Final Exam of ECE301, Section 1 and Section 2
10:30am–12:30pm, Friday, December 19, 2014, EE 129.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

I certify that I have neither given nor received unauthorized aid on this exam.

Signature: Date:

Question 1: [17%, Work-out question]

1. [1%] What does the acronym "AM-SSB" stand for?

Amplitude Modulation Single Side Band.

Prof. Wang wanted to transmit an AM-SSB signal. To that end, he wrote the following MATLAB code.

```
% Initialialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read two different .wav files
[x1, f_sample, N]=wavread('x1');
x1=x1';
[x2, f_sample, N]=wavread('x2');
x2=x2';

% Step 0: Initialize several parameters
W_1=pi*4000;
W_2=pi*6000;
W_3=pi*12000;
W_4=pi*6000;
W_5=pi*2000;
W_6=?????;
W_7=????;


% Step 1: Make the signals band-limited.
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);

% Step 2: Multiply x1_new and x2_new with a cosine wave.
x1_h=x1_new.*cos(W_2*t);
x2_h=x2_new.*cos(W_3*t);

% Step 3: Keep one of the two side bands
h_one=1/(pi*t).*(sin(W_4*t))-1/(pi*t).*(sin(W_5*t));
h_two=1/(pi*t).*(sin(W_6*t))-1/(pi*t).*(sin(W_7*t));
```

```
x1_sb=ece301conv(x1_h, h_one);
x2_sb=ece301conv(x2_h, h_two);

% Step 4: Create the transmitted signal
y=x1_sb+x2_sb;
wavwrite(y', f_sample, N, 'y.wav');
```

2. [1.5%] What is the bandwidth (Hz) of the signal x1_new? 2000 Hz

3. [2.5%] Is this AM-SSB transmitting an upper-side-band signal or a lower-side-band signal?

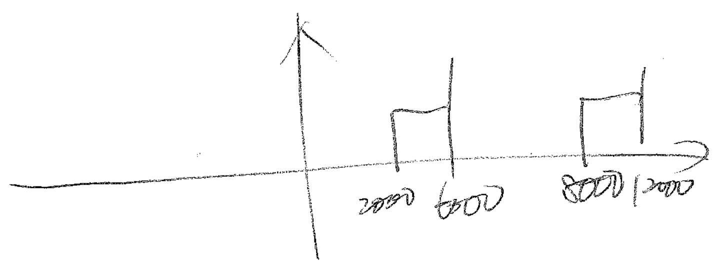
   Lower.

4. [4%] What should the values of W_6 and W_7 be in the MATLAB code?

   12000π 8000π

Knowing that Prof. Wang used the above code to generate the "y.wav" file, a student tried to demodulate the output waveform "y.wav" by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[y, f_sample, N]=wavread('y');
y=y';

% Initialize several parameters
W_8=????;        4000π
W_9=????;        6000π
W_10=????;       12000π

% Create the low-pass filter.
h_M=1/(pi*t).*(sin(W_8*t));

% demodulate signal 1
y1=4*y.*cos(W_9*t);
x1_hat=ece301conv(y1,h_M);

sound(x1_hat,f_sample)

% demodulate signal 2
y2=4*y.*cos(W_10*t);
x2_hat=ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```



5. [3%] Continue from the previous question. What should the values of W_8 to W_10 in the MATLAB code?

6. [5%] It turns out that using the above MATLAB code, we can hear the sound properly when playing "sound(x2_hat,f_sample)" but there is some problem when playing "sound(x1_hat,f_sample)". Please (i) Describe why there is a problem when playing "sound(x1_hat,f_sample)" and (ii) Describe how we can fix the code so that we can hear x1_hat properly. (i) The bandwidth of x_2 will

   Hint: If you do not know the answers of Q1.2 to Q1.6, please simply draw the AMSSB modulation and demodulation diagrams and mark carefully all the parameter values. You will receive 9 points for Q1.2 to Q1.6.

(ii) Pass it through a BPF with 2000π to 6000π before demod.

corrupt the bandwidth of x_1 during the demod.

Question 2: [16%, Work-out question]

Consider a continuous time signal $x(t) = \sin(2\pi \cdot 500 \cdot t)$ and we use a digital voice recorder to convert the continuous time signal $x(t)$ to its discrete time counter part $x[n]$. Each $x[n]$ is represented by 16 bits (or equivalently 2 bytes). The digital voice recorder we use has a setting that allows you to change its sampling frequency.

