Midterm #1 of ECE301, Section 1 and Section 2
6:30-7:30pm, Wednesday, September 10, 2014, EE 129,

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name: Solution

Student ID:

I certify that I have neither given nor received unauthorized aid on this exam.

Signature: Date:

Question 1: [20%, Work-out question, Learning Objective 3] Consider two continuous-time signals $x(t)$ and $y(t)$.

$$x(t) = \begin{cases} e^{jt} & \text{if } 1 \le t \le 5 \\ 0 & \text{otherwise} \end{cases}$$

$$y(t) = \begin{cases} |1-e^t| & \text{if } 2 \le t \\ 0 & \text{if } t < 2 \end{cases}$$

Compute the expression of

$$z(t) = \int_{s=-\infty}^{\infty} x(t-s)y(s)ds.$$

$$x(t-s) = \begin{cases} e^{j(t-s)} & \text{if } t-5 \le s \le t-1 \\ 0 & \text{otherwise} \end{cases} \qquad y(s) = \begin{cases} |1-e^s| = e^s - 1 & \text{if } s \ge 2 \\ 0 & \text{otherwise} \end{cases}$$

① if $t < 3$ $\qquad z(t) = 0$.

② if $3 \le t \le 7$

$$z(t) = \int_2^{t-1} (e^s - 1)\, e^{jt} e^{-js}\, ds$$

$$= e^{jt} \int_2^{t-1} e^{-js} - e^{s(1-j)}\, ds \qquad \times -1$$

$$= e^{jt} \left[\frac{e^{-js}}{-j} - \frac{e^{s(1-j)}}{(1-j)} \right]_2^{t-1} \qquad \times -1$$

$$= -e^{jt} \frac{e^{-j(t-1)} - e^{-2j}}{-j} + e^{jt} \frac{e^{(1-j)(t-1)} - e^{2(1-j)}}{(1-j)}$$

$$= +\frac{1}{j}\left(e^{j} - e^{j(t-2)} \right) + \frac{1}{(1-j)} \left(e^{j+(t-1)} - e^{j(t-2)+2} \right)$$

③ if $t > 7$

$$z(t) = \int_{t-5}^{t-1} (1-e^s)\, e^{jt} e^{-js}\, ds \qquad \times -1$$

$$= -e^{jt} \frac{e^{-j(t-1)} - e^{-j(t-5)}}{-j} + e^{jt} \frac{e^{(t-1)(1-j)} - e^{(t-5)(1-j)}}{(1-j)}$$

$$= \frac{e^{j} - e^{5j}}{j} + e^{t} \frac{e^{j-1} - e^{5(j-1)}}{(1-j)}$$

Question 2: [15%, Work-out question, Learning Objectives 1] Given a discrete-time signal

$$x[n] = \begin{cases} \frac{(1+3j)e^{j2\omega n}}{2-j} & \text{if } 0 \leq n \leq 49 \\ 0 & \text{if } 50 \leq n \leq 99 \\ x[n] \text{ is periodic with period } 100 & \end{cases}$$

Compute the average power of $x[n]$ over the range of $n = -60$ to 30. Hint: You may need to answer something like "what is the value of $\sum_{k=4}^{10} \pi$?"

You need to compute $\frac{1}{30-(-60)+1} \sum_{n=-60}^{30} |x[n]|^2 = \frac{1}{91} \sum_{n=-60}^{30} |x[n]|^2$

$$\sum_{n=-60}^{30} |x[n]|^2 = \sum_{n=-60}^{-51} \left| \frac{(1+3j)}{2-j} e^{j2\omega n} \right|^2 + \sum_{n=0}^{30} \left| \frac{(1+3j)}{2-j} e^{j2\omega n} \right|^2$$

Note that $\left| \frac{(1+3j)}{2-j} e^{j2\omega n} \right|^2 = \left| \frac{(1+3j)}{2-j} \right|^2 \left| e^{j2\omega n} \right|^2 = \left| \frac{(1+3j)}{2-j} \right|^2$

Since $|e^{j2\omega n}| = 1$ regardless of n values.

