Midterm #2 of ECE301, Section 1 and Section 2
6:30-7:30pm, Wednesday, October 8, 2014, EE 129,

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

FALL 2014 MT2 SOLUTION

Name:

Student ID:

I certify that I have neither given nor received unauthorized aid on this exam.

Signature: Date:

Question 1: [20%, Work-out question, Learning Objectives 2 and 3]

Consider the following two signals $x(t)$ and $y(t)$:

$$x(t) = \begin{cases} e^{-t} & \text{if } -1 < t \\ 0 & \text{otherwise} \end{cases} = \begin{cases} e^{-t}, & \text{if } t+1 > 0 \\ 0, & ow \end{cases} = e^{-t} u(t+1)$$

$$y(t) = \delta(t - 0.7) + e^{-t}\mathcal{U}(t-3)$$

Compute the convolution integral $z(t) = x(t) * y(t)$.

Hint 1: You should first rewrite $y(t) = y_1(t) + y_2(t)$ for some $y_1(t)$ and $y_2(t)$ and then find $z_1(t) = x(t) * y_1(t)$ and $z_2(t) = x(t) * y_2(t)$, separately. Hint 2: Once you have found $z_1(t)$ and $z_2(t)$, you can simply express $z(t)$ by $z_1(t)$ and $z_2(t)$. No need to simplify the expression.

$$z_1(t) = x(t) * \delta(t-0.7) = x(t-0.7) = e^{-(t-0.7)} u(t-0.7+1)$$

$$z_2(t) = x(t) * e^{-t}u(t-3) = \int_{-\infty}^{\infty} e^{-(t-\tau)} u(t-\tau+1) e^{-\tau} u(\tau-3)\, d\tau =$$

$$= \int_{3}^{\infty} e^{-t} e^{\tau} e^{-\tau} u(t-\tau+1)\, d\tau = e^{-t} \int_{3}^{\infty} u(t-\tau+1)\, d\tau =$$

$$= \begin{cases} e^{-t} \int_{3}^{t+1} 1\, d\tau, & \text{if } t+1 \geq 3 \\ 0, & ow \end{cases} = \begin{cases} e^{-t}(t+1-3), & \text{if } t \geq 2 \\ 0, & ow \end{cases}$$

$$= e^{-t}(t-2)\, u(t-2)$$

$$z(t) = z_1(t) + z_2(t) = e^{-(t-0.7)} u(t+0.3) + e^{-t}(t-2)\, u(t-2)$$

Question 2: [20%, Work-out question, Learning Objectives 1, 2, 3, and 5]
Consider two DT-LTI systems. The impulse response of System 1 is

$$h_1[n] = \begin{cases} 2 & \text{if } n = 0 \text{ or } n = 3 \\ 0 & \text{if } n = 1 \text{ or } n = 2 \\ 0 & \text{if } n \le -1 \text{ or } n \ge 4 \end{cases} = 2\delta[n] + 2\delta[n-3]$$

The input-output relationship of System 2 is

$$y_2[n] = \frac{x_2[n] + x_2[n+3]}{2}.$$

We now build a new system by "serially concatenating Systems 1 and 2". Namely, we feed the output of System 1 to the input of system 2:

$$x_1[n] \overset{\text{System 1}}{\longrightarrow} y_1[n] = x_2[n] \overset{\text{System 2}}{\longrightarrow} y_2[n] \tag{1}$$

1. [3%] What is the definition of "impulse response"?
2. [10%] Plot the impulse response $h_{\text{serial}}[n]$ of the new concatenated LTI system for the range of $n = -5$ to 5.

   Hint: It will be much easier if you compute $h_{\text{serial}}[n]$ directly by the definition of the impulse response.
3. [7%] What is the output $y[n]$ when the input is $x[n] = e^{2+j\frac{2\pi}{3}n}$?

