Midterm #1 of ECE301, Prof. Wang's section

6:30–7:30pm Wednesday, September 12, 2012, ME 1061,

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Solutions

Name:

Student ID:

E-mail:

Signature:

Question 1: [20%, Work-out question, Learning Objective 3] Consider two signals $x(t)$ and $y(t)$.

$$x(t) = \begin{cases} e^{-(1+j)t} & \text{if } 1 \le t \\ 0 & \text{if } t < 1 \end{cases}$$

$$y(t) = \begin{cases} 0 & \text{if } 3 \le t \\ \cos(t - \frac{\pi}{2}) - j\sin(t - \frac{\pi}{2}) & \text{if } t < 3 \end{cases}$$

Compute the expression of

$$z(t) = \int_{s=-\infty}^{\infty} x(t-s)y(s)ds.$$

$$y(s) = \begin{cases} 0 & \text{if } s \geq 3 \\ e^{-j(s-\frac{\pi}{2})} & \text{if } s < 3 \end{cases}$$

$$x(t-s) = \begin{cases} e^{-(1+j)(t-s)} & \text{if } t-s \geq 1 \\ 0 & \text{if } t-s < 1 \end{cases}$$

$$= \begin{cases} e^{-(1+j)t} e^{(1+j)s} & \text{if } s \leq t-1 \\ 0 & \text{if } s > t-1 \end{cases}$$

$$z(t) = \int_{-\infty}^{3} e^{-j(s-\frac{\pi}{2})} x(t-s)\, ds$$

<u>Case 1:</u> $t-1 \leq 3 \iff t \leq 4$

$$z(t) = e^{-(1+j)t} e^{j\frac{\pi}{2}} \int_{-\infty}^{t-1} e^{-js} e^{(1+j)s}\, ds$$

$$= j\, e^{-(1+j)t}\, e^{t-1} = j e^{-(1+jt)}$$

<u>Case 2:</u> $t-1 > 3 \iff t > 4$

$$z(t) = j e^{-(1+j)t} \int_{-\infty}^{3} e^{s} ds = j\, e^{-(1+j)t} e^{3} = j\, e^{[3-(1+j)t]}$$

$$\Rightarrow z(t) = \begin{cases} j e^{-(1+jt)} & t \leq 4 \\ j e^{[3-(1+j)t]} & t > 4 \end{cases}$$

Question 2: [15%, Work-out question, Learning Objectives 1 and 4] $x[n] = e^{-\sqrt{2}n}(\mathcal{U}[n-50] - \mathcal{U}[n-200]) + 100\delta[n+50]$ where $\mathcal{U}[n]$ and $\delta[n]$ are the unit step and the unit impulse signals, respectively. Compute the expression of

$$g(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}.$$

Hint: You may need to use the formula

$$\sum_{k=1}^{K} ar^{k-1} = \frac{a(1-r^K)}{1-r} \text{ when } r \neq 1.$$

$$g(\omega) = \sum_{n=-\infty}^{\infty} e^{-\sqrt{2}n}\left(u[n-50] - u[n-200]\right)e^{-j\omega n} + 100\sum_{n=-\infty}^{\infty}\delta[n+50]\,e^{-j\omega n}$$

$$= \sum_{n=50}^{199} e^{-(\sqrt{2}+j\omega)n} + 100\,e^{j\omega 50}$$

$$= \frac{e^{-50(\sqrt{2}+j\omega)} - e^{-200(\sqrt{2}+j\omega)}}{1 - e^{-(\sqrt{2}+j\omega)}} + 100\,e^{j\omega 50}$$

Question 3: [10%, Work-out question, Learning Objective 1] Consider the following signal

$$x(t) = \cos(t) + 2jt.$$

Compute the value of

$$\int_{t=-5}^{5} |x(t)|^2 dt.$$

Hint: You may need the trigonometric formula

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}.$$

$$|x(t)|^2 = |\cos(t) + 2jt|^2$$

$$= \cos^2(t) + 4t^2$$

$$\int_{-5}^{5} |x(t)|^2 dt = \int_{-5}^{5} \cos^2(t) dt + 4 \int_{-5}^{5} t^2 dt$$

$$= \frac{1}{2} \int_{-5}^{5} dt + \frac{1}{2} \int_{-5}^{5} \cos(2t) dt + \frac{8}{3} \times 5^3$$

$$= 5 + \frac{1}{2} \sin(10) + \frac{1000}{3}$$

Question 4: [20%, Work-out question, Learning Objective 1] The input-output relationship of a system is described by the following.

$$y[n] = \begin{cases} n^2\cos(n)x[n] & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases} \tag{1}$$

1. Is the above system linear? Please write down your detailed reasoning. This is NOT a yes/no question.

Consider any DT signals $x_1[n]$ and $x_2[n]$.
Denote their corresponding outputs by $y_1[n]$ and $y_2[n]$.

