Final Exam of ECE301, Prof. Wang's section

3:20–5:20pm Wednesday, May 03, 2012, CL50 224.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have two hours to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

E-mail:

Signature:

Solution

Question 1: [14%, Work-out question]

1. [1%] What does the acronym "AM-SSB" stands for? Ans: Amplitude Modulation using Single-Side Band.

Prof. Wang wanted to transmit an AM-SSB upper-side-band signal. To that end, he wrote the following MATLAB code.

```
% Initialialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read two different .wav files
[x1, f_sample, N]=wavread('x1');
x1=x1';
[x2, f_sample, N]=wavread('x2');
x2=x2';

% Step 0: Initialize several parameters
W_1=????;
W_2=????;
W_3=????;
W_4=????;
W_5=????;
```

$W1 = 2\pi \times 2K.$

$W2 = 2\pi \times 3K.$

$W3 = 2\pi \times 6.5K$

$W4 = 2\pi \times 3K.$

$W5 = 2\pi \times 5.5K$

```
% Step 1: Make the signals band-limited.
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);

% Step 2: Multiply x_new with a cosine wave.
x1_h=x1_new.*cos(W_2*t);
x2_h=x2_new.*cos(W_3*t);
h1=1/(pi*t).*(sin(W_4*t));
h2=1/(pi*t).*(sin(W_5*t));

% Step 3: Keep one of the side bands
x1_sb=x1_h-ece301conv(x1_h, h1);
x2_sb=x2_h-ece301conv(x2_h, h2);

% Step 4: Create the transmitted signal
y=x1_sb+x2_sb;
wavwrite(y', f_sample, N, 'y.wav');
```

2. [7.5%] Suppose we also know that Prof. Wang intended to use frequency bands 3K–5K Hz and 5.5K–7.5K Hz for transmitting x1 and x2, respectively. What should the values of W_1 to W_5 be in the MATLAB code?

Knowing that Prof. Wang used the above code to generate the "y.wav" file, a student tried to demodulate the output waveform "y.wav" by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[y, f_sample, N]=wavread('y');
y=y';

% Initialize several parameters
W_6=????;
W_7=????;
W_8=????;

% Create the low-pass filter.
h_M=1/(pi*t).*(sin(W_6*t));

% demodulate signal 1
y1=2*y.*cos(W_7*t);
x1_hat=ece301conv(y1,h_M);

sound(x1_hat,f_sample)

% demodulate signal 2
y2=2*y.*cos(W_8*t);
x2_hat=ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```

$W6 = 2\pi \times 2K$

$W7 = 2\pi \times 3K$

$W8 = 2\pi \times 5.5K$

3. [4.5%] Continue from the previous question. What should the values of W_6 to W_8 in the MATLAB code?

   Hint: If you do not know the answers to Q1.2 and Q1.3, please simply draw the AMSSB modulation and demodulation diagrams and mark carefully all the parameter values. You will receive 9 points for Q1.2 and Q1.3.

4. [1%] Does the demodulated signal "x2_hat" sound the same as the original signal "x2_new"?

The same quality but weaker signal.

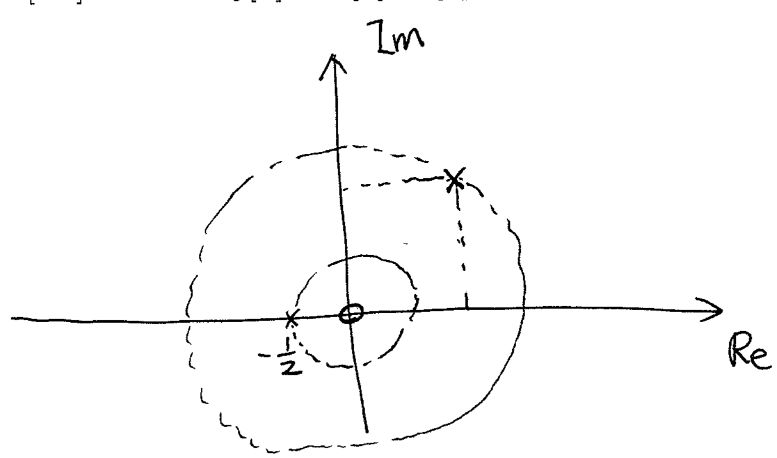
Question 2: [13%, Work-out question] For a given signal $x[n]$, we know that the expression of its Z-transform is $X(z) = \frac{1}{(1-(1+j)z^{-1})(2+z^{-1})}$. Answer the following questions:

