Final Exam of ECE301, Prof. Wang's section

8–10am Tuesday, May 6, 2014, EE 129.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

E-mail:

Signature:

Question 1: [15%, Work-out question]

1. [1%] What does the acronym "AM-DSB" stands for?

Amplitude-Modulation ~~Single~~ Double Side Band.

Prof. Wang wanted to transmit an AM-SSB signal. To that end, he wrote the following MATLAB code.

```
% Initialialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read two different .wav files
[x1, f_sample, N]=wavread('x1');
x1=x1';
[x2, f_sample, N]=wavread('x2');
x2=x2';

% Step 0: Initialize several parameters
W_1=pi*3000;
W_2=pi*4000;
W_3=pi*12000;
W_4=pi*7000;
W_5=pi*4000;
W_6=?????;
W_7=????;


% Step 1: Make the signals band-limited.
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);

% Step 2: Multiply x1_new and x2_new with a cosine wave.
x1_h=x1_new.*cos(W_2*t);
x2_h=x2_new.*cos(W_3*t);

% Step 3: Keep one of the two side bands
h_one=1/(pi*t).*(sin(W_4*t))-1/(pi*t).*(sin(W_5*t));
h_two=1/(pi*t).*(sin(W_6*t))-1/(pi*t).*(sin(W_7*t));
```

```
x1_sb=ece301conv(x1_h, h_one);
x2_sb=ece301conv(x2_h, h_two);

% Step 4: Create the transmitted signal
y=x1_sb+x2_sb;
wavwrite(y', f_sample, N, 'y.wav');
```

2. [1.5%] What is the bandwidth (Hz) of the signal x1_new?
3. [2.5%] Is this AM-SSB transmitting an upper-side-band signal or a lower-side-band signal?
4. [4%] What should the values of W_6 and W_7 be in the MATLABL code?

2. 1500 Hz

3. Upper side band.

4. $W_6 = 15000 \cdot pi$

$W_7 = 12000 \cdot pi$

Knowing that Prof. Wang used the above code to generate the "y.wav" file, a student tried to demodulate the output waveform "y.wav" by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[y, f_sample, N]=wavread('y');
y=y';

% Initialize several parameters
W_8=????;
W_9=????;
W_10=????;

% Create the low-pass filter.
h_M=1/(pi*t).*(sin(W_8*t));

% Create two band-pass filters.
hBPF_1=1/(pi*t).*(sin(pi*4000*t))-1/(pi*t).*(sin(pi*1000*t));
hBPF_2=1/(pi*t).*(sin(pi*15000*t))-1/(pi*t).*(sin(pi*12000*t));


% demodulate signal 1
y1BPF=ece301conv(y,hBPF_1);
y1=4*y1BPF.*cos(W_9*t);
x1_hat=ece301conv(y1,h_M);

sound(x1_hat,f_sample)

% demodulate signal 2
y2BPF=ece301conv(y,hBPF_2);
y2=4*y2BPF.*cos(W_10*t);
x2_hat=ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```

5. [3%] Continue from the previous question. What should the values of W_8 to W_10 in the MATLAB code?

6. [4%] It turns out that using the above MATLAB code, we can hear the sound properly when playing "sound(x2_hat,f_sample)" but there is some problem when

playing “sound(x1_hat,f_sample)”. Please (i) describe how it will sound when playing “sound(x1_hat,f_sample)” and (ii) Describe how we can fix the code so that we can hear x1_hat propertly.

Hint: If you do not know the answers of Q1.2 to Q1.6, please simply draw the AMSSB modulation and demodulation diagrams and mark carefully all the parameter values. You will receive 9 points for Q1.2 to Q1.6.

5. $W_8 = \cancel{1500}\ 3000\pi$

$W_9 = 4000\pi$

$W_{10} = 12000\pi$

6. Silent

$$h_{BPF\,1} = \frac{\sin(7000\pi \cdot t)}{\pi t} - \frac{\sin(4000\pi \cdot t)}{\pi t}$$

Question 2: [27%, Work-out question] We sample a continuous-time signal $x(t)$ with a sampling frequency 3Hz. The sampled value array $x[n] = \delta[n-1]$.

