Final Exam of ECE301, Prof. Wang's section

1–3pm Tuesday, December 11, 2012, Lily 1105.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have ~~one~~ 2 hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.

Question 1: [15%, Work-out question]

1. [1%] What does the acronym "AM-SSB" stands for? Amplitude Modulation Single-Side Band.

Prof. Wang wanted to transmit an AM-SSB lower-side-band signal. To that end, he wrote the following MATLAB code.

```
% Initialialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read two different .wav files
[x1, f_sample, N]=wavread('x1');
x1=x1';
[x2, f_sample, N]=wavread('x2');
x2=x2';

% Step 0: Initialize several parameters
W_1=????;
W_2=????;
W_3=????;
W_4=????;
W_5=????;
```

$W_1 = 2\pi \times 1000$ $W_4 = 2\pi \times 3000$

$W_2 = 2\pi \times 3000$ $W_5 = 2\pi \times 7000$

$W_3 = 2\pi \cdot 7000$

```
% Step 1: Make the signals band-limited.
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);

% Step 2: Multiply x1_new and x2_new with a co-sine wave.
x1_h=x1_new.*cos(W_2*t);
x2_h=x2_new.*cos(W_3*t);

% Step 3: Keep the lower side bands
h1=1/(pi*t).*(sin(W_4*t));
h2=1/(pi*t).*(sin(W_5*t));
x1_sb=ece301conv(x1_h, h1);
x2_sb=ece301conv(x2_h, h2);

% Step 4: Create the transmitted signal
y=x1_sb+x2_sb;
wavwrite(y', f_sample, N, 'y.wav');
```

2. [7.5%] Suppose we also know that Prof. Wang intended to use frequency bands 2K–3K Hz and 6K–7K Hz for transmitting x1 and x2, respectively. What should the values of W_1 to W_5 be in the MATLAB code?

Knowing that Prof. Wang used the above code to generate the "y.wav" file, a student tried to demodulate the output waveform "y.wav" by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[y, f_sample, N]=wavread('y');
y=y';

% Initialize several parameters
W_6=????;
W_7=????;
W_8=????;
```

$W_6 = 2\pi \times 1000$

$W_7 = 2\pi \times 3000$

$W_8 = 2\pi \times 7000$

```
% Create the low-pass filter.
h_M=1/(pi*t).*(sin(W_6*t));

% Create two band-pass filters.
hBPF_1=1/(pi*t).*(sin(2*\pi*3000*t))-1/(pi*t).*(sin(2*\pi*2000*t));
hBPF_2=1/(pi*t).*(sin(2*\pi*7000*t))-1/(pi*t).*(sin(2*\pi*6000*t));


% demodulate signal 1
y1BPF=ece301conv(y,hBPF_1);
y1=4*y1BPF.*sin(W_7*t);
x1_hat=ece301conv(y1,h_M);

sound(x1_hat,f_sample)

% demodulate signal 2
y2BPF=ece301conv(y,hBPF_2);
y2=4*y2BPF.*sin(W_8*t);
x2_hat=ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```

3. [4.5%] Continue from the previous question. What should the values of W_6 to W_8 in the MATLAB code?

Hint: If you do not know the answers to Q1.2 and Q1.3, please simply draw the AMSSB modulation and demodulation diagrams and mark carefully all the parameter values. You will receive 9 points for Q1.2 and Q1.3.

4. [2%] However, even with the correct values of W_6 to W_8, there is still some problem with the above MATLAB code. Please answer (1) What would the student hear using this MATLAB code? (2) How to change the code so that the student can demodulate the signals correct?

(1) Silence

(2) chang "sin" to "cos".

Question 2: [12.5%, Work-out question] Consider a continuous-time signal $x(t) = \cos(6\pi t)$, and we sample it with the sampling frequency 2 Hz.

1. [1%] What is the sampling period? (Make sure you write down the correct unit.)

2. [2.5%] Sampling converts the continuous time signal $x(t)$ to a discrete-time array $x[n]$. Plot $x[n]$ for the range of $-2 \le n \le 2$.