Answer the following question.

1. [1%] Suppose we set the sampling frequency to 1000 Hz. What is the sampling period T? ~~0.0001 sec~~ 0.001 sec.
2. [1%] Continue from the previous question. What is the relationship between $x(t)$ and $x(n)$? $x[n] = x(0.001 \cdot n)$
3. [1.5%] Continue from the previous question. Suppose we start our recording at time $t = -1$ and the recording lasts for for 4 seconds, i.e., from $t = -1$ to 3. What is the file size after we finish recording? Please pay attention to the unit of your answer. 8000 bytes
4. [2.5%] Continue from the previous question. Plot $x[n]$ for the range of $n = -3$ to 3.
5. [2.5%] Continue from the previous question. We use "zero-order hold" to reconstruct the original signal. We denote the output by $\hat{x}_{\text{ZOH}}(t)$. Plot $\hat{x}_{\text{ZOH}}(t)$ for the range of $t = -1$ to 3.

   [Alternative question:] If you do not know the answer to this question, you can assume that the sampling period is $T = 1/3$ and the sampled values are $x[n] = \delta[n] + 2\delta[n-4]$. Plot the Zero-Order-Hold output $\hat{x}_{\text{ZOH}}(t)$ for the range of $t = -2$ to 2. If your answer is correct, you will receive 2.5 points.
6. [2.5%] Continue from the previous question (NOT the alternative question). Suppose we use the optimal band-limited reconstruction and denote the output by $\hat{x}_{\text{sinc}}(t)$. Plot $\hat{x}_{\text{sinc}}(t)$ for the range of $t = -1$ to 3.

   [Alternative question:] If you do not know the answer to this question, you can assume that the sampling period is $T = 1/3$ and the sampled values are $x[n] = \delta[n] + 2\delta[n-2]$. Plot the optimal band-limited reconstruction output $\hat{x}_{\text{sinc}}(t)$for the range of $t = -2$ to 2. If your answer is correct, you will receive 2.5 points.
7. [2%] Continue from the previous question (NOT the alternative question). Is $\hat{x}_{\text{sinc}}(t) = x(t)$? Justify your answer by one quick sentence. No. Since the sampling freq is not large enough.

   [Alternative question:] If you do not know the answer to this question, please under what condition will a sampling system explain the so-called "aliasing" effect. If your answer is correct, you will receive 1.5 points.

Consider another signal $y(t) = \frac{\sin(100t) - \sin(20t)}{\pi t}$. Answer the following questions:

8. [2%] What is the value of the "Nyquist frequency" of $y(t)$? Your answer should be something like 44.1 kHz. There is no need to explain how you find the Nyquist frequency. [There is one more question in the back of this page.]

   200 rad/sec.

9. [1%] Please choose a sampling period T that ensures that the system is not under-sampled. Your answer should be something like $T = 0.5$ sec. There is no need to explain why you choose such a T value.

$$T \leq \frac{2\pi}{W_{Nyquist}} \approx$$

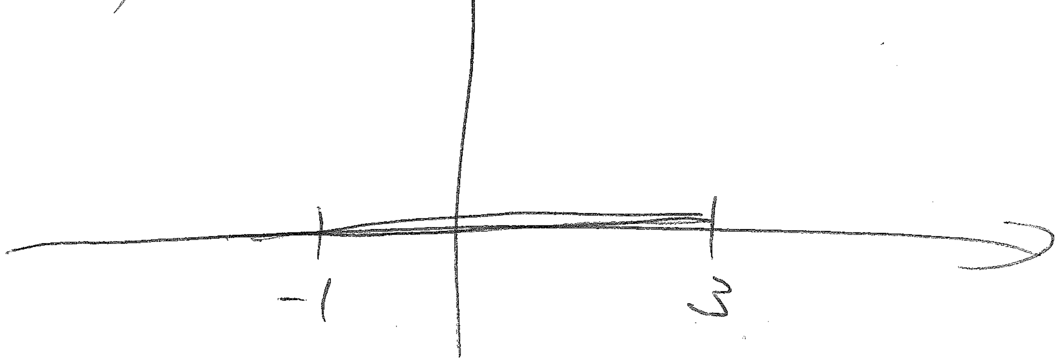
2\pi / W_Nyquist = 0.01\pi ~= 0.03

We thus choose T=0.025 sec.