1. [1%] What is the sampling period? (Make sure you write down the correct unit.)

2. [3%] We use $x_{\text{sync}}(t)$ to denote the reconstructed signal based on the optimal band-limited reconstruction. Plot $x_{\text{sync}}(t)$ for the range of $-1 \leq t \leq 1$.

We sample another continuous-time signal $y(t)$ with a sampling frequency 3Hz. The sampled value array is

$$y[n] = \begin{cases} 0 & \text{if } n = 0 \text{ or } 2 \\ 1 & \text{if } n = 1 \\ -1 & \text{if } n = -1 \\ \text{periodic with period 4} & \end{cases} \tag{1}$$

3. [4%] We use $y_{\text{LIN}}(t)$ to denote the reconstructed signal based on linear interpolation. Plot $y_{\text{LIN}}(t)$ for the range of $-1 \leq t \leq 1$.
   We use $y_{\text{ZOH}}(t)$ to denote the reconstructed signal based on Zero-Order Hold. Plot $y_{\text{ZOH}}(t)$ for the range of $-1 \leq t \leq 1$.

4. [3%] Which one of the following statements can possibly be true?
   (i) $y(t) = \sum_{k=\infty}^{\infty} \delta(t-1-4k) - \delta(t-3-4k)$;
   (ii) $y(t) = \cos(\frac{\pi t}{2})$;
   (iii) $y(t) = \sin(\frac{3\pi t}{2})$;
   (iv) $y(t) = \cos(2\pi t)$; and
   (v) None of the above.

   Hint 1: This is a multiple choice question. You do not need to justify your answer.

   Hint 2: This question basically asks you to look at the sampled values $y[n]$ and deduce which of the above three signals can possibly be the original $y(t)$ signal?

5. [6%] Find out the DTFT of $y[n]$.

   Hint: You should be able to answer this sub-question without using the answers to the previous 2 sub-questions.

6. [5%] Let $y_p(t)$ denote the impulse-train-sampled signal with the sampling period 3 Hz. Plot $Y_p(j\omega)$ for the range of $-12\pi < \omega < 12\pi$.

   Hint: You would need to use the DTFT of $y[n]$ found in the previous sub-question to find the answer for this sub-question. If you do not know the answer to the previous sub-question, you can assume that the DTFT of $y[n]$ is

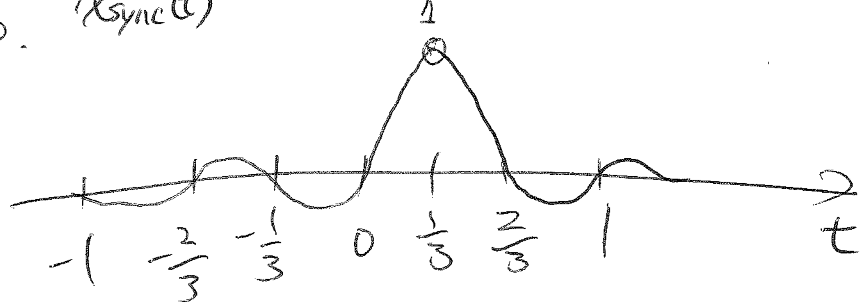
$$Y(e^{j\omega}) = \begin{cases} 1 & \text{if } 0 \leq |\omega| \leq \frac{\pi}{2} \\ 0 & \text{if } \frac{\pi}{2} < |\omega| \leq \pi \\ \text{periodic with period } 2\pi & \end{cases} \tag{2}$$

7. [2.5%, advanced] We use $y_{\text{sync}}(t)$ to denote the reconstructed signal based on the optimal reconstruction. Write down the expression of $y_{\text{sync}}(t)$.

