Midterm #1 of ECE301, Prof. Wang's section
8–9pm Monday, February 3, 2014, ME 1061,

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Solution

Name:

Student ID:

E-mail:

Signature:

Question 1: [20%, Work-out question, Learning Objective 3] Consider two continuous-time signals $x(t)$ and $y(t)$.

$$x(t) = \begin{cases} e^{-t}(\cos(2t) - j\sin(2t)) & \text{if } 1 \le t \\ 0 & \text{if } t < 1 \end{cases}$$

$$y(t) = \begin{cases} e^{jt} & \text{if } 1 \le t \\ 0 & \text{if } t < 1 \end{cases}$$

Compute the expression of

$$z(t) = \int_{s=-\infty}^{\infty} x(t-s)y(s)ds.$$

$$x(t-s) = \begin{cases} e^{-(t-s)}\left(\cos(2(t-s)) - j\sin(2(t-s))\right) & \text{if } 1 \le t-s \\ 0, \; ow \end{cases}$$

$$= \begin{cases} e^{-(t-s)} e^{-j2(t-s)}, & s \le t-1 \\ 0, & ow \end{cases}$$

$$z(t) = \int_1^{\infty} e^{js} x(t-s)\, ds.$$

case 1 $\quad t-1 < 1 \Rightarrow t < 2 \qquad z(t) = 0$

case 2 $\quad t-1 \geq 1 \Rightarrow t \geq 2 \qquad z(t) = \int_1^{t-1} e^{js} e^{-(t-s)} e^{-j2(t-s)} ds =$

$$= e^{-t} e^{-j2t} \int_1^{t-1} e^{js} e^{s} e^{+j2s} ds = e^{-t(1+j2)} \int_1^{t-1} e^{s(1+j3)} ds =$$

$$= e^{-t(1+2j)} \left. \frac{e^{s(1+j3)}}{1+j3} \right|_1^{t-1} = \frac{e^{-t(1+2j)}}{1+j3}\left(e^{(t-1)(1+j3)} - e^{1(1+j3)}\right)$$

$$z(t) = \begin{cases} \frac{e^{-t(1+j2)}}{1+j3}\left(e^{(t-1)(1+j3)} - e^{(1+j3)}\right), & t \geq 2 \\ 0, & t < 2 \end{cases}$$

Question 2: [15%, Work-out question, Learning Objective 4] Given a discrete-time signal

$$x[n] = \begin{cases} 2^n & \text{if } n \le 1 \\ 0 & \text{if } n > 1 \end{cases}$$

Define

$$y(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}.$$

Find the value of $y(e^{j\frac{\pi}{4}})$. Hint: You may need to use the formula

$$\sum_{k=1}^{\infty} a r^{k-1} = \frac{a}{1-r} \text{ when } |r| < 1.$$

$$y(z) = \sum_{n=-\infty}^{1} 2^n z^{-n} \Rightarrow y(e^{j\frac{\pi}{4}}) = \sum_{n=-\infty}^{1} 2^n e^{-j\frac{\pi}{4}n} =$$

$$= \sum_{n=-1}^{\infty} 2^{-n} e^{j\frac{\pi}{4}n} = \sum_{n=-1}^{\infty} \left(\frac{e^{j\frac{\pi}{4}}}{2}\right)^n$$

$\underline{k = n+2}$

$$y(z) = \sum_{k=1}^{\infty} \left(\frac{e^{j\frac{\pi}{4}}}{2}\right)^{k-2} = \sum_{k=1}^{\infty} \left(\frac{e^{j\frac{\pi}{4}}}{2}\right)^{k-1} \left(\frac{e^{j\frac{\pi}{4}}}{2}\right)^{-1} =$$

$$= \frac{2}{e^{j\frac{\pi}{4}}} \sum_{k=1}^{\infty} \left(\frac{e^{j\frac{\pi}{4}}}{2}\right)^{k-1} = \frac{2}{e^{j\frac{\pi}{4}}} \frac{1}{1 - \frac{e^{j\frac{\pi}{4}}}{2}} =$$

$$= \frac{2 \cdot \frac{2}{e^{j\frac{\pi}{4}}}}{2 - e^{j\frac{\pi}{4}}} = 4 e^{-j\frac{\pi}{4}} \frac{1}{2 - e^{j\frac{\pi}{4}}}$$