   Hint: If you do not know the answer to the previous question, you can assume $h[n]$ being

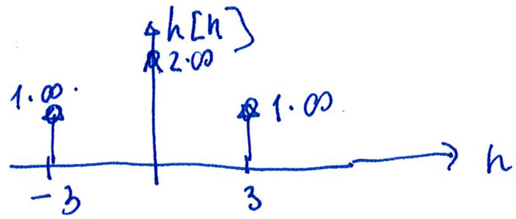
$$h[n] = \begin{cases} 2 & \text{if } 0 \le n \le 1 \\ -1 & \text{if } 2 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$$

1. output of the system to $x[n] = \delta[n]$

2. $h_2[n] = \frac{1}{2}\delta[n] + \frac{1}{2}\delta[n+3]$

$h[n] = h_1[n] * h_2[n] = \left(\frac{1}{2}\delta[n] + \frac{1}{2}\delta[n+3]\right)\left(2\delta[n] + 2\delta[n-3]\right) =$

$= \delta[n] + \delta[n-3] + \delta[n+3] + \delta[n] = 2\delta[n] + \delta[n-3] + \delta[n+3]$



3. $x[n] = e^{2+j\frac{2\pi}{3}n}$

$$y[n] = x[n] * h[n] = e^{2+j\frac{2\pi}{3}n} * (2\delta[n] + \delta[n-3] + \delta[n+3])$$

$$= 2e^{2+j\frac{2\pi}{3}n} + e^{2+j\frac{2\pi}{3}(n-3)} + e^{2+j\frac{2\pi}{3}(n+3)} =$$

$$= 2e^{2+j\frac{2\pi}{3}n} + e^{2+j\frac{2\pi}{3}n-j2\pi} + e^{2+j\frac{2\pi}{3}n+j2\pi} =$$

$$= 4e^{2+j\frac{2\pi}{3}n}$$

alternative
<u>optional</u>: $h[n] = 2\delta[n] + 2\delta[n-1] - \delta[n-2] - \delta[n-3]$

$$y[n] = x[n] * h[n] = 2e^{2+j\frac{2\pi}{3}n} + 2e^{2+j\frac{2\pi}{3}(n-1)} - e^{2+j\frac{2\pi}{3}(n-2)} - e^{2+j\frac{2\pi}{3}(n-3)}$$

Question 3: [20%, Work-out question, Learning Objectives 1, 2, 3, and 5]
Consider the following system:

$$y(t) = \begin{cases} 0 & \text{if } t < 0 \\ \int_{-\infty}^{t} e^{-t+s} x(s) ds & \text{if } t \geq 0 \end{cases}$$

1. [10%] Find the impulse response $h(t)$ of the above system.
2. [10%] Find out the output when the input is e^{jt}.

1) $h(t) = \begin{cases} 0, & \text{if } t<0 \\ \int_{-\infty}^{t} e^{-t+s} \delta(s) ds, & t \geq 0. \end{cases} =$

$= \begin{cases} 0, \text{ if } t<0 \\ e^{-t} \int_{-\infty}^{t} \delta(s) ds, \ t \geq 0 \end{cases} = e^{-t} u(t)$

2) $h(t) = e^{-t} u(t)$ is not LTI system

$y(t) = \begin{cases} 0, \text{ if } t<0 \\ \int_{-\infty}^{t} e^{-t+s} e^{js} ds, \text{ if } t \geq 0 \end{cases} = \begin{cases} 0, \text{ if } t<0 \\ e^{-t} \int_{-\infty}^{t} e^{s(1+j)} ds, \ t \geq 0 \end{cases}$

$= \left[e^{-t} \cdot \int_{-\infty}^{t} e^{s(1+j)} ds \right] \cdot u(t) = e^{-t} u(t) \cdot \frac{1}{1+j} e^{(1+j)s} \Big|_{-\infty}^{t} =$

$= e^{-t} u(t) \frac{1}{1+j} e^{(1+j)t} = \frac{1}{1+j} e^{jt} u(t)$

Question 4: [20%, Work-out question, Learning Objective 4]
We know that

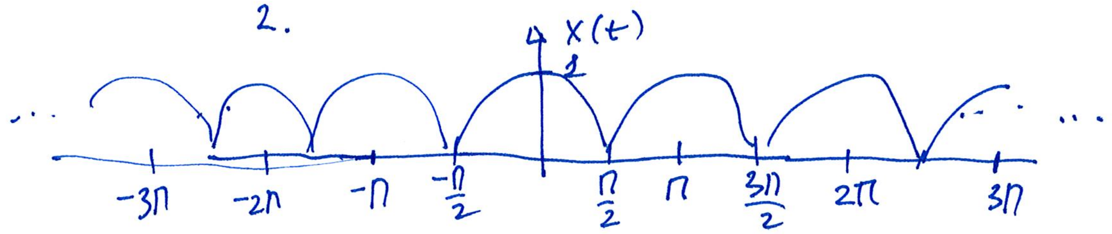
$$x(t) = \begin{cases} \cos(t) & \text{if } |t| < \frac{\pi}{2} \\ x(t) \text{ is periodic with period } \pi & \end{cases} \tag{2}$$