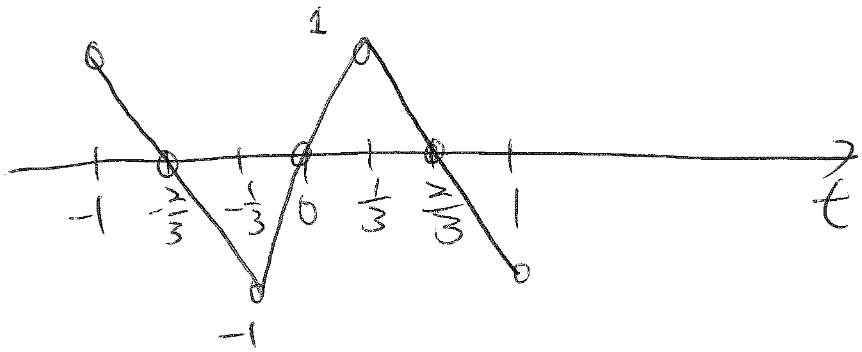
8. [2.5%, advanced] We now perform some discrete-time signal processing with $w[n] = y[n] + y[n+3]$ and use $w_{\text{sync}}(t)$ to denote the reconstructed signal based on the optimal reconstruction. Write down the expression of $w_{\text{sync}}(t)$.

1. $\frac{1}{3}$ sec.

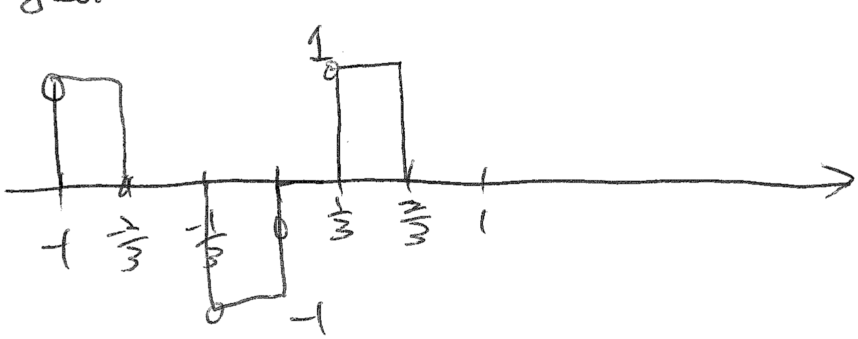
2. $x_{\text{sync}}(t)$



3. $y_{\text{LIN}}(t)$



4. $y_{\text{ZOH}}(t)$



4. $y(t) = \sin\left(\frac{3\pi}{2}t\right)$

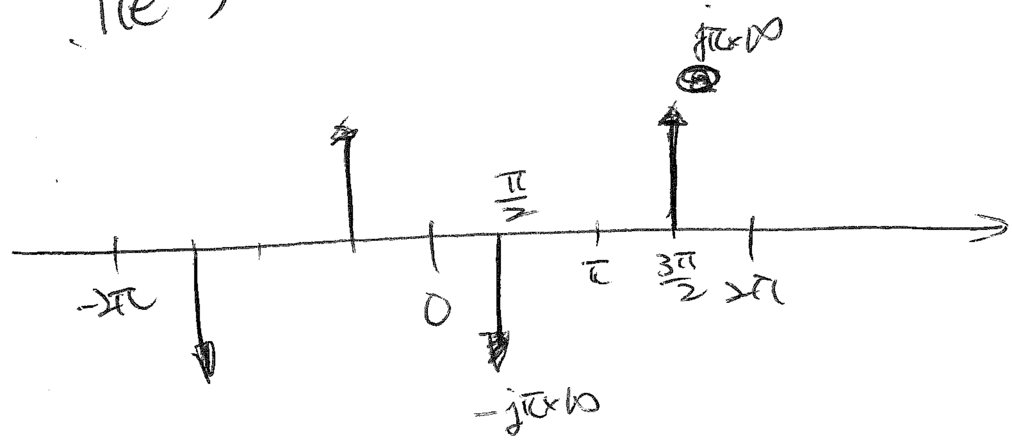
5. $a_k = \frac{1}{4}\sum_{n=0}^{3} x[n]\, e^{-jk\frac{2\pi}{4}n}$