1. [2%] Draw the pole-zero plot.
2. [2%] If we also know that the DTFT of $x[n]$ exists, what is the ROC of the Z-transform of $x[n]$.
3. [2%] Find the expression of $x[n]$. Hint: The partial fraction expression of $X(z)$ is $X(z) = \frac{\frac{1+j}{3+2j}}{1-(1+j)z^{-1}} + \frac{\frac{1}{3+2j}}{2+z^{-1}}$.

Consider two signals $w[n] = 3^n\mathcal{U}[-n+1]$ and $h[n] = 0.2^n\mathcal{U}[n]$. Answer the following questions:

4. [3%] Find the Z-transform of $w[n]$.
5. [4%] We know $y[n] = w[n] * h[n]$. Find the Z-transform of $y[n]$.

1.



2.

$\Rightarrow$ ROC: $\frac{1}{2} < |z| < \sqrt{2}$

3. $\frac{1+j}{3+2j} \cdot (-1) \cdot (1+j)^n u[-n-1]$

$+ \frac{1}{3+2j} \cdot \frac{1}{2} \cdot \left(\frac{-1}{2}\right)^n u[n]$

4. $w[n] = 3\delta[n-1] + \delta[n] + 3^n u[-n-1]$

$\Rightarrow W(z) = 3z^{-1} + 1 - \frac{1}{1-3z^{-1}}$ and the ROC is $0 < |z| < 3$

5. $H(z) = \frac{1}{1-0.2z^{-1}}$ & the ROC is

$|z| > 0.2$

$\Rightarrow Y(z) = \left(3z^{-1} + 1 - \frac{1}{1-3z^{-1}}\right)\left(\frac{1}{1-0.2z^{-1}}\right)$

& the corresponding ROC is

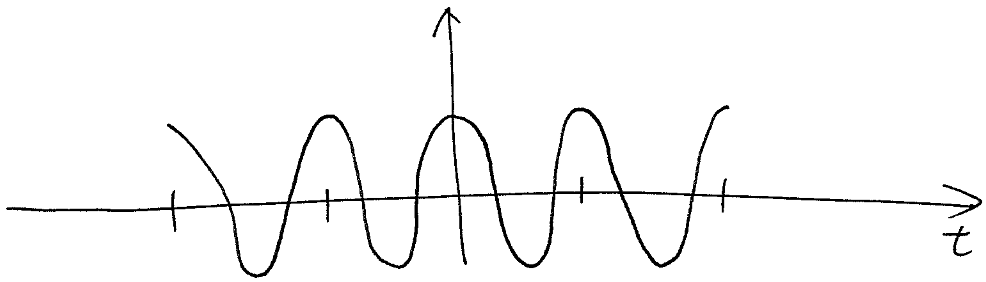
$0.2 < |z| < 3$

Question 3: [13%, Work-out question]

