Midterm #2 of ECE301, Prof. Wang's section
6:30–7:30pm Monday, February 25, 2014, ME 1061,

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

E-mail:

Signature:

Question 1: [24%, Work-out question, Learning Objectives 1, 2, and 3] Consider two continuous-time signals $x(t)$ and $y(t)$.

$$x(t) = \mathcal{U}(t-2) - \mathcal{U}(t+2) + \delta(t)$$
$$h(t) = e^{-t}\mathcal{U}(t-1) = \begin{cases} e^{-t} & \text{if } t \geq 1 \\ 0 & \text{if } t < 1 \end{cases}$$

1. [4%] Plot $x(t)$ for the range of $-3 \leq t \leq 3$.

2. [20%] Compute the expression of

$$y(t) = x(t) * h(t).$$

Hint: You may want to express $x(t)$ as $x(t) = x_1(t) + x_2(t)$, compute the corresponding output $y_1(t)$ and $y_2(t)$ separately, and then assemble the final answer.

Question 2: [16%, Work-out question, Learning Objectives 1, 2, and 3] Consider the following system:

$$y[n] = x[n]e^{x[n-1]}$$

1. [4%] Plot the impulse response $h[n]$ for the range of $-4 \leq n \leq 4$.
2. [4%] When the input is

$$x[n] = \begin{cases} 1 & \text{if } -1 \leq n \leq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

   Plot the output signal $y[n]$ for the range of $-4 \leq n \leq 4$.
3. [4%] Continue from the previous question. Plot the signal $z[n] = x[n] * h[n]$.
4. [4%] Is $y[n]$ identical to $z[n]$? Write one sentence or two to justify your answer for this yes/no question.

Question 3: [24%, Work-out question, Learning Objectives 1, 2, 3, and 5] Consider the following LTI system

$$y(t) = \int_{s=t-2}^{t} x(s)e^{-(t-s)}ds.$$

1. [8%] Compute the impulse response $h(t)$.

2. [16%] Compute the output $y(t)$ when the input is $x(t) = e^{-jt} + e^{j(\pi t + \frac{\pi}{3})}$. (Hint: If you do not know the answer of the first sub-question, you can assume that $h(t) = e^{-|t|}$. You will still get full credit if your answer is correct.)

Question 4: [16%, Work-out question, Learning Objectives 4 and 5] Consider the following signal.

$$x(t) = \cos(\frac{4\pi}{3}t) + \sin(5\pi t) \tag{2}$$

Compute the Fourier series representation of $x(t)$.

Question 5: [20%, Multiple Choices] The following questions are multiple-choice questions and there is no need to justify your answers. Consider the following two systems:

System 1: When the input is $x_1(t)$, the output is

$$y_1(t) = \int_{-\infty}^{2t} x_1(s)ds. \tag{3}$$

System 2: When the input is $x_2[n]$, the output is

$$y_2[n] = \begin{cases} x_2[n/5] & \text{if } \frac{n}{5} \text{ is an integer} \\ y_2[n-1] & \text{if } \frac{n}{5} \text{ is not an integer} \end{cases} \tag{4}$$

Answer the following questions

1. [4%, Learning Objective 1] Is System 1 memoryless? Is System 2 memoryless?
2. [4%, Learning Objective 1] Is System 1 causal? Is System 2 causal?
3. [4%, Learning Objective 1] Is System 1 stable? Is System 2 stable?
4. [4%, Learning Objective 1] Is System 1 linear? Is System 2 linear?
5. [4%, Learning Objective 1] Is System 1 time-invariant? Is System 2 time-invariant?

Discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \tag{1}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n} \tag{2}$$

Continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \tag{3}$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt \tag{4}$$

Continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \tag{5}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{6}$$

Discrete-time Fourier transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(j\omega) e^{j\omega n} d\omega \tag{7}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \tag{8}$$

Laplace transform

$$x(t) = \frac{1}{2\pi} e^{\sigma t} \int_{-\infty}^{\infty} X(\sigma + j\omega) e^{j\omega t} d\omega \tag{9}$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \tag{10}$$

Z transform

$$x[n] = r^n \mathcal{F}^{-1}(X(re^{j\omega})) \tag{11}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{12}$$