3. [3%] We use $x_{\text{lin}}(t)$ to denote the reconstructed signal based on linear interpolation. Plot $x_{\text{lin}}(t)$ for the range of $-2 \le t \le 2$.
   We use $x_{\text{ZOH}}(t)$ to denote the reconstructed signal based on Zero-Order Hold. Plot $x_{\text{ZOH}}(t)$ for the range of $-2 \le t \le 2$.

   Hint: If you do not know the answer to Q2.2, you can assume that

$$x[n] = \begin{cases} 1 & \text{if } n = 4k \text{ for some integer } k \\ -1 & \text{if } n = 4k+2 \text{ for some integer } k \\ 0 & \text{if } n \text{ is odd} \end{cases} \tag{1}$$

4. [3%] Let $x_p(t)$ denote the impulse-train-sampled signal with the sampling ~~period~~ freq 2 Hz. Plot $X_p(j\omega)$ for the range of $-8\pi < \omega < 8\pi$.

5. [3%, advanced] We use $x_{\text{sync}}(t)$ to denote the reconstructed signal based on the optimal reconstruction. Write down the expression of $x_{\text{sync}}(t)$. Plot $x_{\text{sync}}(t)$ for the range of $-2 \le t \le 2$.

   Hint 1: If you do not know the answer to Q2.2, you can use the same assumption as specified in Q2.3.

