Midterm #3 of ECE301, Prof. Wang's section

8–9pm Thursday, November 17, 2011, EE 170,

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, e-mail address, and signature in the space provided on this page, **NOW!**

2. This is a closed book exam.

3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.

4. There are 16 pages in the exam booklet. Use the back of each page for rough work.

5. Neither calculators nor help sheets are allowed.

Name: Solutions

Student ID:

E-mail:

Signature:

Question 1: [15%, Work-out question, Outcomes 3, 4, and 5]

1. [15%] A discrete-time signal $x[n]$ is described as follows

$$x[n] = \begin{cases} 2 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \text{ or } -1 \\ 0 & \text{if } n = 2 \end{cases} \tag{1}$$

and $x[n]$ is periodic with period 4.

Find the Fourier transform $X(e^{j\omega})$ and plot it for the range of $\omega = -2\pi$ to 2π.

$$X(e^{j\omega}) = \sum_n x[n] e^{-j\omega n}$$

$$X'(e^{j\omega}) = (1) e^{-j\omega(-1)} + 2 e^{-j\omega(0)} + (1) e^{-j\omega(1)}$$

$$= 2 + 2\cos(\omega) = \underline{2(1+\cos\omega)}$$

ω	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\pi/2$	π	$\frac{3\pi}{2}$
$X'(e^{j\omega})$	4	2	0	2	4	2	0	2

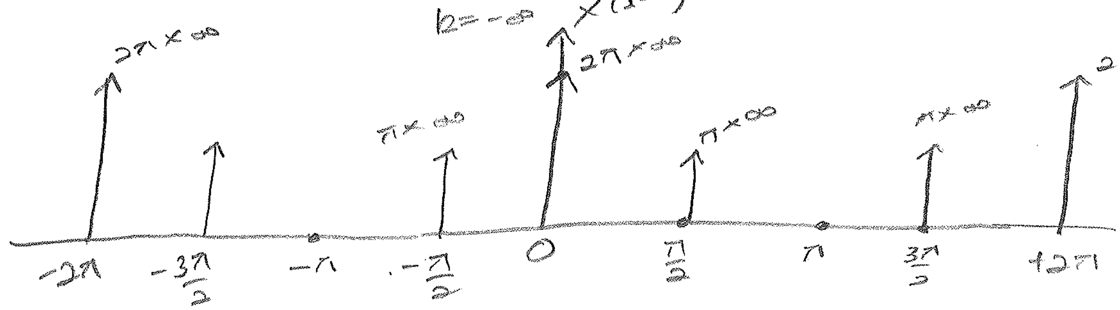
where $x'[n]$ is $x[n] = \begin{cases} 2 & \text{if } n=0 \\ 1 & \text{if } n = 1/-1 \\ 0 & \text{if } n=2 \end{cases}$ and not periodic

$$\Rightarrow x[n] = x'[n] * \sum_{k=-\infty}^{\infty} \delta[n-4k]$$

$$\Rightarrow X(e^{j\omega}) = \frac{\pi}{2} \sum_{k=-\infty}^{\infty} X(e^{j\omega}) \delta(\omega - k\tfrac{\pi}{2})$$

$$= \frac{\pi}{2} \sum_{k=-\infty}^{\infty} X(e^{j(k\pi/2)}) \delta(\omega - k\tfrac{\pi}{2})$$

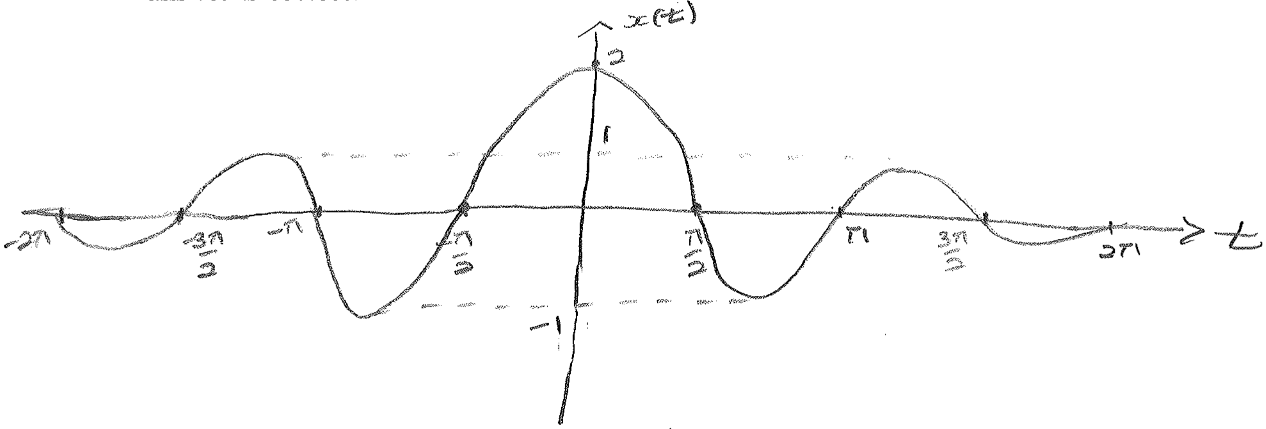
$$= \pi \sum_{k=-\infty}^{\infty} (1+\cos(k\tfrac{\pi}{2})) \delta(\omega - k\pi/2)$$