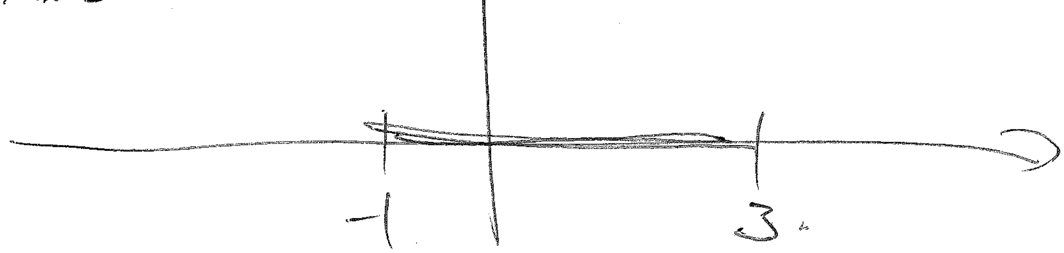
(Please ignore the previous answer of T=0.05 sec)

Choose $T =$ [illegible] sec.

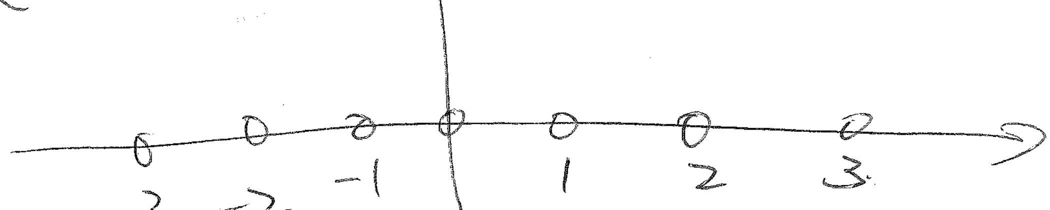
5. $\hat{x}_{ZOH}(t)$



6. $\hat{x}_{sinc}(t)$

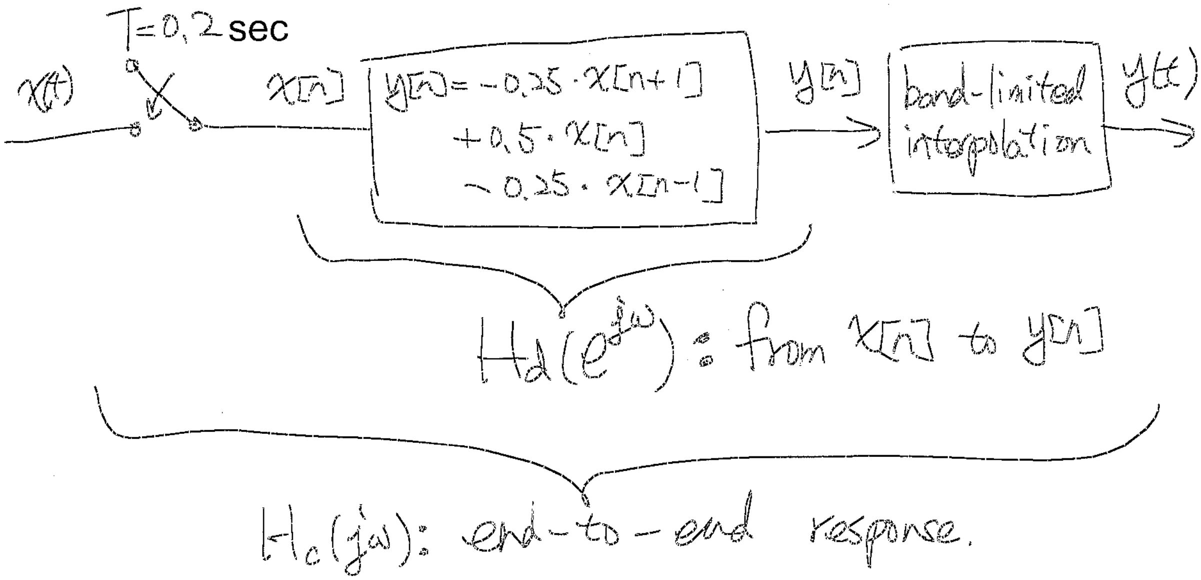


4. $x[n] =$



Question 3: [14%, Work-out question]

Consider the following discrete-time processing system for continuous-time signals.