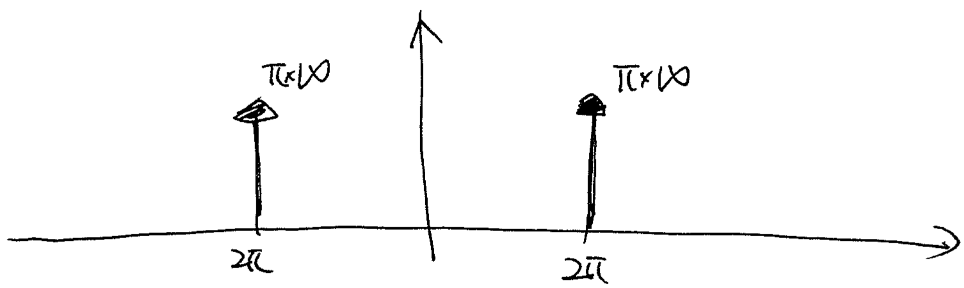
1. [1.5%] Consider a signal $x(t) = \cos(2\pi t)$. Plot $x(t)$ for the range of $-2 \le t \le 2$.
2. [1.5%] Plot $X(j\omega)$ for the range of $-6\pi \le \omega \le 6\pi$.
3. [2.5%] We pass $x(t)$ through an *impulse train sampling* system with sampling period $T = \frac{2}{3}$ sec. That is $x_p(t) = x(t)p(t)$ where $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - \frac{2k}{3})$. Plot $x_p(t)$ for the range of $-2 \le t \le 2$.
4. [4%] Plot $X_p(j\omega)$ for the range of $-6\pi \le \omega \le 6\pi$.
5. [3.5%] For reconstruction, we pass $x_p(t)$ through a low-pass filter with cutoff frequency $W = \frac{\omega_s}{2}$, where ω_s is the sampling frequency. We then multiply it by T. That is, the overall reconstructed signal is $\hat{x}(t) = T \cdot (x_p(t) * h_{\mathrm{LPF},\omega_s/2}(t))$.

   What is the expression of the reconstructed signal $\hat{x}(t)$.

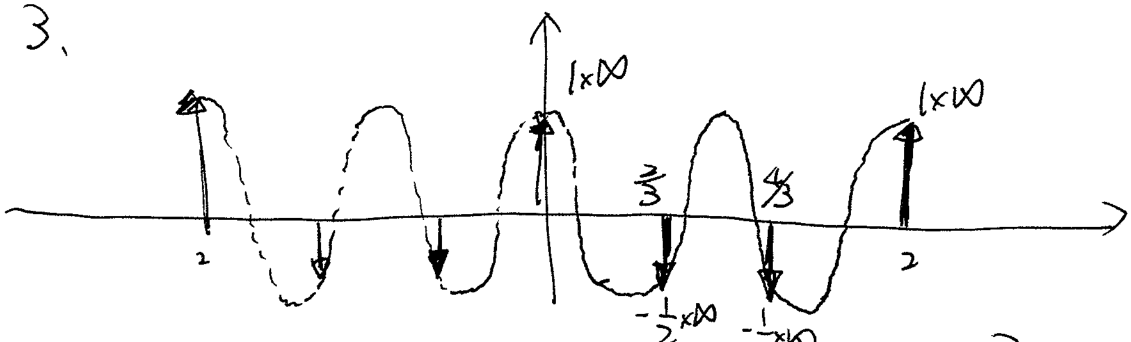
1.



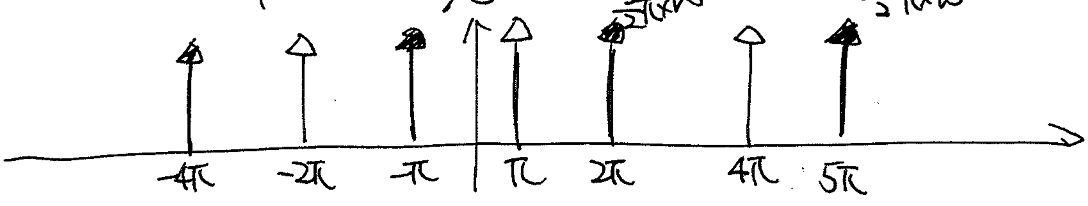
2.