$= \frac{1}{4} e^{-jk\frac{2\pi}{4}} - \frac{1}{4} e^{+jk\frac{2\pi}{4}}$

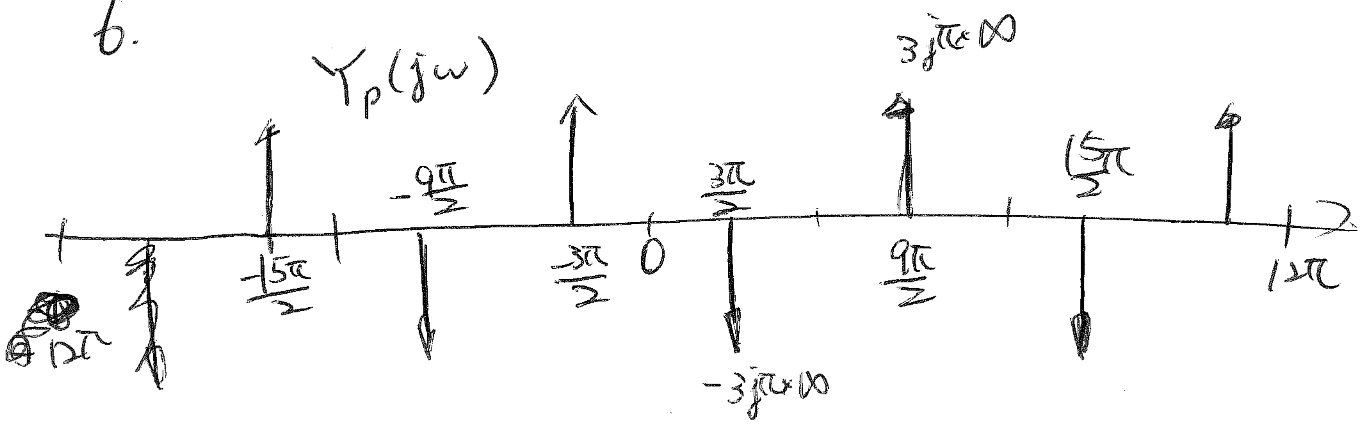
$= \frac{1}{4}\cdot(-2j)\sin\left(k\frac{\pi}{2}\right)$

$= \frac{-j}{2}\sin\left(k\frac{\pi}{2}\right)$

$Y(e^{j\omega})$



6. $Y_p(j\omega)$



7. $\sin\left(\frac{3\pi}{2}t\right)$

8. $\sin\left(\frac{3\pi}{2}t\right) + \sin\left(\frac{3\pi}{2}(t+1)\right)$ $\sin\left(\frac{3\pi}{2}\right) + \sin\left(\frac{3\pi}{2}(t+1)\right)$

Question 3: [14%, Work-out question] Consider the following *impulse-train sampling* and reconstruction system, which contains the following 2 steps.

Step 1: For any given signal $x(t)$, we first perform impulse train sampling with sampling period 0.5 seconds. The final impulse-train sampled signal is denoted by $x_p(t)$.

Step 2: Use a perfect band-limited reconstruction to convert $x_p(t)$ back to $\hat{x}(t)$.

1. [2%] Write down the mathematical relationship between $x(t)$ and $x_p(t)$. Your answer should look like "$x_p(t) = x(t)\cdots$".

2. [3%] Suppose the CTFT of $x(t)$ is

$$X(j\omega) = \begin{cases} 1 & \text{if } -3\pi \le \omega \le 3\pi \\ 0 & \text{otherwise} \end{cases}. \tag{3}$$

   Plot $X_p(j\omega)$, the CTFT of $x_p(t)$, for the range of $-6\pi \le \omega \le 6\pi$.

