Midterm #1 of ECE301, Prof. Wang's section
6:30–7:30pm Wednesday, October 10, 2012, ME 1061,

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, e-mail address, and signature in the space provided on this page, **NOW!**

2. This is a closed book exam.

3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.

4. Use the back of each page for rough work.

5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

E-mail:

Signature:

Question 1: [20%, Work-out question, Learning Objectives 2 and 3] Consider the following two signals $x[n]$ and $y[n]$:

$$x[n] = \begin{cases} e^{(1+j)n} & \text{if } n \leq 1 \\ 0 & \text{if } n > 1 \end{cases}$$

$$y[n] = \begin{cases} 0 & \text{if } 3 \leq n \\ -2 & \text{if } n < 3 \end{cases}$$

Compute the convolution sum $z[n] = x[n] * y[n]$.

Hint: You may need the following formula: When $|r| < 1$, we have $\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$.

When $r \neq 1$, we have $\sum_{k=1}^{K} ar^{k-1} = \frac{a(1-r^K)}{1-r}$.

Question 2: [20%, Work-out question, Learning Objectives 2, 3, 5, and 6] Consider an LTI system. Suppose we now that when the input $x[n] = \mathcal{U}[n]$, the output is

$$y[n] = \begin{cases} 1+n & \text{if } 0 \le n \le 1 \\ 3-n & \text{if } 2 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$$

1. [3%] What is the definition of "impulse response"?

2. [7%] Plot the impulse response $h[n]$ of the given LTI system for the range of $n = -2$ to 5.

3. [10%] What is the output $y[n]$ when the input is $x[n] = \cos(\frac{3\pi}{2}n + 1)$? If you do not know the answer to the previous question, you can assume $h[n]$ being

$$h[n] = \begin{cases} 1 & \text{if } -1 \le n \le 0 \\ -1 & \text{if } 1 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$$

Question 3: [25%, Work-out question, Learning Objective 4] Consider the following two signals

$$x(t) = \sin(3t) + e^{j\frac{2}{7}t}$$

$$y(t) = \begin{cases} \cos\left(\frac{\pi}{2}t\right) & \text{if } -1 \leq t \leq 1 \\ y(t) \text{ is periodic with period } 2 \end{cases}$$

1. [9%] Find out the Fourier Series representation of $x(t)$.

2. [6%] Plot $y(t)$ for the range of $-3 \leq t \leq 3$.

3. [10%] Find out the Fourier Series coefficients b_0 and b_1 of $y(t)$.

Question 4: [15%, Work-out question, Learning Objective 4] We know that

$$x(t) = \begin{cases} \delta(t) & \text{if } -\pi \leq t \leq \pi \\ x(t) \text{ is periodic with period } 2\pi \end{cases} \tag{1}$$

1. [5%] Plot $x(t)$ for the range of $-3\pi \leq t \leq 3\pi$.

2. [5%] Find out the Fourier series coefficients a_k of the given signal $x(t)$.

3. [5%] Consider another signal

$$y(t) = \begin{cases} \delta(t) - \delta(t - \pi/2) & \text{if } -\pi \leq t \leq \pi \\ y(t) \text{ is periodic with period } 2\pi \end{cases}$$

Express the Fourier series coefficients b_k of the given signal $y(t)$ in terms of a_k.

Question 5: [20%, Multiple Choices] The following questions are multiple-choice questions and there is no need to justify your answers. Consider the following two systems:

System 1: When the input is $x_1(t)$, the output is

$$y_1(t) = \int_{-\infty}^{t} x_1(2s)ds. \tag{2}$$

System 2: When the input is $x_2[n]$, the output is

$$y_2[n] = \begin{cases} y_2[n-1] \cdot x_2[n] & \text{if } n \geq 1 \\ x_2[n+1] & \text{if } n \leq 0 \end{cases} \tag{3}$$

Answer the following questions

1. [4%, Learning Objective 1] Is System 1 memoryless? Is System 2 memoryless?

2. [4%, Learning Objective 1] Is System 1 causal? Is System 2 causal?

3. [4%, Learning Objective 1] Is System 1 stable? Is System 2 stable?

4. [4%, Learning Objective 1] Is System 1 linear? Is System 2 linear?

5. [4%, Learning Objective 1] Is System 1 time-invariant? Is System 2 time-invariant?

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$x(t)$ $\Big\}$ Periodic with period T and	a_k
		$y(t)$ $\Big\}$ fundamental frequency $\omega_0 = 2\pi/T$	b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} x(t) = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a_{-k}^*
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\displaystyle\int_T x(\tau)y(t - \tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\displaystyle\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\dfrac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\dfrac{2\pi}{T}a_k$
Integration		$\displaystyle\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\dfrac{1}{jk\omega_0}\right)a_k = \left(\dfrac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a_{-k}^* \\ \mathfrak{Re}\{a_k\} = \mathfrak{Re}\{a_{-k}\} \\ \mathfrak{Im}\{a_k\} = -\mathfrak{Im}\{a_{-k}\} \\ |a_k| = |a_{-k}| \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t)\text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t)\text{ real}] \end{cases}$	$\mathfrak{Re}\{a_k\}$ $j\mathfrak{Im}\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

three examples, we illustrate this. The last example in this section then demonstrates how properties of a signal can be used to characterize the signal in great detail.

Example 3.6

Consider the signal $g(t)$ with a fundamental period of 4, shown in Figure 3.10. We could determine the Fourier series representation of $g(t)$ directly from the analysis equation (3.39). Instead, we will use the relationship of $g(t)$ to the symmetric periodic square wave $x(t)$ in Example 3.5. Referring to that example, we see that, with $T = 4$ and $T_1 = 1$,

$$g(t) = x(t - 1) - 1/2.$$

Thus, in general, *none* of the finite partial sums in eq. (3.52) yield the exact values of $x(t)$, and convergence issues, such as those considered in Section 3.4, arise as we consider the problem of evaluating the limit as the number of terms approaches infinity.

3.7 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

There are strong similarities between the properties of discrete-time and continuous-time Fourier series. This can be readily seen by comparing the discrete-time Fourier series properties summarized in Table 3.2 with their continuous-time counterparts in Table 3.1.

TABLE 3.2   PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$x[n]$ Periodic with period N and $y[n]$ fundamental frequency $\omega_0 = 2\pi/N$	a_k Periodic with b_k period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n}x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a_{-k}^*
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\dfrac{1}{m}a_k \begin{pmatrix} \text{viewed as periodic} \\ \text{with period } mN \end{pmatrix}$
Periodic Convolution	$\displaystyle\sum_{r=\langle N\rangle} x[r]y[n-r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\displaystyle\sum_{l=\langle N\rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\displaystyle\sum_{k=-\infty}^{n} x[k] \begin{pmatrix} \text{finite valued and periodic only} \\ \text{if } a_0 = 0 \end{pmatrix}$	$\left(\dfrac{1}{(1 - e^{-jk(2\pi/N)})}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a_{-k}^* \\ \mathcal{R}e\{a_k\} = \mathcal{R}e\{a_{-k}\} \\ \mathcal{I}m\{a_k\} = -\mathcal{I}m\{a_{-k}\} \\ |a_k| = |a_{-k}| \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}v\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{O}d\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\mathcal{R}e\{a_k\}$ $j\mathcal{I}m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$$