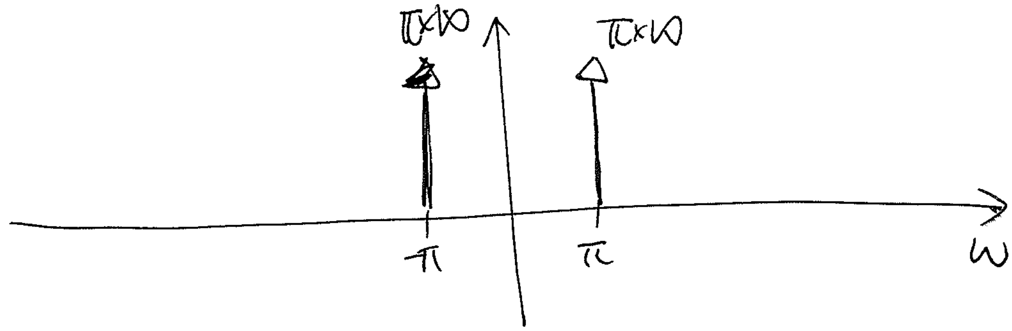
3.



4. $W_s = \frac{2\pi}{T} = \frac{2\pi}{2/3} = 3\pi$ $\quad \frac{1}{T} = \frac{3}{2}$



5.



$\Rightarrow \quad \cos(\pi t)$

Question 4: [11%, Work-out question] Consider a continuous time signal $x(t) = \sin(2\pi \cdot 2000 \cdot t)$ and we use a digital voice recorder to convert the continuous time signal $x(t)$ to its discrete time counter part $x[n]$ with sampling frequency 44.1K Hz. The array $x[n]$ is stored as a .wav file.

We can now do some discrete-time signal processing $y[n] = x[n] * h[n]$ where $h[n]$ is the impulse response of the discrete-time signal processing and then play the "processed signal" by the MATLAB command.

```
sound(y,44100);
```

which converts the discrete-time signal $y[n]$ to its continuous time counterpart $y(t)$.

Answer the following question.

1. [2%] Suppose that we do not do any processing, i.e., $y[n] = x[n]$. Answer the following question: Is the reconstructed output $y(t)$ the same as the original signal $x(t)$? Please use one to two sentences to explain your answer.

2. [3%] Continue from the above question. Suppose when Prof. Wang tried to use the MATLAB command, he made a mistake and entered the following wrong command

   ```
   sound(y,22050);
   ```

   How does $y(t)$ sound when compared to the original signal $x(t)$? Please write down the expression of $y(t)$.

3. [3%] Suppose Prof. Wang decided to do some discrete-time signal processing and chose $h[n] = \delta[n - 22050]$. Furthermore, this time he used the right command

   ```
   sound(y,44100);
   ```

   How does $y(t)$ look like when compared to the original signal $x(t)$? Please write down the expression of $y(t)$.

4. [Advanced 3%] Prof. Wang found an old MP3 player which can only play files of the sampling frequency 22.05KHz but he knew that the $y[n]$ was sampled at the frequency 44.1K Hz. How could he generate another file $y'[n]$ from his original file $y[n]$ such that the new file $y'[n]$ can be properly played in the old MP3 player. (That is, when playing the new file $y'[n]$ in the old MP3 player, which can only support 22.05K Hz, it sounds exactly the same as when played by the MATLAB command.)

   ```
   sound(y,44100);
   ```

   Please use 1 to 3 sentences to describe how you would generate the new file $y'[n]$ from the old file $y[n]$.

1. Yes : $\because$ $W_s > 2W_M$

   $44.1K > 2 \cdot 2K$.

2. It sounds "slower" slow-motion. $y(t) = \sin(2\pi \cdot 1000t)$

3. It is as if delayed by $\frac{22050}{44100} = \frac{1}{2}$ sec.

$\Rightarrow y(t) = x(t - 0{,}5)$

4. $y'[n]$ is obtained from $y[n]$ by deleting the ~~odd~~ values of odd indices.