3. [4%] Draw the diagram how to derive $\hat{x}(t)$ from $x_p(t)$. Your diagram (flow chart) needs to be carefully labeled.

4. [5%] Continue from the previous question. Plot $\hat{X}(j\omega)$, the CTFT of $\hat{x}(t)$, for the range of $-6\pi \le \omega \le 6\pi$.

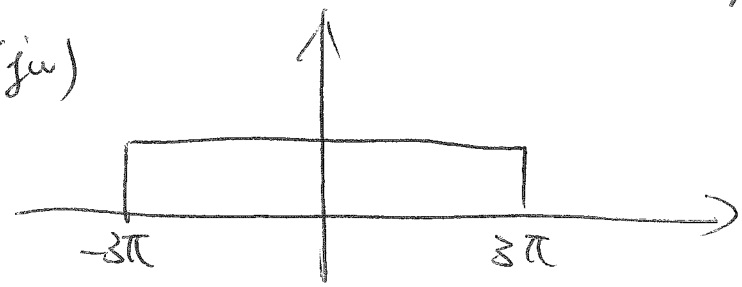
   Hint: If you do not know the answer, you can assume

$$y(t) = (x(t)\cos(4\pi t)) * \left(\frac{\sin(2\pi t)}{\pi t}\right) \tag{4}$$

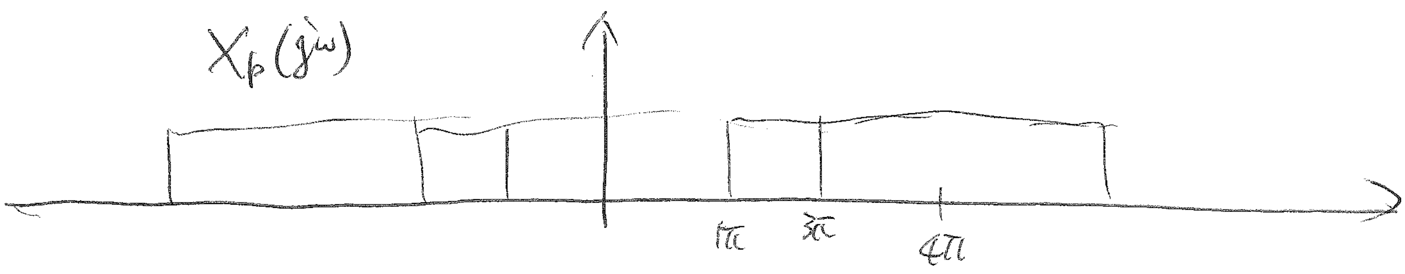
   and plot $Y(j\omega)$ for the range of $-6\pi \le \omega \le 6\pi$. You will get 4 points for this sub-question if your answer is correct.