Question 3: [10%, Work-out question, Learning Objective 1] Consider the following signal

$$x[n] = e^{(3+j)n}.$$

Define $x_{\text{Re}}[n]$ as the real part of $x[n]$ and define $x_{\text{Im}}[n]$ as the imaginary part of $x[n]$. (That is, $x[n] = x_{\text{Re}}[n] + jx_{\text{Im}}[n]$.) Also define

$$y[n] = x_{\text{Re}}[n] + 2x_{\text{Im}}[n].$$

Find the average power of $y[n]$ in the interval $[3, 5]$. (Hint 1: interval $[3, 5]$ contains both end points $n = 3$ and $n = 5$. Hint 2: There is no need to simplify any trigonometric expressions. For example, your answer can be something like $\sqrt{2}\cos^2(3) + \cdots$)

$$x[n] = e^{3n} e^{jn} = e^{3n}(\cos n + j\sin n)$$

$$x_{RE}[n] = e^{3n}\cos n \quad , \quad x_{im}[n] = e^{3n}\sin n$$

$$y[n] = e^{3n}\cos n + 2e^{3n}\sin n$$

$$P_{ave} = \frac{1}{3}\sum_{n=3}^{5} |y[n]|^2 = \frac{1}{3}\sum_{n=3}^{5} |e^{3n}\cos n + 2e^{3n}\sin n|^2 =$$

$$= \frac{1}{3}\sum_{n=3}^{5} e^{6n}(\cos n + 2\sin n)^2 = \frac{1}{3}\Big[e^{18}(\cos 3 + 2\sin 3)^2 +$$

$$+ e^{24}(\cos 4 + 2\sin 4)^2 + e^{30}(\cos 5 + 2\sin 5)^2\Big]$$

Question 4: [20%, Work-out question, Learning Objective 1] Consider the following discrete-time signal

$$y[n] = \begin{cases} \cos(\frac{\pi(n+3)}{4}) & \text{if } -1 \leq n \leq 2 \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

Let $y_{\text{even}}[n]$ denote the even part of $y[n]$. Plot $y_{\text{even}}[n]$ for the range of $n = -2$ to 2. (Hint: If you do not know the answer, you should plot $y[n]$ for the range of $n = -2$ to 2 instead. You will get 8 points if you plot $y[n]$ correctly.)

$$y_{even}[n] = \frac{y[n]+y[-n]}{2} = \frac{\cos(\frac{\pi(n+3)}{4}) + \cos(\frac{\pi}{4}(-n+3))}{2}$$

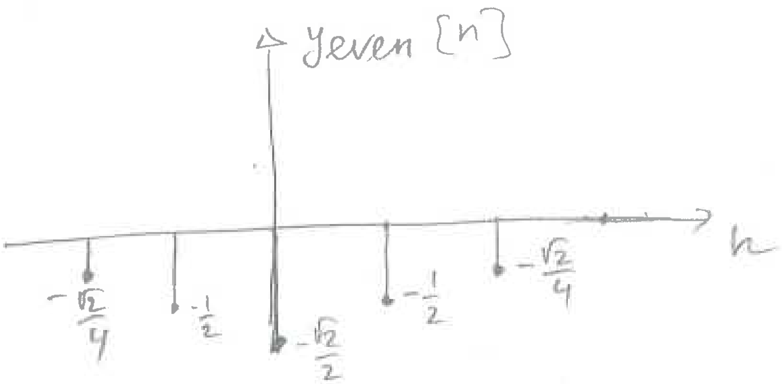
$$y_{even}[-2] = y_{even}[2] = \frac{y[2]+y[-2]}{2} = \frac{\cos(\frac{\pi}{4}\cdot(2+3)) + 0}{2}$$