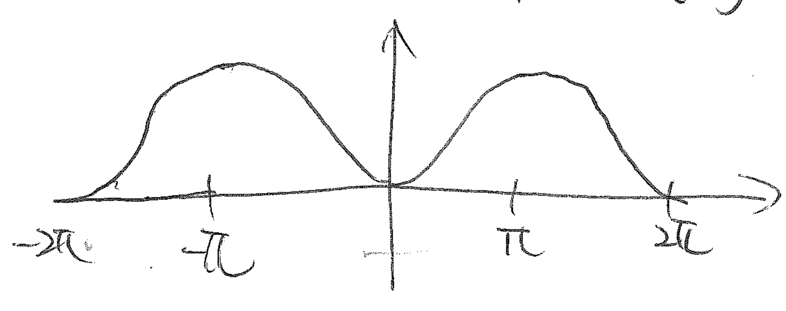
1. [6%] Find the discrete-time frequency response $H_d(e^{j\omega})$ and plot $H_d(e^{j\omega})$ for the range of $-2\pi \le \omega \le 2\pi$.

2. [8%] Find the continuous-time frequency response $H_c(j\omega)$ plot $H_c(j\omega)$ for the range of $-20\pi < \omega < 20\pi$.

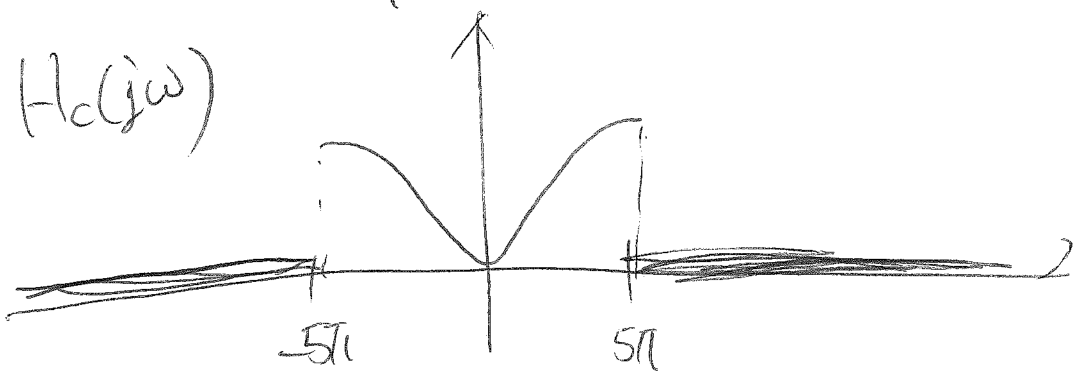
   Hint: If you do not know the answer to this question, you can answer the following questions instead: (i) If the input is $x(t) = 5$, what is the output? (ii) What type of filter is the overall system? A low-pass filter? A high-pass filter? Or a band-pass filter? Use a single sentence to justify your answer. If both your answers are correct, you will still get 4 points.

$$Hd(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= 0.5 - 0.5\cos(\omega)$$



$$W_s = \frac{2\pi}{T} = 10\pi$$



$$H_c(j\omega) = \begin{cases} 0.5 - 0.5\cos(\omega \cdot 0.2) & \text{if } |\omega| < 5\pi \\ 0 & \text{otherwise.} \end{cases}$$

Question 4: [13%, Work-out question]

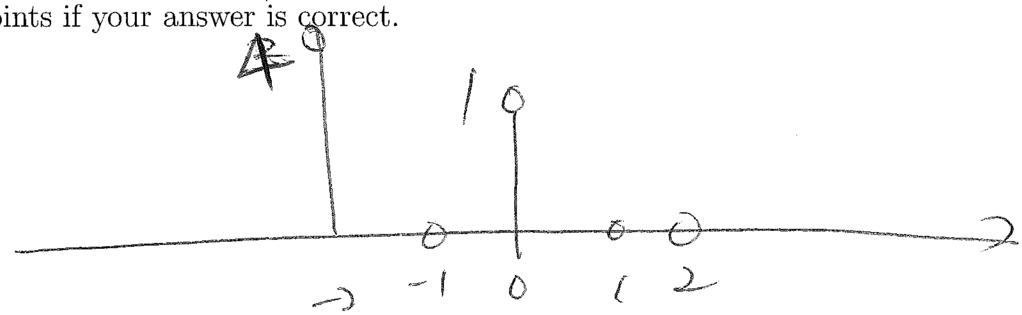
Consider the following discrete-time signal

$$x[n] = \begin{cases} 2^{-n} & \text{if } n \leq 0 \text{ and } n \text{ is even} \\ 0 & \text{if } n \leq 0 \text{ and } n \text{ is odd} \\ 0 & \text{if } n > 0 \end{cases} \tag{1}$$

1. [1%] Plot $x[n]$ for the range of $n = -2$ to 2.
2. [9%] Find the Z-transform of $x[n]$ and carefully specify and draw its ROC.
3. [3%] Draw a separate figure to describe the pole-zero chart of the Z-transform.