That is

$$y'[n] = y[2n]$$

Question 5: [12%, Work-out question] Consider a continuous-time linear time invariant system satisfying $y(t) = 0.5\frac{d}{dt}y(t) + x(t-0.5) + x(t+0.5)$. Find the output $y(t)$ when the input is $x(t) = e^{j\pi t} + \cos(2t) + \sin(3t)$.

$$Y(j\omega) = 0{,}5\, j\omega\, Y(j\omega) + \left(e^{+j\omega\times 0{,}5} + e^{-j\omega\times 0{,}5}\right) X(j\omega)$$

$$\Rightarrow H(j\omega) = \frac{2\cos(0{,}5\,\omega)}{1 - 0{,}5\, j\omega}$$

$$x(t) = e^{j\pi t} + \frac{1}{2}e^{j2t} + \frac{1}{2}e^{-j2t} + \frac{1}{2j}e^{+j3t} - \frac{1}{2j}e^{-j3t}$$

$$\Rightarrow y(t) = \frac{2\cos(0{,}5\pi)}{1 - 0{,}5\, j\cdot\pi}\, e^{j\pi t}$$

$$+ \frac{1}{2}\cdot\frac{2\cos(1)}{1 - 0{,}5\, j\times 2}\cdot e^{j2t}$$

$$+ \frac{1}{2}\,\frac{2\cos(-1)}{1 - 0{,}5\, j\times(-2)}\cdot e^{-j2t}$$

$$+ \frac{1}{2j}\,\frac{2\cos(1{,}5)}{1 - 0{,}5\, j\times 3}\, e^{+j3t}$$

$$- \frac{1}{2j}\,\frac{2\cos(-1{,}5)}{1 - 0{,}5\, j\times 3\times(-1)}\, e^{-j3t}.$$

Question 6: [12%, Work-out question] Consider two discrete-time signals

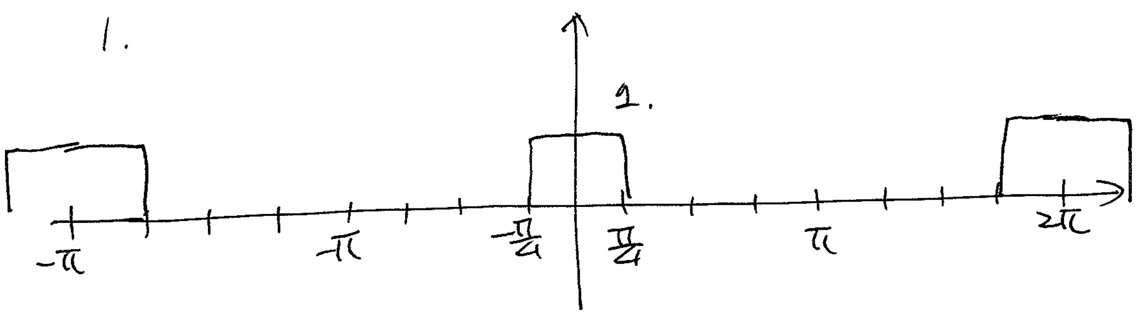
$$w[n] = \frac{\sin(\frac{\pi \cdot n}{4})}{\pi \cdot n} \tag{1}$$