1. 0.5 second

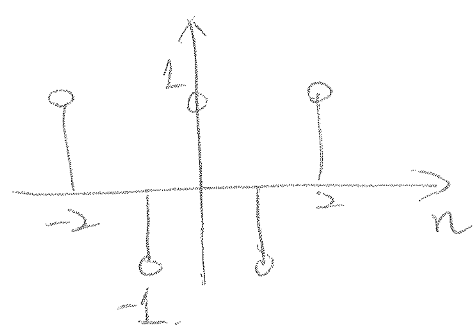
2. $X[-2] = \cos(-6\pi) = 1.$

   $X[-1] = \cos(-3\pi) = -1$

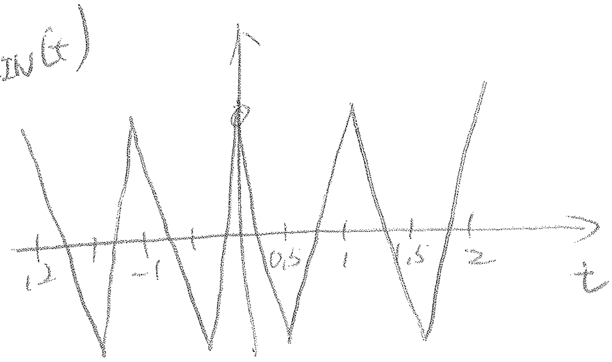
   $X[0] = 1$

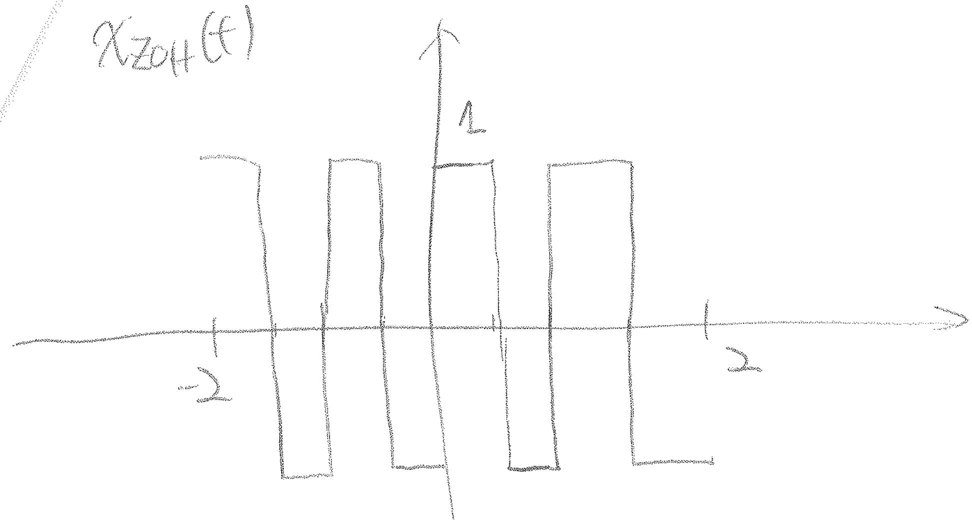
   $X[+1] = -1$

   $X[2] = 1$



3. $X_{LIN}(t)$





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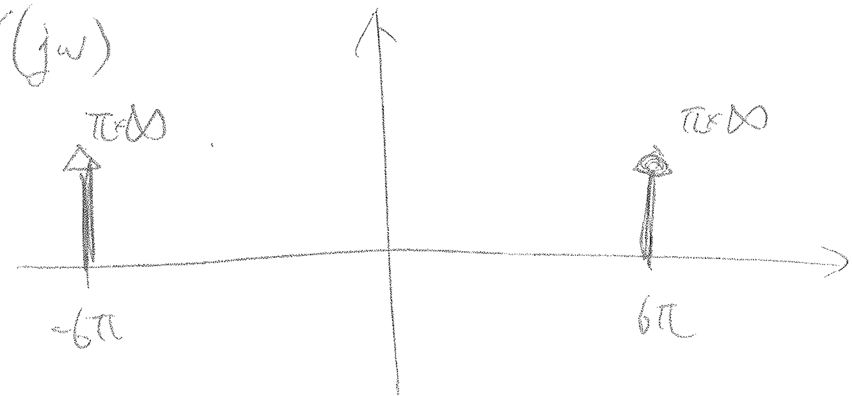
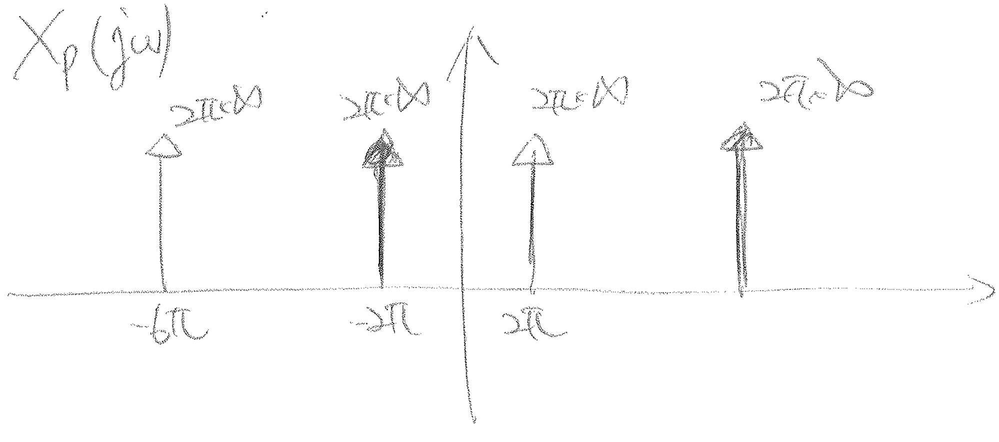
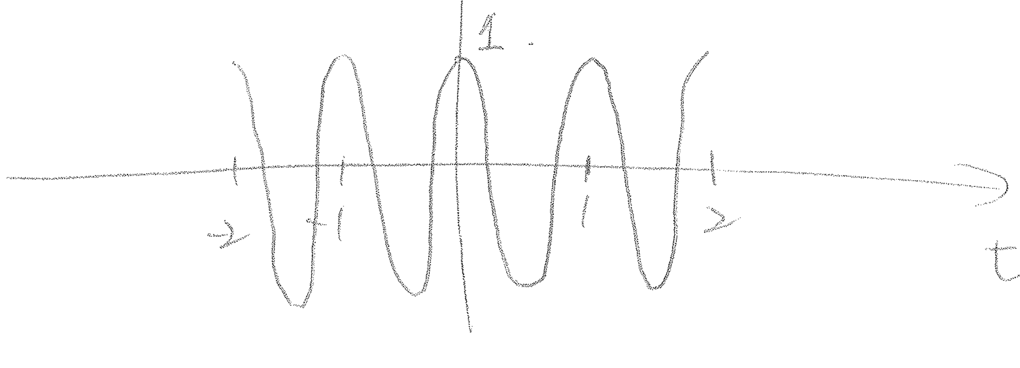
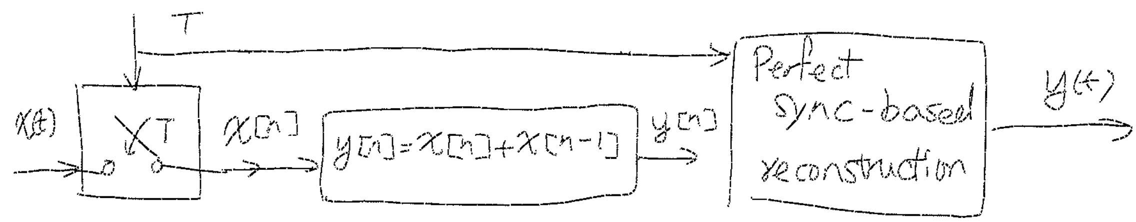