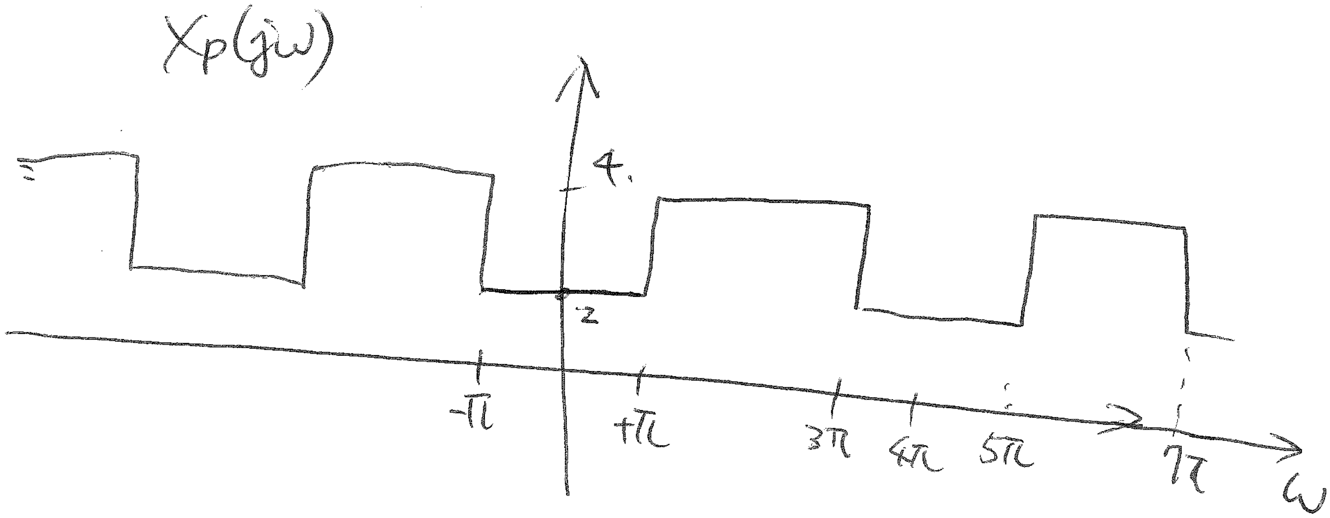
1. $x_p(t) = x(t)\cdot\left(\sum_{k=-\infty}^{\infty} \delta(t - k\cdot 0.5)\right)$

2. $X(j\omega)$



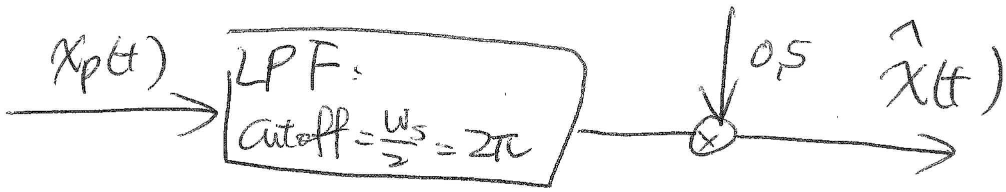
$X_p(j\omega)$





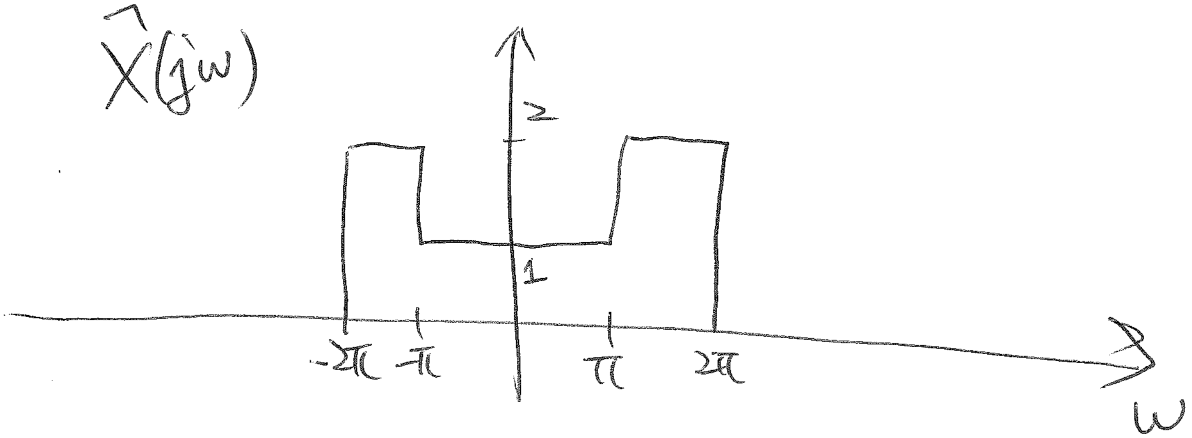
Xp(jω)
4
2
-π
+π
3π
4π
5π
7π
ω

3.



xp(t)
LPF:
cutoff = ws/2 = 2π
0.5
x̂(t)

4.