and

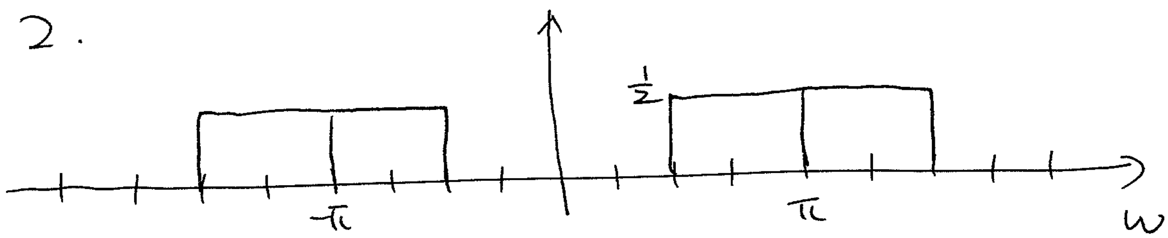
$$h[n] = \frac{\sin(\frac{\pi \cdot n}{4})\cos(\frac{3\pi \cdot n}{4})}{\pi \cdot n}. \tag{2}$$

1. [3%] Plot the DTFT $W(e^{j\omega})$ for the range of $-4\pi \le \omega \le 4\pi$.
2. [7%] Plot the DTFT $H(e^{j\omega})$ for the range of $-4\pi \le \omega \le 4\pi$.
3. [2%] If $h[n]$ is the impulse response of a discrete-time LTI system. Is such a system a low-pass filter, a band-pass filter, or a high-pass filter?

1.



2.



3. High-pass filter.

Question 7: [10%, Work-out question] Consider a continuous time LTI system. We know that for this particular system, when the input is

$$x(t) = \begin{cases} 1+t & \text{if } -1 \le t < 0 \\ 1-t & \text{if } 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

the output is

$$y(t) = \begin{cases} 1+t & \text{if } -1 \le t < 0 \\ 1 & \text{if } 0 \le t < 2 \\ 3-t & \text{if } 2 \le t < 3 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

1. [4%] Plot the output $y(t)$ for the range of $-4 \le t \le 4$ when the input is

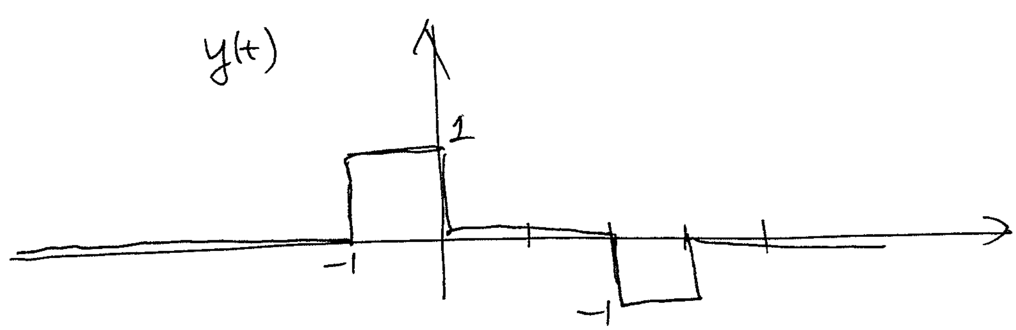
$$x(t) = \begin{cases} 1 & \text{if } -1 \le t < 0 \\ -1 & \text{if } 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases} . \tag{5}$$

2. [6%] Plot the output $y(t)$ for the range of $-4 \le t \le 4$ when the input is

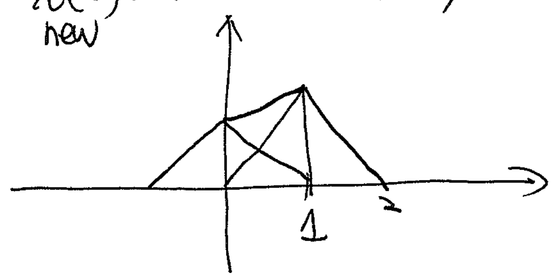
$$x(t) = \begin{cases} 1+t & \text{if } -1 \le t < 0 \\ 1+0.5t & \text{if } 0 \le t < 1 \\ 3-1.5t & \text{if } 1 \le t < 2 \\ 0 & \text{otherwise} \end{cases} . \tag{6}$$