5. $x_{sync}(t) = \cos(2\pi t)$



Question 3: [8%, Work-out question] Consider the following discrete-time signal processing system. Namely, the continuous time input $x(t)$ is sampled first and then processed in discrete-time. In the end, we reconstruct the continuous time output $y(t)$ from the processed array $y[n]$.



Suppose we know that the sampling period is 0.1 second. Determine the end-to-end frequency response $H(j\omega)$ of the above system.

Hint: If you do not know the answer to the above question, you can answer the following two sub-questions instead: Q1: when $x(t) = 1$, what is the output $y(t)$? Q2: when $x(t) = \cos(5\pi t)$, what is the output $y(t)$? You will get 3.5 points and 3 points, respectively, if your answers are correct.

$$H(e^{jw}) = 1 + e^{-jw}$$

$$\frac{\pi}{T} = \frac{w_s}{2} = 10\pi$$

$$H(jw) = \begin{cases} 1 + e^{-jw \cdot 0.1} & \text{if } -10\pi < w < 10\pi \\ 0 & \text{otherwise.} \end{cases}$$

Q1: $y(t) = 2.$

Q2: $y(t) = \cos(5\pi t) + \cos(5\pi(t - T))$

Question 4: [10%, Work-out question]

1. [1%] What is the acronym "ROC" stands for (when considering the Z-transform)?

We know that

$$x[n] = \begin{cases} 2^n & \text{if } n \geq 0 \text{ and } n \text{ is even} \\ 0.2^n & \text{if } n \geq 0 \text{ and } n \text{ is odd} \\ 0 & \text{if } n < 0 \end{cases} \tag{2}$$

2. [9%] Find the Z-transform $X(z)$, write down the expression of the corresponding ROC, and plot the ROC.

   Hint 1: You may need to use the formula: $\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$ if $|r| < 1$.

   Hint 2: If you do not know how to solve this question, you can assume

$$x[n] = \begin{cases} 2^n & \text{if } n \geq 0 \\ 3^n & \text{if } n < 0 \end{cases}. \tag{3}$$

   You will get 9 points if your answer is correct.

