Midterm #2 of ECE301, Prof. Wang's section

6:30–7:30pm Tuesday, February 28, 2012, FRNY G140,

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. There are 10 pages in the exam booklet. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

E-mail:

Signature:

Question 1: [35%, Work-out question, Outcomes 2 and 3] Consider a discrete-time linear time-invariant system with the impulse response being

$$h[n] = 2^{-|n|}. \tag{1}$$

Find out the output $y[n]$ when the input is

1. [3%] Find out the output $y[n]$ when the input is

$$x[n] = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

2. [20%] Find out the output $y[n]$ when the input is

$$x[n] = \begin{cases} 2^n & \text{if } -10 \le n \le 10 \\ 0 & \text{otherwise} \end{cases}. \tag{3}$$

3. [12%] Find out the output $y[n]$ when the input is $x[n] = e^{j(n+2)}$.

Hint: The geometric series formulas are

$$\text{if } |r| < 1, \text{ then } \sum_{k=1}^{\infty} a_0 r^{k-1} = \frac{a_0}{1-r} \tag{4}$$

$$\text{if } r \neq 1, \text{ then } \sum_{k=1}^{K} a_0 r^{k-1} = \frac{a_0(1-r^K)}{1-r}. \tag{5}$$

Question 2: [15%, Work-out question, Outcome 4] $y[n] = \sin\left(\frac{4\pi n}{5}\right) - e^{j\frac{2\pi n}{3}}$. Find the Fourier series of $y[n]$.

Question 3: [15%, Work-out question, Outcome 4]

1. [5%] $x(t)$ is periodic with period $T = 4$ and

$$x(t) = \delta(t-1) - t^2 \cdot \delta(t-2) \quad \text{if } 0 \leq t \leq 4. \tag{6}$$

   Plot $x(t)$ for the range of $t = -6$ to 6.

2. [10%] Compute the Fourier series of $x(t)$.

Question 4: [15%, Work-out question, Outcome 4]

Consider a signal $z(t)$, which is periodic with period $T = 4$ and

$$z(t) = \begin{cases} 1 & \text{if } -1 \leq t \leq 1 \\ 0 & \text{if } 1 \leq t < 3 \end{cases}. \tag{7}$$

We know that the Fourier series coefficients of $z(t)$ are

$$b_k = \begin{cases} \frac{1}{2} & \text{if } k = 0 \\ \frac{\sin\left(\frac{k\pi}{2}\right)}{k\pi} & \text{if } k \neq 0 \end{cases}. \tag{8}$$

Answer the following questions

1. [12%, 2%, 1%] Compute the values of (i) $\sum_{k=-\infty}^{\infty} b_k \cdot e^{-jk\frac{\pi}{3}}$; (ii) $\sum_{k=-\infty}^{\infty} b_k \cdot \cos(k\frac{\pi}{3})$; and (iii) $\sum_{k=-\infty}^{\infty} b_k \cdot \sin(k\frac{\pi}{3})$.

Question 5: [20%, Multiple Choices] The following questions are multiple-choice questions and there is no need to justify your answers. Consider the following two systems:

System 1: When the input is $x_1(t) = 0$, the output is $y_1(t) = 0$. For general input $x_1(t)$, the output is

$$y_1(t) = y_1(t-1) + x_1(2t). \tag{9}$$

System 2: When the input is $x_2[n]$, the output is

$$y_2[n] = x_2[-n^2]. \tag{10}$$

Answer the following questions

1. [4%, Outcome 1] Is System 1 memoryless? Is System 2 memoryless?
2. [4%, Outcome 1] Is System 1 causal? Is System 2 causal?
3. [4%, Outcome 1] Is System 1 stable? Is System 2 stable?
4. [4%, Outcome 1] Is System 1 linear? Is System 2 linear?
5. [4%, Outcome 1] Is System 1 time-invariant? Is System 2 time-invariant?

Discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \tag{1}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n} \tag{2}$$

Continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \tag{3}$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt \tag{4}$$

Continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \tag{5}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{6}$$

Discrete-time Fourier transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(j\omega) e^{j\omega n} d\omega \tag{7}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \tag{8}$$

Laplace transform

$$x(t) = \frac{1}{2\pi} e^{\sigma t} \int_{-\infty}^{\infty} X(\sigma + j\omega) e^{j\omega t} d\omega \tag{9}$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \tag{10}$$

Z transform

$$x[n] = r^n \mathcal{F}^{-1}(X(re^{j\omega})) \tag{11}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{12}$$

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$x(t)$, $y(t)$ — Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k, b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t}x(t) = e^{jM(2\pi/T)t}x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k\rvert = \lvert a_{-k}\rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

three examples, we illustrate this. The last example in this section then demonstrates how properties of a signal can be used to characterize the signal in great detail.

Example 3.6

Consider the signal $g(t)$ with a fundamental period of 4, shown in Figure 3.10. We could determine the Fourier series representation of $g(t)$ directly from the analysis equation (3.39). Instead, we will use the relationship of $g(t)$ to the symmetric periodic square wave $x(t)$ in Example 3.5. Referring to that example, we see that, with $T = 4$ and $T_1 = 1$,

$$g(t) = x(t-1) - 1/2. \quad (3.6\ldots)$$