Question 2: [25%, Work-out question, Outcomes 3, 4, and 5]

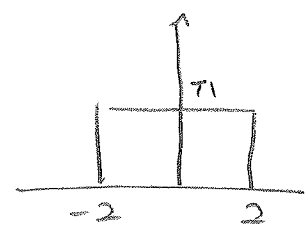
1. [6%] $x(t) = \frac{\sin(2t)}{t}$, plot $x(t)$ for the range of $t = -2\pi$ to 2π. Carefully mark the height of the main lobe and the locations it intersects the horizontal axis.
2. [4%] Find the Fourier transform $X(j\omega)$ and plot it for the range of $\omega = -12$ to 12.
3. [15%] $y(t) = x(t) \cdot \sin(3t) \cdot \sin(6t)$. Find the Fourier transform $Y(j\omega)$ and plot it for the range of $\omega = -12$ to 12. If you do not know the answer to the previous question, you can write $Y(j\omega)$ in terms of $X(j\omega)$. You will get 12 points if your answer is correct.

(1)



(2) $x(t) = \frac{\sin(2t)}{t}$

$\frac{\sin 2t}{\pi t} \xleftrightarrow{FT} \begin{cases} 1 & |\omega| < 2 \\ 0 & |\omega| > 2 \end{cases}$



$\Rightarrow x(t) = \frac{\sin(2t)}{t} \xleftrightarrow{FT} \begin{cases} \pi, & |\omega| < 2 \\ 0, & |\omega| > 2 \end{cases}$

(3)

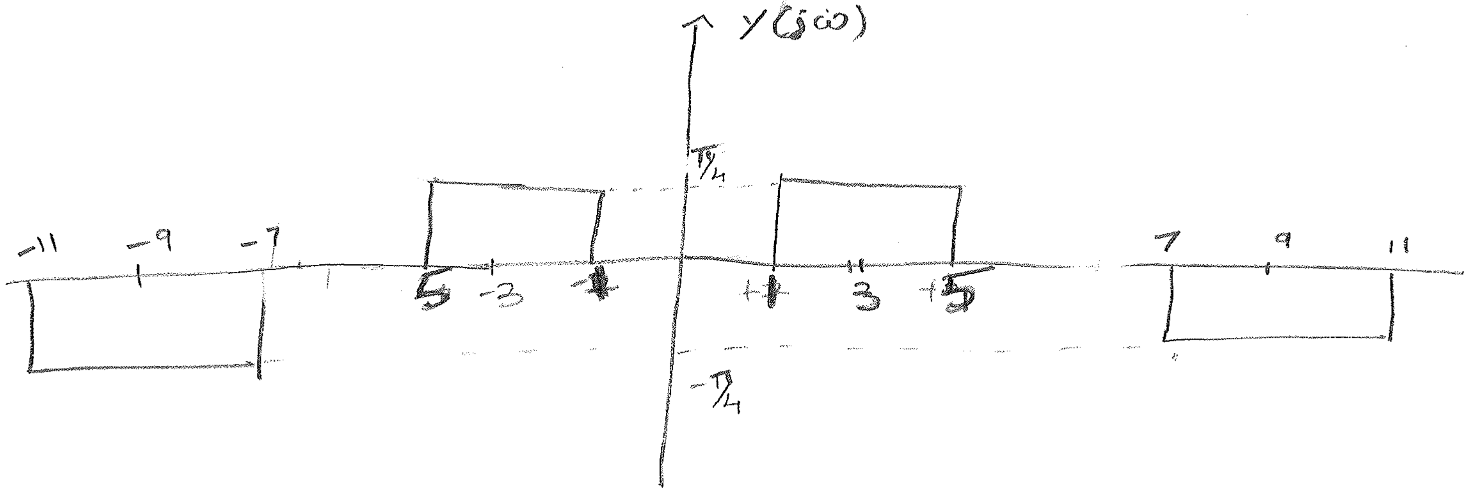
$\sin(3t) \xleftrightarrow{FT} \frac{\pi}{j}[\delta(\omega-3) - \delta(\omega+3)]$

$\sin(6t) \xleftrightarrow{FT} \frac{\pi}{j}[\delta(\omega-6) - \delta(\omega+6)]$