1. Region of Convergence

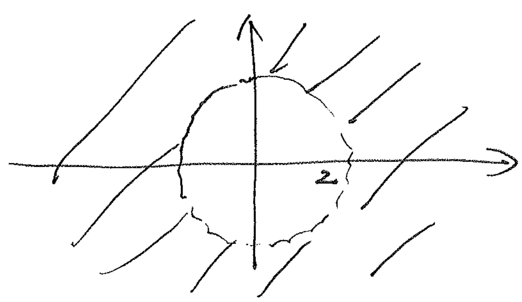
2. $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$$= \sum_{k=0}^{\infty} 2^{2k} z^{-2k} + \sum_{k=1}^{\infty} 0.2^{(2k-1)} z^{-(2k-1)}$$

(if $|2^2 z^{-2}| < 1$ and $|0.2^2 z^{-2}| < 1$

$$= \frac{1}{1-2^2 z^{-2}} + \frac{0.2 z^{-1}}{1-0.2^2 z^{-2}}$$

ROC:



$|z| > 2$

Alternative

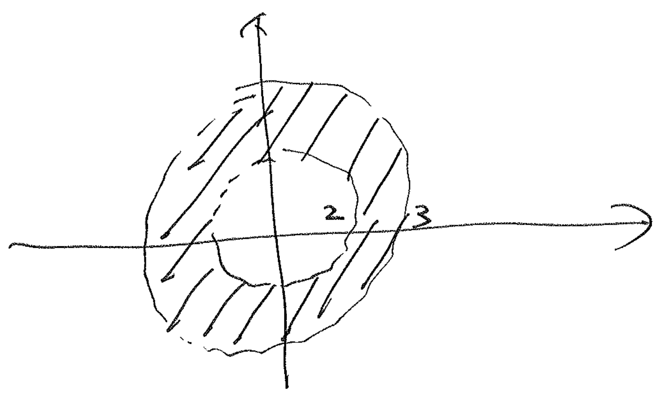
$$X(z) = \sum_{n=-\infty}^{-1} 3^n z^{-n} + \sum_{n=0}^{\infty} 2^n z^{-n}$$

$$= \sum_{n=1}^{\infty} 3^{-n} z^n + \sum_{n=0}^{\infty} 2^n z^{-n}$$

if $|3^{-1}z| < 1$ & $|2z^{-1}| < 1$

$$= \frac{3^{-1}z}{1-3^{-1}z} + \frac{1}{1-2z^{-1}}$$

ROC: $2 < |z| < 3$



uestion 5: [10%, Work-out question] Consider a continuous-time differential equation system:

$$y(t) = \frac{1}{3}y(t-10) - \frac{1}{2}y(t-20) + x(t). \tag{3}$$

When the input is $x(t) = \sum_{k=1}^{\cancel{100}\ 6} \left(\frac{1}{2}\right)^k \cos\left(\frac{k\pi}{5}t\right)$, find the corresponding output $y(t)$.

$$H(j\omega) = \frac{1}{1 - \frac{1}{3}e^{-j\omega \times 10} + \frac{1}{2}e^{-j\omega \times 20}}$$

$k=1$

$$H(j\tfrac{k\pi}{5}) = \frac{1}{1 - \frac{1}{3}e^{-j\frac{\pi}{5}\times 10} + \frac{1}{2}e^{-j\omega\frac{\pi}{5}\times 20}} = \frac{6}{7}$$