X̂(jω)
2
1
-2π
-π
π
2π
ω

Question 4: [7%, Work-out question]

1. [1%] What is the acronym "ROC" stands for (when considering the Z-transform)?

We know that

$$x[n] = \left(\frac{1}{4}\right)^n u[2-n] \tag{5}$$

2. [6%] Find the Z-transform $X(z)$, write down the expression of the corresponding ROC, and plot the ROC.

   Hint: You may need to use the formula: $\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$ if $|r| < 1$.

1. Region of Convergence.

2.
$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

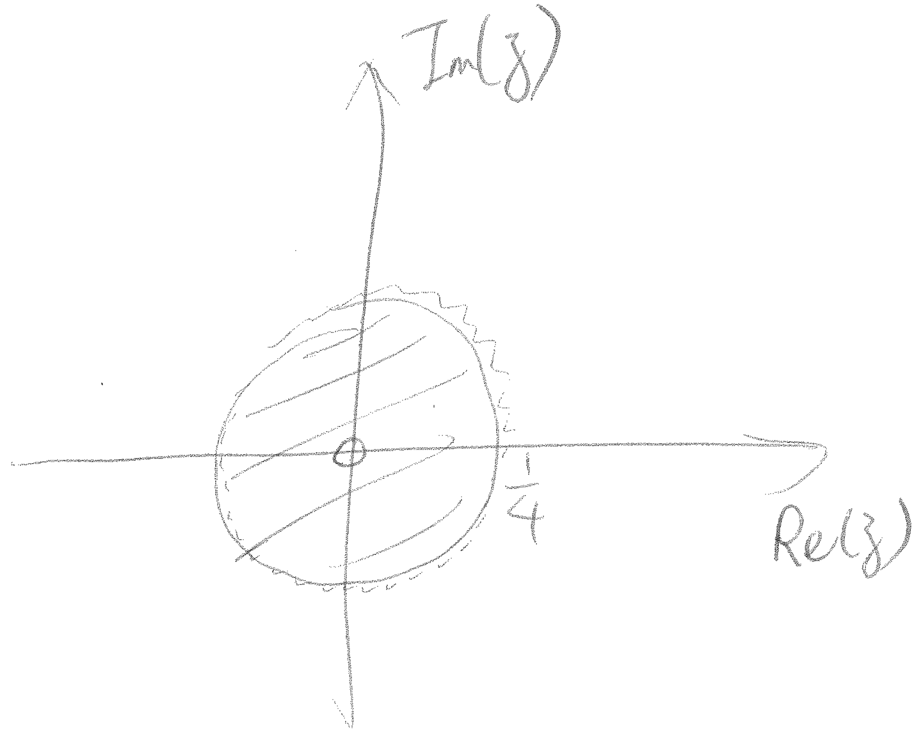
$$= \sum_{n=-\infty}^{2} \left(\frac{1}{4}\right)^n z^{-n}$$

$$= \sum_{n=-2}^{\infty} \left(\frac{1}{4}\right)^{-n} z^{n}$$

$$= \sum_{n=-2}^{\infty} (4z)^{n}$$

if $|4z| < 1 \Leftrightarrow$ ROC : $|z| < \frac{1}{4}$.

$$= \frac{(4z)^{-2}}{1-4z}$$


Im(z)
Re(z)
1/4

Question 5: [12%, Work-out question] Consider a continuous-time system:

$$y(t) = x(t+3) + \int_{t-2}^{t+2} x(s)ds. \tag{6}$$

1. [7%] Find the frequency response of the system $H(j\omega)$.

2. [5%] When the input is $x(t) = \sum_{k=1}^{6} \left(\frac{1}{2}\right)^k \cos\left(\frac{k\pi}{5}t\right)$, find the corresponding output $y(t)$.

