Midterm #1 of ECE301, Prof. Wang's section
6:30–7:30pm Thursday, February 2, 2012, FRNY G140,

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. There are 10 pages in the exam booklet. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name: Solution

Student ID:

E-mail:

Signature:

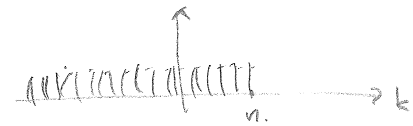
Question 1: [20%, Work-out question, Outcomes 1, and 3] $x[n]$ is $e^{-(1+2j)n}$ if $1 \leq n \leq 20$, and $x[n] = 0$ otherwise. $\mathcal{U}[n]$ is the unit step signal. Compute the expression of

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\mathcal{U}[n-k].$$

You may need the following formula:

$$\text{if } r \neq 1, \text{ then } \sum_{k=1}^{K} ar^{k-1} = \frac{a(1-r^K)}{1-r}. \tag{1}$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,u[n-k] = \sum_{k=1}^{20} e^{-(1+2j)k}\,\underline{u[-(k-n)]}.$$



① if $n > 20$

$$y[n] = \sum_{k=1}^{20}\left(e^{-(1+2j)}\right)^k = e^{-(1+2j)}\sum_{k=1}^{20}\left(e^{-(1+2j)}\right)^{k-1} = \frac{1-e^{-(1+2j)20}}{1-e^{-(1+2j)}}\cdot e^{-(1+2j)}$$

② if $0 < n \leq 20$.

$$y[n] = \sum_{k=1}^{n}\left(e^{-(1+2j)}\right)^k = \frac{1-e^{-(1+2j)n}}{1-e^{-(1+2j)}}\cdot e^{-(1+2j)}$$

③ if $n \leq 0$, $y[n] = 0$.

$$\therefore\quad y[n] = \begin{cases} \dfrac{1-e^{-(1+2j)20}}{1-e^{-(1+2j)}}\cdot e^{-(1+2j)} & n > 20 \\[2ex] \dfrac{1-e^{-(1+2j)n}}{1-e^{-(1+2j)}}\cdot e^{-(1+2j)} & 0 < n \leq 20 \\[2ex] 0 & n \leq 0. \end{cases}$$

Question 2: [15%, Work-out question, Outcome 5] $x(t) = e^{-\sqrt{2}\cdot t}$ if $5 \leq t$, and $x(t) = 0$ otherwise. Compute the expression of

$$g(\omega) = \int_{s=-\infty}^{\infty} x(s)e^{-j\omega s} ds$$

$$g(w) = \int_{s=-\infty}^{\infty} x(s)\, e^{-jws}\, ds$$

$$= \int_{5}^{\infty} e^{-\sqrt{2}s}\, e^{-jws}\, ds = \int_{5}^{\infty} e^{-(\sqrt{2}+jw)s}\, ds$$

$$= \left[\frac{e^{-(\sqrt{2}+jw)s}}{-(\sqrt{2}+jw)} \right]_{5}^{\infty} = 0 - \frac{e^{-(\sqrt{2}+jw)5}}{-(\sqrt{2}+jw)} = \frac{e^{-(\sqrt{2}+jw)5}}{\sqrt{2}+jw}$$

Question 3: [20%, Work-out question, Outcomes 1 and 5] $x(t) = 1$ if $-1 \le t \le 1$ and $x(t) = 0$ otherwise. Let

$$y[n] = \frac{1}{4}\int_{t=-2}^{2} x(t)e^{-j\cdot n\frac{2\pi}{4}t}dt \tag{2}$$

1. [5%] Find the value of $y[0]$.
2. [10%] Find the expression of $y[n]$ in terms of n when $n \neq 0$.
3. [5%] Plot $y[n]$ for the range of $-2 \le n \le 2$.

1. $$y[0] = \frac{1}{4}\int_{t=-2}^{2} x(t)\,dt = \frac{1}{4}\int_{-1}^{1} 1\,dt = \frac{1}{4}\left(1-(-1)\right) = \frac{1}{2}$$

2. $$y[n] = \frac{1}{4}\int_{-2}^{2} x(t)\,e^{-jn\frac{2\pi}{4}t}\,dt$$

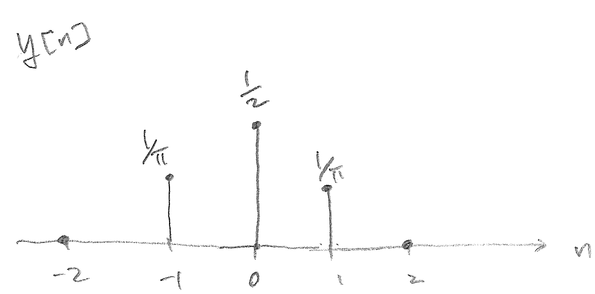
$$= \frac{1}{4}\int_{-1}^{1} e^{-jn\frac{2\pi}{4}t}\,dt = \frac{1}{4}\left[\frac{e^{-jn\frac{2\pi}{4}t}}{-jn\frac{2\pi}{4}}\right]_{-1}^{1}$$

$$= \frac{1}{4}\left(\frac{e^{-jn\frac{\pi}{2}} - e^{+jn\frac{\pi}{2}}}{-jn\frac{\pi}{2}}\right)$$

$$= \frac{1}{-j2n\pi}\left(\cancel{\cos\left(-\frac{n\pi}{2}\right)} + j\sin\left(-\frac{n\pi}{2}\right) - \cancel{\cos\left(\frac{n\pi}{2}\right)} - j\sin\left(\frac{n\pi}{2}\right)\right)$$

$$= \frac{-2j\sin\left(\frac{n\pi}{2}\right)}{-j2n\pi} = \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi}$$

3.

