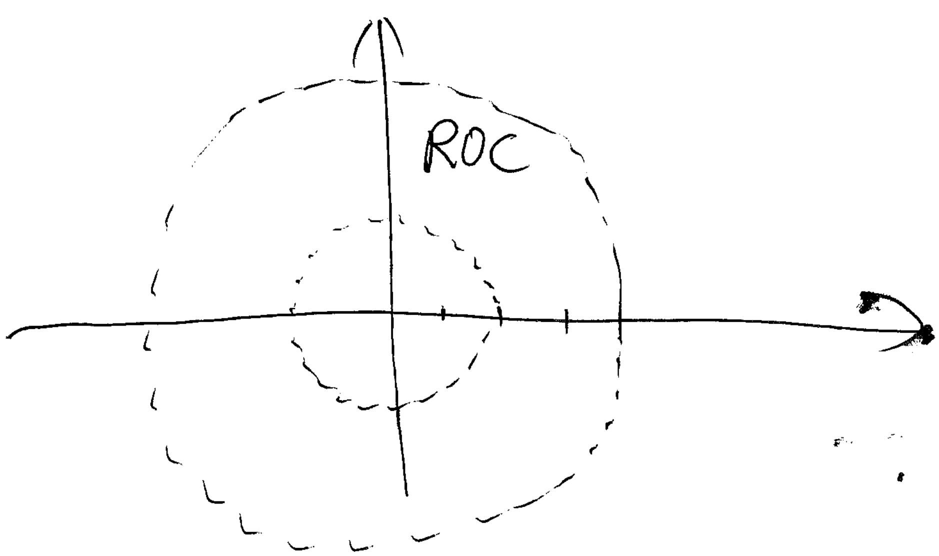
$= \dfrac{2 \cdot z^{-1}}{1 - 2z^{-1}}$ if $\left|\dfrac{2}{z}\right| < 1$

$\Leftrightarrow 2 < |z|$



4. $X(z) = \dfrac{1}{1 - \frac{z}{4}} + \dfrac{2 \cdot z^{-1}}{1 - 2z^{-1}}$

if $2 < |z| < 4$.



Question 6: [10%, Work-out question]

Consider a CT LTI system with impulse response

$$h(t) = \begin{cases} 2^{-|t|} & \text{if } -10 \le t \le 10 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Find the output $y(t)$ when the input is $x(t) = \sum_{k=1}^{2} \cos(3k\pi t)$.

$$H(jw) = \int_{-10}^{0} 2^{t} e^{-jwt}\, dt + \int_{0}^{10} 2^{-t} e^{-jwt}\, dt$$

$$= \frac{1 - \cancel{2^{-10} e^{jw}}}{\ln 2 - jw} + \frac{1 - \cancel{2^{-10} e^{-jw}}}{\ln 2 + jw}$$

$$= \frac{(\ln 2 + jw)(1 - \cancel{2^{-10} e^{jw}}) + (\ln 2 - jw)(1 - \cancel{2^{-10} e^{-jw}})}{(\ln 2)^2 + w^2}$$

$$= \frac{2 \ln 2}{(\ln 2)^2 + w^2}$$

$$\Rightarrow y(t) = \frac{2\ln 2 \cdot \cos(3\pi t)}{(\ln 2)^2 + (3\pi)^2} + \frac{2\ln(2) \cdot \cos(6\pi t)}{(\ln 2)^2 + (6\pi)^2}$$

Question 7: [12%, Work-out question]

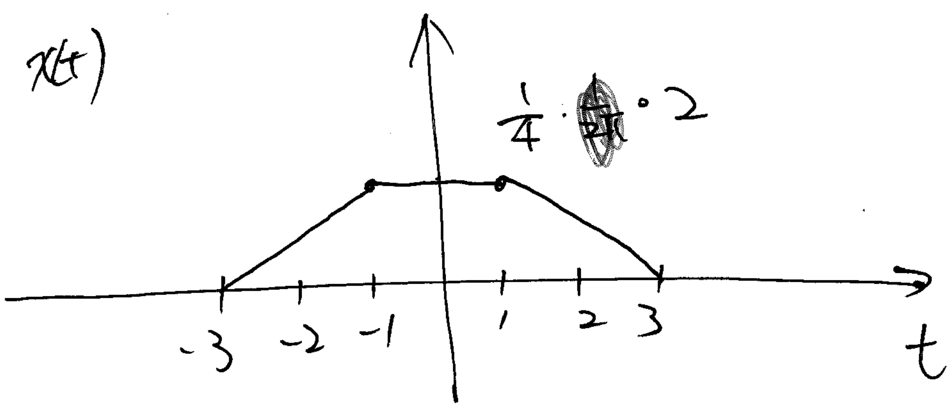
Consider $X(j\omega) = \frac{\sin(2\omega)\sin(\omega)}{\omega^2}$. Find its inverse Fourier transform $x(t)$ and plot it for the range of $-4 < t < 4$.

$$X(j\omega) = \frac{1}{4}\left(\frac{2\sin(2\omega)}{\omega}\right)\cdot\left(\frac{2\sin(\omega)}{\omega}\right)$$

$$x(t) = \frac{1}{4}\cdot\cancel{\frac{1}{2\pi}}\cdot\mathcal{F}^{-1}\left(\frac{2\sin(2\omega)}{\omega}\right) * \mathcal{F}^{-1}\left(\frac{2\sin(\omega)}{\omega}\right)$$

$x(t)$

$=$



Question 8: [15%, Multiple-choice question] Consider two signals

$$h_1(t) = \begin{cases} \cos(t) & \text{if } |t| < 0.5\pi \\ 0 & \text{if } 0.5\pi \le |t| < 1.5\pi \\ \cos(t) & \text{if } 1.5\pi \le |t| < 2.5\pi \\ 0 & \text{if } 2.5\pi \le |t| < 3.5\pi \\ \vdots & \vdots \\ \cos(t) & \text{if } (k+0.5)\pi \le |t| < (k+1.5)\pi \text{ for some odd integer } k \\ 0 & \text{if } (k+0.5)\pi \le |t| < (k+1.5)\pi \text{ for some even integer } k \end{cases} \tag{5}$$

and

$$h_2[n] = \sum_{k=0}^{\max(n,0)} k2^{-k} \tag{6}$$

1. [1.25%] Is $h_1(t)$ periodic? Yes.
2. [1.25%] Is $h_2[n]$ periodic? No
3. [1.25%] Is $h_1(t)$ even or odd or neither? Even
4. [1.25%] Is $h_2[n]$ even or odd or neither? Neither
5. [1.25%] Is $h_1(t)$ of finite energy? No
6. [1.25%] Is $h_2[n]$ of finite energy? No

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless? No
2. [1.25%] Is System 2 memoryless? No
3. [1.25%] Is System 1 causal? No
4. [1.25%] Is System 2 causal? Yes.
5. [1.25%] Is System 1 stable? No.
6. [1.25%] Is System 2 stable? No.