   Hint: If you do not know how to solve this question, you can assume that

$$H(j\omega) = \begin{cases} \omega + \pi & \text{if } -\pi \le \omega \le 0 \\ \pi - \omega & \text{if } 0 \le \omega \le \pi \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

   You will still receive full credit if your answer is correct using the given $H(j\omega)$.

1. $h(t) = \delta(t+3) + u(t+2) - u(t-2)$

$$H(j\omega) = e^{-j\omega\cdot(-3)} + \frac{2\cdot\sin(\omega\cdot 2)}{\omega}$$

2. $$y(t) = \sum_{k=1}^{6} \left(\frac{1}{2}\right)^k \cos\left(\frac{k\pi}{5}(t+3)\right) + \sum_{k=1}^{6} \left(\frac{1}{2}\right)^k \frac{\sin\left(\frac{k\pi}{5}(t+2)\right) - \sin\left(\frac{k\pi}{5}(t-2)\right)}{k\pi/5}$$

Question 6: [9%, Work-out question] Consider two discrete-time signals

$$x[n] = \begin{cases} e^{j\frac{\pi}{40}n} & \text{if } 0 \leq n < 40 \\ 0 & \text{if } 40 \leq n < 80 \\ \text{periodic with period } 80 \end{cases} \tag{8}$$

1. [5%] Denote the DTFS $x[n]$ by a_k. Find the general expression of a_k for all possible k values.
   Hint: You may need to use the formula $\sum_{k=1}^{K} ar^{k-1} = \frac{a(1-r^K)}{1-r}$ when $r \neq 1$.
2. [2%] Find out the value of $\sum_{k=0}^{79} a_k$.
3. [2%] Find out the value of $\sum_{k=0}^{79} |a_k|^2$.

1\. $a_k = \frac{1}{80} \sum_{n=0}^{79} x[n] e^{-jk\frac{2\pi}{80} \times n}$

$= \frac{1}{80} \sum_{n=0}^{39} e^{-jk\frac{2\pi}{80} \times n} \cdot e^{j\frac{\pi}{40}n}$

if $k = 1$, $a_1 = \frac{1}{80} \sum_{n=0}^{39} 1 = \frac{1}{2}$.

if $k \neq 1$. $a_k = \frac{1}{80} \cdot \sum_{n=0}^{39} e^{j\frac{\pi}{40}(1-k)n}$

$= \frac{1}{80} \cdot \frac{1 - e^{j\frac{\pi}{40} \cdot (1-k) \times 40}}{1 - e^{j\frac{\pi}{40} \cdot (1-k)}}$

$= \frac{1}{80} \cdot \frac{1 - e^{j\pi(1-k)}}{1 - e^{j\frac{\pi}{40}(1-k)}}$

2. $\sum_{k=0}^{79} a_k = x[0] = 1.$

3. $\sum_{k=0}^{79} |a_k|^2 = \frac{1}{N} \sum_{n=0}^{79} |x[n]|^2$

$$= \frac{1}{80} \cdot \sum_{n=0}^{39} 1 = \frac{1}{2}$$

Question 7: [15%, Multiple-choice question] Consider two signals

$$h_1(t) = \begin{cases} t\sin(t^2) & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases} \tag{9}$$

and

$$h_2[n] = \cos(3n) + \sin(2n) \tag{10}$$

1. [1.25%] Is $h_1(t)$ periodic? No
2. [1.25%] Is $h_2[n]$ periodic? No
3. [1.25%] Is $h_1(t)$ even or odd or neither? Neither
4. [1.25%] Is $h_2[n]$ even or odd or neither? neither
5. [1.25%] Is $h_1(t)$ of finite ~~energy?~~ ~~power~~ energy No.
6. [1.25%] Is $h_2[n]$ of finite ~~energy~~ power? ~~No~~ Yes.

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless? No
2. [1.25%] Is System 2 memoryless? No.
3. [1.25%] Is System 1 causal? Yes
4. [1.25%] Is System 2 causal? No.
5. [1.25%] Is System 1 stable? No.
6. [1.25%] Is System 2 stable? No.