   [Alternative question:] If you do not know the answer to the previous question, you can assume $X(z) = \frac{z^2+z}{1+z^2}$ and plot the corresponding pole-zero chart. You will get 3 points if your answer is correct.

1.



2. $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$$= \sum_{k=-\infty}^{0} x[2k] z^{-(2k)}$$

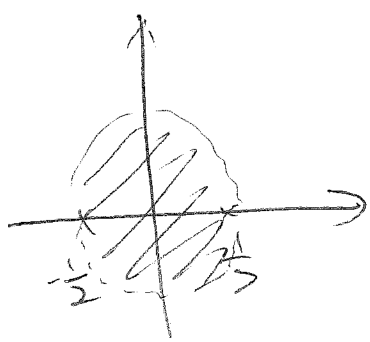
$$= \sum_{k=\infty}^{0} 2^{-2k} z^{-2k}$$

$$= \frac{1}{1-4z^2}$$

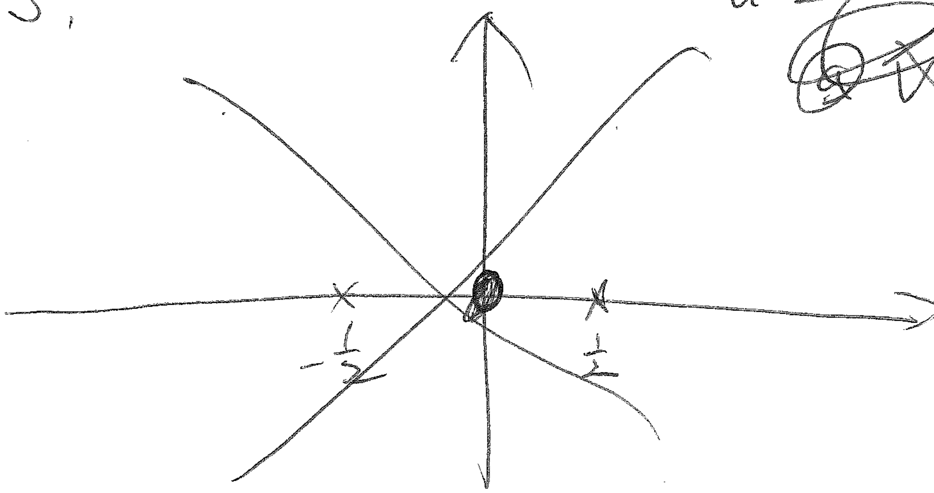
if $|4z^2| < 1$

$\Leftrightarrow |z| < \frac{1}{2}$.

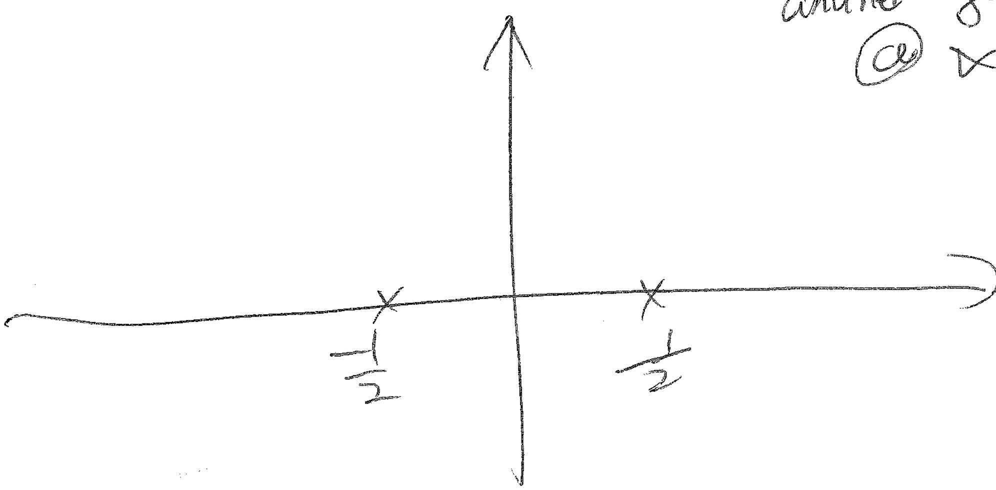
ROC



3.