$k=2$

$$H(j\tfrac{k\pi}{5}) = \frac{6}{7}$$

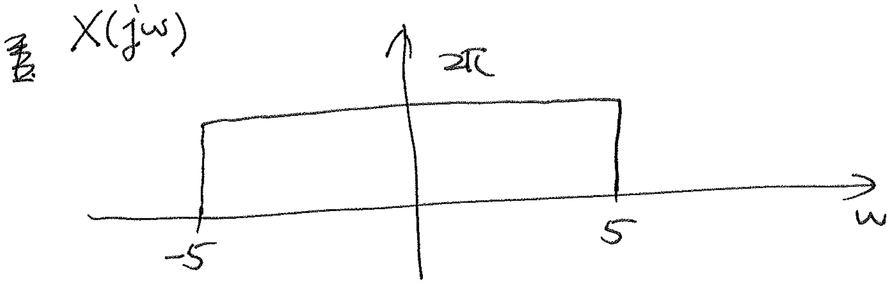
$k=3, \ \cdots \ 6$

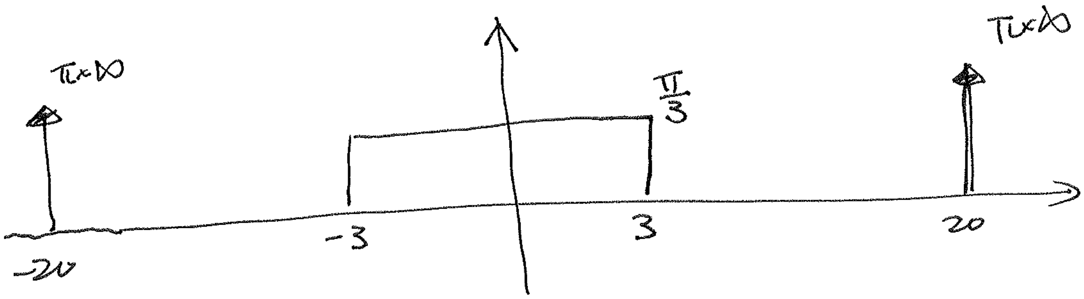
$$H(j\tfrac{k\pi}{5}) = \frac{6}{7}$$

$$\Rightarrow y(t) = \frac{6}{7}x(t) \ \#$$

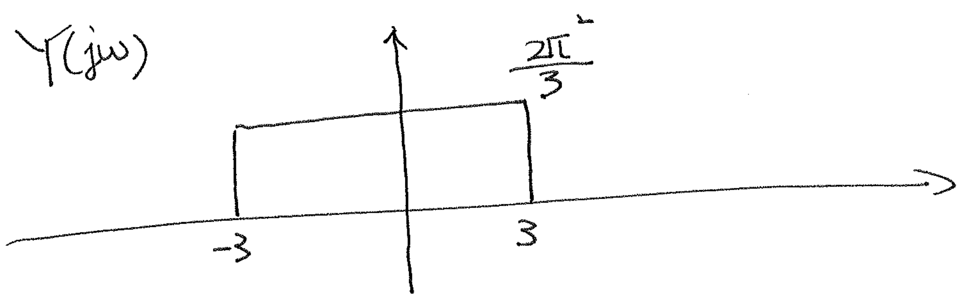
Question 6: [12%, Work-out question] Suppose $x(t) = \frac{2\sin(5t)}{t}$ and $h(t) = \frac{\sin(3t)}{3t} + \cos(20t)$.

1. [4%] Suppose $y(t) = x(t) * h(t)$. Plot $Y(j\omega)$ for the range of $-20 < \omega < 20$.
2. [4%] Suppose $z(t) = x(t) \cdot h(t)$. Plot $Z(j\omega)$ for the range of $-20 < \omega < 20$.
3. [4%] Find $\int_{-\infty}^{\infty} x(t)dt$ and $\int_{-\infty}^{\infty} (x(t))^2 dt$.





1.



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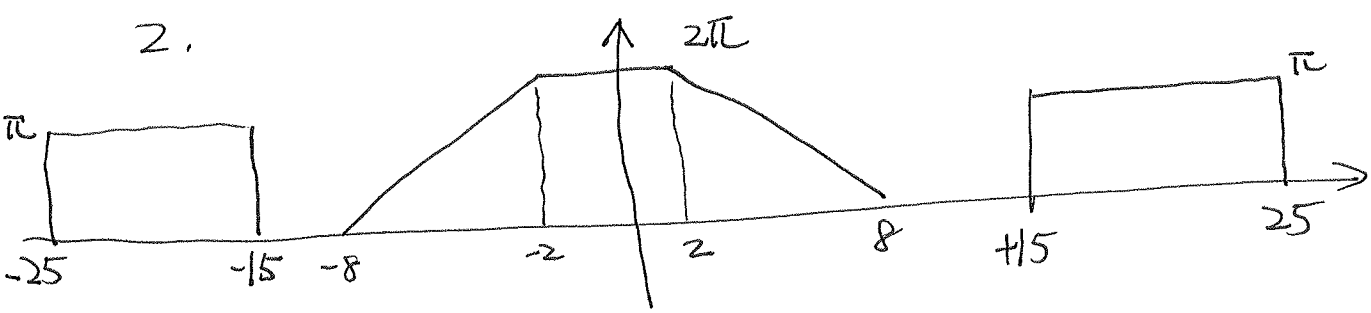