As a result,

$$\frac{1}{91} \sum_{n=-60}^{30} |x[n]|^2 = \frac{1}{91} \cdot 41 \cdot \left| \frac{1+3j}{2-j} \right|^2$$

$$= \frac{41}{91} \left| \frac{-1+7j}{5} \right|^2 = \frac{41}{91} \left(\frac{1}{25} + \frac{49}{25} \right) = \frac{82}{91}$$

Question 3: [10%, Work-out question, Learning Objective 1] Compute the following values

$$\sum_{m=-\infty}^{108} \frac{2\pi(1.5)^{m+20}e^{j\pi m}}{5}.$$

Hint: You may need to use the formula $\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$ if $|a| \cancel{<1}$. $|r|<1$

$$\sum_{m=-\infty}^{108} \frac{2\pi\,(1.5)^{m+20}\,e^{j\pi m}}{5}$$

by change of variables $n = 109-m$.

$$= \frac{2\pi}{5}(1.5)^{20} \sum_{m=-\infty}^{108} \left(1.5\,e^{j\pi}\right)^m = \frac{2\pi}{5}(1.5)^{20} \sum_{n=1}^{\infty} \left(1.5\,e^{j\pi}\right)^{109-n}$$

$$= \frac{2\pi}{5}(1.5)^{128}\left(e^{j\pi}\right)^{108} \sum_{n=1}^{\infty} \left(\frac{1}{1.5e^{j\pi}}\right)^{n-1}$$

$$= \frac{2\pi}{5}(1.5)^{128}\left(e^{j\pi}\right)^{108} \frac{1}{1-\frac{1}{1.5e^{j\pi}}}$$

since $\left|\frac{1}{1.5e^{j\pi}}\right| = \frac{2}{3} < 1$

※ $e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1$.

$$= \frac{2\pi}{5}(1.5)^{128}\left(e^{j\pi}\right)^{108} \cdot \frac{3}{5}$$

$$= \frac{6\pi}{25}(1.5)^{128}(-1)^{108} = \frac{6\pi}{25}\left(\frac{3}{2}\right)^{128}$$

Question 4: [20%, Work-out question, Learning Objectives 1, 5] We know a linear system that does the following.

$$y(t) = \begin{cases} 3x(t) & \text{if } x(t) \text{ is an odd signal} \\ 0 & \text{if } x(t) \text{ is an even signal} \end{cases} \quad (1)$$

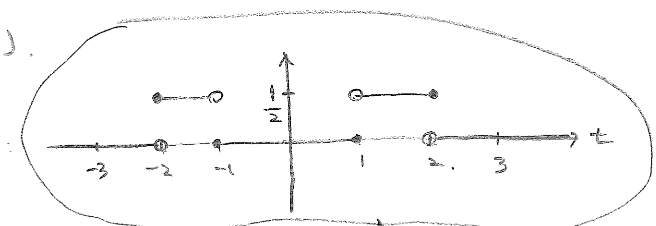
Consider the following continuous-time signal

$$x(t) = \begin{cases} 1 & \text{if } 0 < t \leq 2 \\ -1 & \text{if } -1 \leq t < 0 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Plot the output $y(t)$ (using the above $x(t)$ as input) for the range of $t = -3$ to 3. (Hint: If you do not know the answer, you should plot the even part of $x(t)$, denoted by $x_{\text{even}}(t)$, for the range of $t = -3$ to 3 instead. You will get 14 points if you plot $x_{\text{even}}(t)$ correctly.)