$$= \frac{\cos(\frac{5\pi}{4})}{2} = -\frac{\sqrt{2}}{4}$$

$$y_{even}[-1] = y_{even}[1] = \frac{y[1]+y[-1]}{2} = \frac{\cos(\frac{\pi}{4}\cdot 4) + \cos(\frac{\pi}{4}\cdot 2)}{2} =$$

$$= \frac{\cos\pi + \cos(\frac{\pi}{2})}{2} = -\frac{1}{2}$$

$$y_{even}[0] = \frac{y[0]+y[0]}{2} = y[0] = \cos(\frac{\pi 3}{4}) = -\frac{\sqrt{2}}{2}$$



Question 5: [15%, Work-out question, Learning Objectives 1] Consider the following system that takes signal $x(t)$ as input and outputs

$$y(t) = -2x(-t+\pi). \tag{2}$$

Show that the above system is linear.

$$y_1(t) = -2x_1(-t+\pi)$$
$$y_2(t) = -2x_2(-t+\pi)$$

$$x_3(t) \triangleq \alpha x_1(t) + \beta x_2(t)$$
$$y_3(t) = -2x_3(-t+\pi) = -2(\alpha x_1(-t+\pi) + \beta x_2(-t+\pi)) =$$
$$= -2\alpha x_1(-t+\pi) - 2\beta x_2(-t+\pi) = \alpha y_1(t) + \beta y_2(t)$$

$\therefore$ linear

Question 6: [20%, Multiple Choices] The following questions are multiple-choice questions and there is no need to justify your answers. Consider two continuous-time signals:

$$x_1(t) = e^{jt} + e^{-jt} + t^2$$
$$x_2(t) = e^{t^2-|t|+\cos(t)} \cdot \sin(t)$$

and two discrete-time signals:

$$x_3[n] = \cos(\pi n^2) + \sin(\frac{\pi n}{2})$$
$$x_4[n] = (n-2)^2 + \cos(n).$$

1. [10%, Learning Objective 1] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not, *respectively*. If it is periodic, write down the fundamental period. Please state explicitly which signal is periodic and which is not.

2. [10%, Learning Objective 1] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them, respectively. Please state explicitly which signal is even, which is odd, and which is neither.

1. $x_1(t+T) = e^{j(t+T)} + e^{-j(t+T)} + (t+T)^2 \neq x_1(t)$ not periodic

$x_2(t)$ not periodic

$x_3[n+N] = \cos(\pi(n+N)^2) + \sin\left(\frac{\pi(n+N)}{2}\right) =$

$= \cos(\pi n^2 + 2\pi n N + N^2\pi) + \sin\left(\frac{\pi n}{2} + \frac{\pi N}{2}\right)$

$\frac{\pi N}{2} = 2\pi K \qquad \pi N = 4\pi k. \qquad N = 4k.$ for $k=1$ $N=4$

$\cos(\pi n^2 + 2\pi \cdot n 4 + 16\pi) = \cos(\pi n^2)$

$\sin\left(\frac{\pi n}{2} + \frac{\pi}{2} \cdot 4\right) = \sin\left(\frac{\pi n}{2}\right)$

periodic with $N=4$

$x_4[n]$ not periodic

$x_4[n+N] = (n+N-2)^2 + \cos(n+N)$

2. a) $x_1(-t) = e^{-jt} + e^{jt} + (-t)^2 = x_1(t)$ $\underline{\text{even}}$

b) $x_2(-t) = e^{(-t)^2 - |-t| + \cos(-t)} \sin(-t) =$

$= e^{t^2 - |t| + \cos t}(-\sin t)$ $\underline{\text{odd}}$

c) $x_3[-n] = \cos(\pi(-n)^2) + \sin\left(\frac{-\pi n}{2}\right) =$

$= \cos(\pi n^2) - \sin\left(\frac{\pi n}{2}\right)$ $\underline{\text{neither}}$

d) $x_4[-n] = (-n-2)^2 + \cos(-n) = \underline{(-n-2)}^2 + \cos n$

$\underline{\text{neither}}$