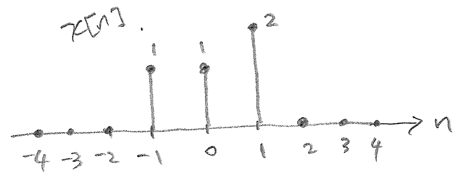


Question 4: [15%, Work-out question, Outcomes 1 and 4] We know that $x[n] = \delta[n-1] + \mathcal{U}[n+1] - \mathcal{U}[n-2]$ and $y[n] = -2x[2-n]$.

1. [5%] Plot $x[n]$ for the range of $-4 \leq n \leq 4$.
2. [5%] Find the total energy of $x[n]$.
3. [5%] Plot $y[n]$ for the range of $-4 \leq n \leq 4$.

1. $x[n] = \delta[n-1] + u[n+1] - u[n-2] = \delta[n-1] + \left(\delta[n+1] + \delta[n] + \delta[n-1]\right)$

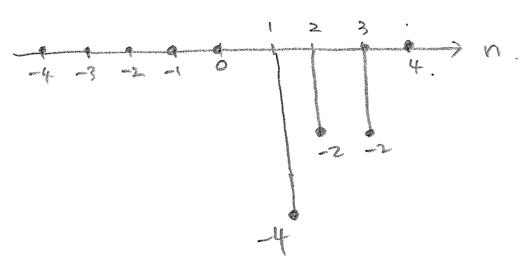
$= \delta[n+1] + \delta[n] + 2\delta[n-1]$.



2. $\sum_{n=-\infty}^{\infty} |x[n]|^2 = |x[-1]|^2 + |x[0]|^2 + |x[1]|^2 = 1 + 1 + 4 = 6$.

3. $y[n] = -2x[2-n] = -2x[-(n-2)]$.

y[n].



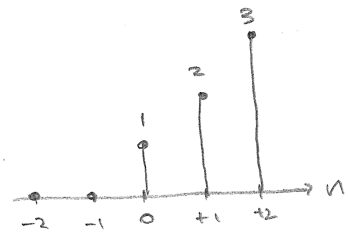
Question 5: [15%, Work-out question, Outcome 1] Let $x[n] = \mathcal{U}[n]$ and $y[n] = \sum_{k=-\infty}^{n} x[k]$.

- [5%] Plot $y[n]$ for the range of $-2 \leq n \leq 2$.

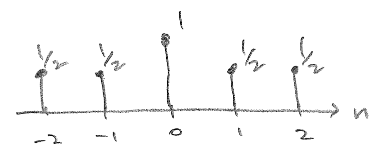
Denote the even and odd parts of $x[n]$ by $x_{\text{even}}[n]$ and $x_{\text{odd}}[n]$, respectively.

- [5%] Plot $x_{\text{even}}[n]$ for the range of $-2 \leq n \leq 2$.
- [5%] Plot $x_{\text{odd}}[n]$ for the range of $-2 \leq n \leq 2$.

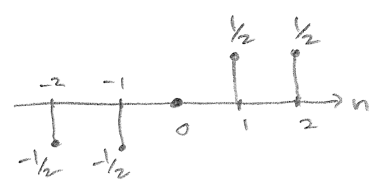
1. $$y[n] = \sum_{k=-\infty}^{n} x[k] = \sum_{k=-\infty}^{n} u[k] = \sum_{k=0}^{n} u[k] = \begin{cases} n+1 & n \geq 0. \\ 0 & n < 0. \end{cases}$$



2. $$x_{even}[n] = \frac{x[n] + x[-n]}{2} = \frac{u[n] + u[-n]}{2}$$



3. $$x_{odd}[n] = \frac{x[n] - x[-n]}{2} = \frac{u[n] - u[-n]}{2}$$



Question 6: [20%, Multiple Choices] The following questions are multiple-choice questions and there is no need to justify your answers. Consider two continuous-time signals:

$$\begin{aligned} x_1(t) &= \sin(e^{|t|}) \\ x_2(t) &= e^{jt}(\cos(2t) + \sin(3t)) \end{aligned}$$

and two discrete-time signals:

$$\begin{aligned} x_3[n] &= \sin(\pi n^3) + \cos(\pi n) \\ x_4[n] &= n^2 e^{2n}. \end{aligned}$$

1. [10%, Outcome 1] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not, *respectively.* If it is periodic, write down the fundamental period. Please state explicitly which signal is periodic and which is not.

2. [10%, Outcome 1] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them, respectively. Please state explicitly which signal is even, which is odd, and which is neither.

1. $x_1(t)$: not periodic.

   $x_2(t)$: periodic with $T = 2\pi$

   $x_3[n]$: periodic with $N = 2$

   $x_4[n]$: not periodic

2. $x_1(t)$: even.

   $x_2(t)$: neither

   $x_3[n]$: even (∵ $\sin(\pi n^3) = 0$ $\forall$ integer n).

   $x_4[n]$: neither