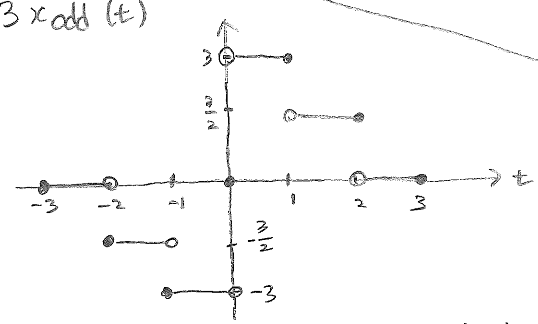
$x(t) = x_{even}(t) + x_{odd}(t).$

$x_{odd}(t) = \dfrac{x(t) - x(-t)}{2}$

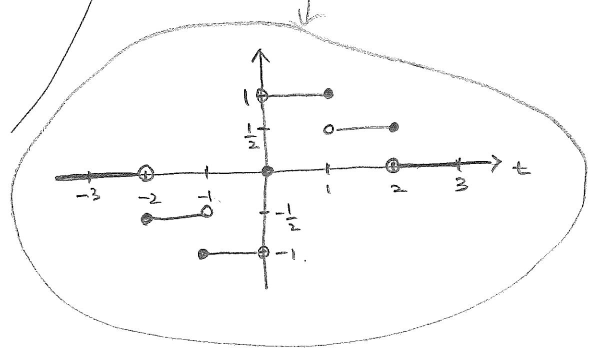


$\therefore \; y(t) = 3x_{odd}(t)$

$=$



For your reference, $x_{even}(t) = \dfrac{x(t) + x(-t)}{2}$



Question 5: [15%, Work-out question, Learning Objectives 1, 6] Consider the following system that takes signal $x[n]$ as input and outputs

$$y[n] = x[2n-1] + 1 \tag{3}$$

Is the above system is linear or not? Carefully explain the steps how you decide whether the system is linear or not.

Is $\alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow \alpha_1 y_1[n] + \alpha_2 y_2[n]$?

where $y_1[n] = x_1[2n-1]+1$

$y_2[n] = x_2[2n-1]+1$

Let $x[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow y[n]$.

Then, $y[n] = x[2n-1]+1 = \alpha_1 x_1[2n-1] + \alpha_2 x_2[2n-1] + 1$.

However, $\alpha_1 y_1[n] + \alpha_2 y_2[n] = \alpha_1 x_1[2n-1] + \alpha_1 + \alpha_2 x_2[2n-1] + \alpha_2$.

different except $\alpha_1 + \alpha_2 = 1$

But this must be true for all α_1, α_2 in order to be "linear".

$\therefore$ System is not linear.

Question 6: [20%, Multiple Choices] The following questions are multiple-choice questions and there is no need to justify your answers. Consider two continuous-time signals:

$$x_1(t) = (e^{2t} + e^{-2t})\cos(t)$$
$$x_2(t) = \cos(3t) + \sin(5t) + e^{j0.2t}$$

and two discrete-time signals:

$$x_3[n] = \sin(\pi n^3) + \cos(\frac{\pi n}{3})$$
$$x_4[n] = e^{j\frac{3\pi}{2}n}\cos(\pi n).$$

1. [10%, Learning Objective 1] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not, *respectively.* If it is periodic, write down the fundamental period. Please state explicitly which signal is periodic and which is not.

2. [10%, Learning Objective 1] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them, respectively. Please state explicitly which signal is even, which is odd, and which is neither.

$x_1(t) = (e^{2t} + e^{-2t})\cos(t)$ N.P. EVEN.

$x_2(t) = \cos(3t) + \sin(5t) + e^{j0.2t}$

Periodic : $T = LCM(\frac{2\pi}{3}, \frac{2\pi}{5}, \frac{2\pi}{0.2}) = 10\pi.$ Neither.

$x_3[n] = \cancel{\sin(\pi n^3)}^{\to 0} + \cos(\frac{\pi n}{3}) = \cos(\frac{\pi n}{3})$

Periodic : $N = LCM(\frac{2\pi}{\pi/3}, 1) = 6.$ EVEN.

$x_4[n] = e^{j\frac{3\pi}{2}n}\cos(\pi n) = e^{j\frac{3\pi}{2}n}\left(\frac{e^{j\pi n} + e^{-j\pi n}}{2}\right)$

$= \frac{1}{2}e^{j\frac{5\pi}{2}n} + \frac{1}{2}e^{j\frac{\pi}{2}n}$

Periodic : $N = LCM(\frac{2\pi}{5\pi/2}, \frac{2\pi}{\pi/2}, 1) = 4.$ Neither.