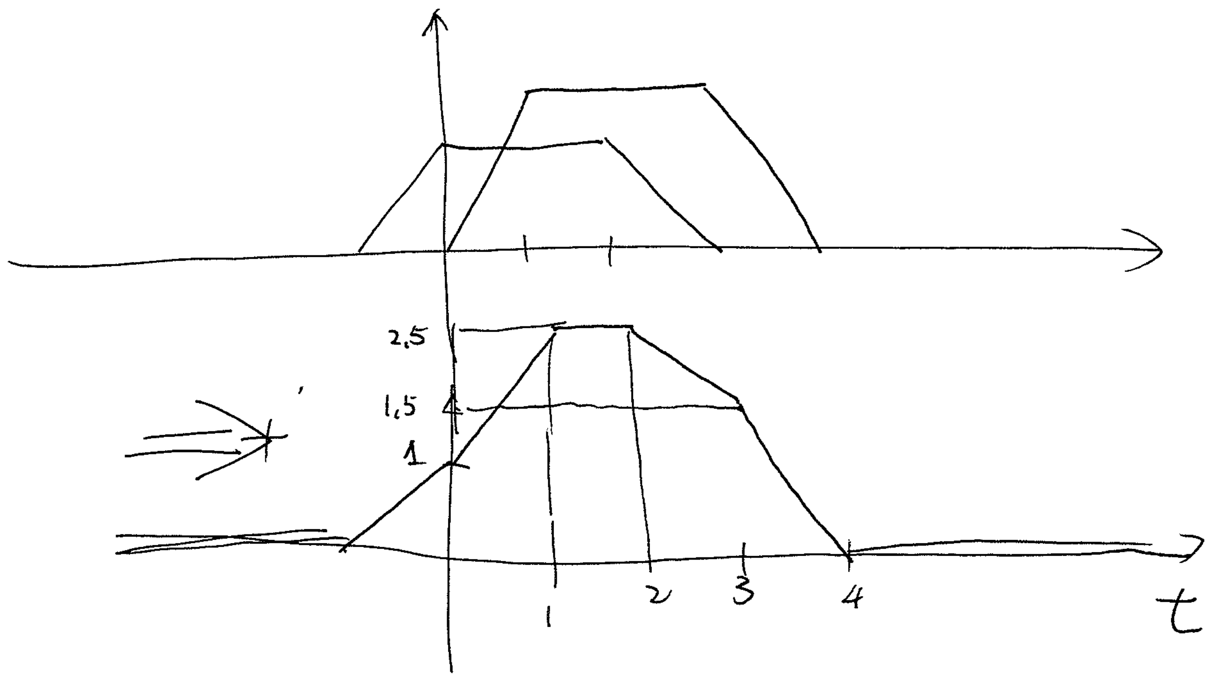
1. Differentiation

$\Rightarrow$



2. $x_{new}(t) = x(t) + 1.5\,x(t-1)$ $\Rightarrow$ $y_{new}(t) = y(t) + 1.5\,y(t-1)$




2,5
1,5
1
1
2
3
4
t

Question 8: [15%, Multiple-choice question] Consider two signals $h_1(t) = e^{-\int_{s=t-1}^{t+1} s^2 ds}$ and $h_2[n] = \min(\cos((\pi n)^2), 0)$

1. [1.25%] Is $h_1(t)$ periodic? No
2. [1.25%] Is $h_2[n]$ periodic? No
3. [1.25%] Is $h_1(t)$ even or odd or neither? Even
4. [1.25%] Is $h_2[n]$ even or odd or neither? Even
5. [1.25%] Is $h_1(t)$ of finite energy? Yes
6. [1.25%] Is $h_2[n]$ of finite energy? No

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless? No
2. [1.25%] Is System 2 memoryless? No
3. [1.25%] Is System 1 causal? No
4. [1.25%] Is System 2 causal? No
5. [1.25%] Is System 1 stable? Yes
6. [1.25%] Is System 2 stable? No.