Consider a third input signal

$$x_3[n] = a_1x_1[n] + a_2x_2[n]$$

where a_1 and a_2 are any reals.

Then,

$$y_3[n] = \begin{cases} n^2\cos(n)\left(a_1x_1[n] + a_2x_2[n]\right) & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$

$$= \begin{cases} a_1 n^2\cos(n)x_1[n] & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases} + \begin{cases} a_2 n^2\cos(n)x_2[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$= a_1y_1[n] + a_2y_2[n]$$

Hence, by definition the system is linear.

Question 5: [15%, Work-out question, Learning Objectives 1 and 6] We know that $x[n] = n\mathcal{U}[n] - (n-1)\mathcal{U}[n-1]$ and

$$y[n] = \begin{cases} 2x[\frac{2-n}{2}] & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}. \tag{2}$$

1. [5%] Plot $x[n]$ for the range of $-3 \le n \le 3$.
2. [5%] Plot $y[n]$ for the range of $-3 \le n \le 3$.
3. [5%] Plot the odd part of $x[n]$ for the range of $-3 \le n \le 3$.

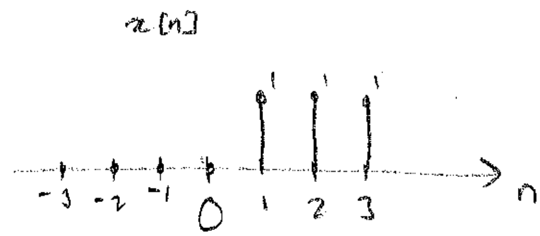
1. $x[n] = n u[n] - (n-1) u[n-1]$

$= n u[n] - (n-1)(u[n] - \delta[n])$

$= u[n] + (n-1)\delta[n]$

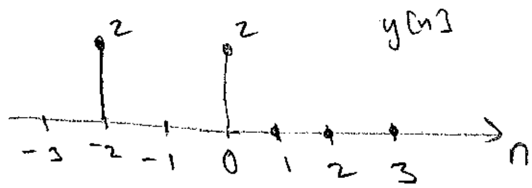
$= u[n] - \delta[n]$

$= u[n-1]$



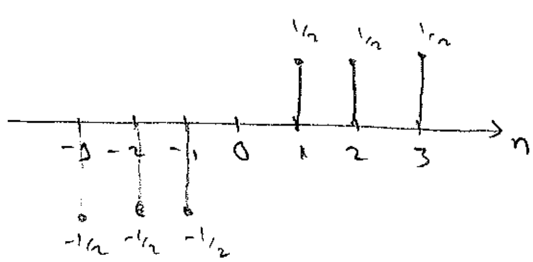
2. $x[1-n] = u[-n]$

$\Rightarrow y[n] = \begin{cases} 2\, u[-\frac{n}{2}] & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases} = \begin{cases} 2\, u[-n] & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$



3. $\text{Odd}\{x[n]\} = \dfrac{x[n] - x[-n]}{2} = \dfrac{u[n-1] - u[-n-1]}{2}$

$= \begin{cases} 1/2 & n \ge 1 \\ 0 & n = 0 \\ -1/2 & n \le -1 \end{cases}$



Question 6: [20%, Multiple Choices] The following questions are multiple-choice questions and there is no need to justify your answers. Consider two continuous-time signals:

$$\begin{aligned} x_1(t) &= \cos(\sin(t)) \\ x_2(t) &= (e^{-|t|})^3 + t^2 + |e^{jt}|^5 \end{aligned}$$

and two discrete-time signals:

$$\begin{aligned} x_3[n] &= \frac{\mathcal{U}[n+3] - \mathcal{U}[n-3]}{2} \\ x_4[n] &= e^{j\frac{5\pi n}{2}} + \sin(3\pi n^2). \end{aligned}$$

1. [10%, Learning Objective 1] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not, *respectively.* If it is periodic, write down the fundamental period. Please state explicitly which signal is periodic and which is not.
2. [10%, Learning Objective 1] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them, respectively. Please state explicitly which signal is even, which is odd, and which is neither.

1. $x_1(t)$ is periodic with fundamental perod π

   $x_2(t)$ is not periodic

   $x_3[n]$ is not periodic

   $x_4[n]$ is periodic with fundamental period 4

2. $x_1(t)$ is even

   $x_2(t)$ is even

   $x_3[n]$ is neither

   $x_4[n]$ is neither