1. [3%] What is the frequency ω_0 of this signal $x(t)$?
2. [7%] Plot $x(t)$ for the range of $-3\pi \le t \le 3\pi$.
3. [10%] Find out the Fourier series coefficient a_5 of the given signal $x(t)$. Hint: You may need to use the formulas:

$$\cos(\theta) = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$
$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt.$$

1. $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$

2.



3. $a_k = \frac{1}{T}\int_{-\pi/2}^{\pi/2} \cos t \, e^{-j2kt} dt = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (e^{jt}+e^{-jt})e^{-j2kt} dt$

$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{jt(1-2k)} + e^{-jt(1+2k)} dt = \frac{1}{2\pi}\frac{1}{j(1-2k)}\left[e^{j\frac{\pi}{2}(1-2k)} - e^{-j\frac{\pi}{2}(1-2k)}\right]$

$+ \frac{1}{2\pi}\frac{1}{(1+2k)(-j)}\left[e^{-j\frac{\pi}{2}(1+2k)} - e^{j\frac{\pi}{2}(1+2k)}\right] = \frac{1}{\pi(1-2k)}\sin\left(\frac{\pi}{2}(1-2k)\right)$

$+ \frac{1}{\pi(1+2k)}\sin\left(\frac{\pi}{2}(1+2k)\right)$

$a_5 = \frac{1}{\pi(1-10)}\sin\left(\frac{\pi}{2}(1-10)\right) + \frac{1}{\pi(1+10)}\sin\left(\frac{\pi}{2}(1+10)\right) =$

$= \frac{1}{-9\pi}\sin\left(\frac{\pi}{2}-5\pi\right) + \frac{1}{\pi \cdot 11}\sin\left(\frac{\pi}{2}+5\pi\right) = +\frac{1}{9\pi} + \frac{1}{11\pi}$

Question 5: [20%, Multiple Choices]

The following questions are multiple-choice questions and there is no need to justify your answers. Consider the following two systems:

System 1: When the input is $x_1(t)$, the output is

$$y_1(t) = e^{-t}x(2t) + 2e^{-j0.25\pi}x(t-1) + x(3t-2). \tag{3}$$

System 2: When the input is $x_2[n]$, the output is

$$y_2[n] = \begin{cases} \left|x_2[\frac{n}{2}]\right| e^{jn} & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases} \tag{4}$$

Answer the following questions

1. [4%, Learning Objective 1] Is System 1 memoryless? Is System 2 memoryless?
2. [4%, Learning Objective 1] Is System 1 causal? Is System 2 causal?
3. [4%, Learning Objective 1] Is System 1 stable? Is System 2 stable?
4. [4%, Learning Objective 1] Is System 1 linear? Is System 2 linear?
5. [4%, Learning Objective 1] Is System 1 time-invariant? Is System 2 time-invariant?

$y_1(t)$	$y_2[n]$
1. with memory	1. with memory
2 not causal	2 not causal
3 not stable	3 stable
4 linear	4 non-linear
5 time-varying	5 time-varying

Discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \tag{1}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n} \tag{2}$$

Continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \tag{3}$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt \tag{4}$$

Continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \tag{5}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{6}$$

Discrete-time Fourier transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(j\omega) e^{j\omega n} d\omega \tag{7}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \tag{8}$$

Laplace transform

$$x(t) = \frac{1}{2\pi} e^{\sigma t} \int_{-\infty}^{\infty} X(\sigma + j\omega) e^{j\omega t} d\omega \tag{9}$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \tag{10}$$

Z transform

$$x[n] = r^n \mathcal{F}^{-1}(X(re^{j\omega})) \tag{11}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{12}$$