3. $\int_{-\infty}^{\infty} x(t)dt = X(j\cdot 0) = 2\pi$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 20\pi$$

Question 7: [12%, Work-out question] Consider two discrete-time signals

$$x[n] = \begin{cases} 3 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ 0 & \text{if } n = 2 \\ \text{periodic with period 3} & \end{cases} \tag{5}$$

and

$$h[n] = \frac{\sin(\frac{\pi \cdot n}{4})\cos(\frac{\pi \cdot n}{4})}{\pi \cdot n}. \tag{6}$$

1. [5%] Find the expression of the DTFT $X(e^{j\omega})$.
2. [4%] Plot the DTFT $H(e^{j\omega})$ for the range of $-4\pi \leq \omega \leq 4\pi$.
3. [3%] Let $y[n] = x[n] * h[n]$. Find the expression of $y[n]$.

1. $a_k = \frac{1}{3}\left(3 \cdot 1 + 1 \cdot e^{-jk\frac{2\pi}{3} \times 1} + 0\right)$

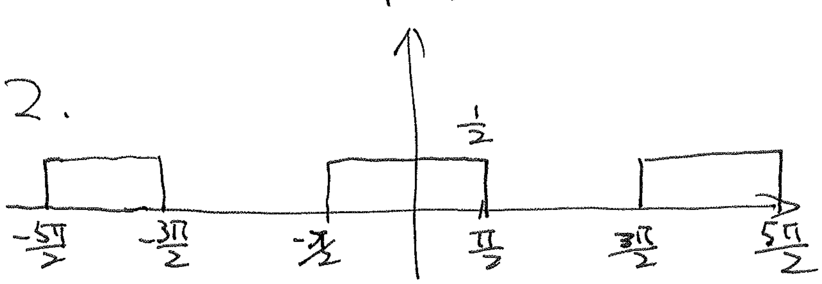
$\Rightarrow a_0 = \frac{1}{3}(3+1) = \frac{4}{3}$

$a_1 = \frac{1}{3}\left(3 + e^{-j\frac{2\pi}{3}}\right)$

$a_2 = \frac{1}{3}\left(3 + e^{+j\frac{2\pi}{3}}\right.$

$$X(e^{j\omega}) = \begin{cases} \sum_{k=0}^{2} 2\pi \cdot a_k \cdot \delta(\omega - k\frac{2\pi}{3}) & \text{if } 0 \leq \omega < 2\pi \\ \text{periodic with period } 2\pi & \end{cases}$$

2.



3. $y[n] = \frac{4}{3} \cdot \frac{1}{2} = \frac{2}{3}$ #

uestion 8: [15%, Multiple-choice question] Consider two signals $h_1(t) = \max(\sin(t), \sin(\sqrt{3}t))$ and

$$h_2[n] = \begin{cases} 2^{-n} & \text{if } n \text{ is even and } n \geq 0 \\ 2^{-n-3} & \text{if } n \text{ is odd and } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases} \quad (7)$$

1. [1.25%] Is $h_1(t)$ periodic? No
2. [1.25%] Is $h_2[n]$ periodic? No
3. [1.25%] Is $h_1(t)$ even or odd or neither? ~~odd~~ neither
4. [1.25%] Is $h_2[n]$ even or odd or neither? neither
5. [1.25%] Is $h_1(t)$ of finite energy? No
6. [1.25%] Is $h_2[n]$ of finite energy? Yes

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless? No
2. [1.25%] Is System 2 memoryless? No
3. [1.25%] Is System 1 causal? No
4. [1.25%] Is System 2 causal? Yes
5. [1.25%] Is System 1 stable? No
6. [1.25%] Is System 2 stable? Yes