~~another pole~~ @ ∞



another zero @ ∞.

Question 5: [13%, Work-out question]

Consider a DT LTI system with impulse response

$$h[n] = \sum_{k=0}^{19} 2^{-(k+1)} \delta[n-k]. \tag{2}$$

1. [13%] Find the output $y[n]$ when the input is $x[n] = \cos(3\pi n) + e^{j0.1\pi n}$.

   Hint 1: If the expression of $h[n]$ is confusing to you, you may want to plot $h[n]$ first so that you understand how $h[n]$ looks like.

   Hint 2: You may need the following formula: $\sum_{n=1}^{N} ar^{n-1} = \frac{a(1-r^N)}{1-r}$ if $r \neq 1$.

$$H(e^{j\omega}) = \sum_{k=0}^{19} 2^{-(k+1)} e^{-jk\omega}$$

$$= \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}e^{-j\omega}\right)^{20}\right)}{1-\frac{1}{2}e^{-j\omega}}$$

$$y[n] = \frac{1}{2}e^{j3\pi n} \cdot \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}e^{-j3\pi}\right)^{20}\right)}{1-\frac{1}{2}e^{-j3\pi}}$$

$$+\frac{1}{2}e^{-j3\pi n} \cdot \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}e^{-j(-3\pi)}\right)^{20}\right)}{1-\frac{1}{2}e^{j3\pi}}$$

$$+ e^{j0.1\pi n} \cdot \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}e^{-j\frac{\pi}{10}}\right)^{20}\right)}{1-\frac{1}{2}e^{-j0.1\pi}}$$

#

Question 6: [12%, True / False questions. No need for justification]

F 1. [1%] [T] [F] Two distinctly different discrete-time signals must have different Z-transform *expressions.*

T 2. [1%] [T] [F] Two distinctly different discrete-time signals can have the same Z-transform *ROCs.*

T 3. [1%] [T] [F] For any invertible system, two distinctly different input signals $x_1(t)$ and $x_2(t)$ must have two different outputs $y_1(t)$ and $y_2(t)$.

F 4. [1.5%] [T] [F] For any signal $x[n]$ that has finite power, we can always find its DTFT $X(e^{j\omega})$

F 5. [1.5%] [T] [F] For any aperiodic signal $x[n]$, its DTFT is also aperiodic.

F 6. [1.5%] [T] [F] Multiplying by t in the time domain causes multiplication by $j\omega$ in the frequency domain.

T 7. [1.5%] [T] [F] Multiplying by e^{jWt} in the time domain places the original signal in another frequency band.

F 8. [1.5%] [T] [F] Consider a signal $y(t) = 5(\mathcal{U}(t+2) - \mathcal{U}(t-2))$. We then have $\int_{-\infty}^{\infty} Y(j\omega)d\omega = 5$.

T 9. [1.5%] [T] [F] Consider a linear system and we know that when the input is $x(t) = \cos(t)$ the output is $y(t) = \sin(2t)$. The system must be time-varying.

Question 7: [15%, Multiple-choice question] Consider two signals

$$h_1(t) = e^{-t^2+1}\cos(t) \tag{3}$$

and

$$h_2[n] = \sin\left(0.1\pi \cdot \left(\int_{n-1}^{n+1} s ds\right)\right) + \cos((n+3.5)\pi) \tag{4}$$

1. [1.25%] Is $h_1(t)$ periodic? No
2. [1.25%] Is $h_2[n]$ periodic? Yes.
3. [1.25%] Is $h_1(t)$ even or odd or neither? even
4. [1.25%] Is $h_2[n]$ even or odd or neither? odd
5. [1.25%] Is $h_1(t)$ of finite energy? Yes.
6. [1.25%] Is $h_2[n]$ of finite energy? No.

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless? No
2. [1.25%] Is System 2 memoryless? No
3. [1.25%] Is System 1 causal? No
4. [1.25%] Is System 2 causal? No
5. [1.25%] Is System 1 stable? Yes.
6. [1.25%] Is System 2 stable? No.