$Y(j\omega) = \frac{1}{4\pi^2} X(j\omega) * [-\pi^2(\delta(\omega-3) - \delta(\omega+3)) * (\delta(\omega-6) - \delta(\omega+6))]$

$= \frac{-\pi^2}{4\pi^2}[X(j(\omega-9)) + X(j(\omega+9)) - X(j(\omega-3)) - X(j(\omega+3))]$

$$Y(j\omega) = \frac{1}{4}\left[X(j(\omega-3)) + X(j(\omega+3)) - X(j(\omega-9)) - X(j(\omega+9))\right]$$



Question 3: [32%, Work-out question, Outcomes 4 and 5]

1. [10%] Compute the Fourier transform $X(j\omega)$ of $x(t) = \frac{(\sin(3\pi(t-0.5)))^2}{(\pi(t-0.5))^2}$ and write down its expression.
2. [6%] Is $X(j\omega)$ periodic? Compute the value of $\int_{-\infty}^{\infty} X(j\omega)d\omega$.
3. [10%] Compute the Fourier transform $X(e^{j\omega})$ of $x[n] = ne^{-n}\mathcal{U}[n]$.
4. [6%] Is $X(e^{j\omega})$ periodic? Compute the value of $\sum_{n=1}^{\infty} ne^{-n}$. (Hint: You need to use the answer from the previous question.)

(1)

$$\frac{\sin(3\pi t)}{\pi t} \xleftrightarrow{FT} \begin{cases} 1 & |\omega| < 3\pi \\ 0 & |\omega| > 3\pi \end{cases}$$

$$\Rightarrow \left(\frac{\sin(3\pi t)}{\pi t}\right)^2 \xleftrightarrow{FT} \begin{cases} -\frac{\omega}{2\pi} + 3 & 0 \le \omega \le 6\pi \\ \frac{\omega}{2\pi} + 3 & -6\pi \le \omega \le 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$\Rightarrow \frac{\sin(3\pi(t-0.5))^2}{(\pi(t-0.5))^2} \xleftrightarrow{FT} \begin{cases} e^{-j\omega(0.5)}\left(-\frac{|\omega|}{2\pi} + 3\right) & 0 \le |\omega| \le 6\pi \\ 0 & \text{otherwise} \end{cases}$$

(2) NO.

$$\int_{-\infty}^{\infty} X(j\omega)d\omega = 2\pi x(0) \quad \frac{(\sin(\frac{3\pi}{2}))^2}{(\frac{\pi}{2})^2}$$

$$= \frac{8}{\pi}$$

(3)

$$e^{-n}u[n] \xleftrightarrow{FT} \frac{1}{1-e^{-1}e^{-j\omega}} = \frac{1}{1-e^{-j\omega-1}}$$

$$\Rightarrow ne^{-n}u[n] \xleftrightarrow{FT} -j\frac{1}{(1-e^{-1}e^{-j\omega})^2}(-e^{-1}(-j)e^{-j\omega})$$

$$= \frac{e^{-j\omega-1}}{(1-e^{-j\omega-1})^2}$$

(4) $\sum_{n=1}^{\infty} n e^{-n} = X(e^{j(0)}) = \frac{e^{-1}}{(1-e^{-1})^2}$

YES

Question 4: [28%, Work-out question, Outcomes 3, 4, and 5] Consider an AM system, which sends the input signal $x(t)$ over a cos carrier of frequency $\frac{1}{\pi} \approx 0.637$ Hz.

More specifically, we denote the input signal as $x(t)$ and use $y(t)$ to denote the AM modulated signal, which will be sent out by the AM transmitter.

1. [4%] What is the value of the carrier frequency ω_c with the unit being (rad/sec)? Write down the input/output relationship (equation) between $x(t)$ and $y(t)$.

2. [12%] Consider the receiver end. To demodulate the original signal from $y(t)$, the receiver first construct $w(t) = 2 \cdot y(t) \cdot \cos(\omega_c t)$ and then passes $w(t)$ through a low-pass filter with cutoff frequency ω_{cutoff}.

   Suppose we also know that $x(t) = \cos(t)$, plot the Fourier transforms $X(j\omega)$, $Y(j\omega)$, and $W(j\omega)$ for the range of $\omega = -6$ to 6.

3. [12%] Suppose that Prof. Wang forgot the expression of the impulse response of a low-pass filter, and decided to pass $w(t)$ through an LTI system with frequency response

$$H(j\omega) = \begin{cases} 1 + 0.5\omega & \text{if } -2 < \omega \leq 0 \\ 1 - 0.5\omega & \text{if } 0 < \omega \leq 2 \\ 0 & \text{otherwise} \end{cases} . \tag{2}$$

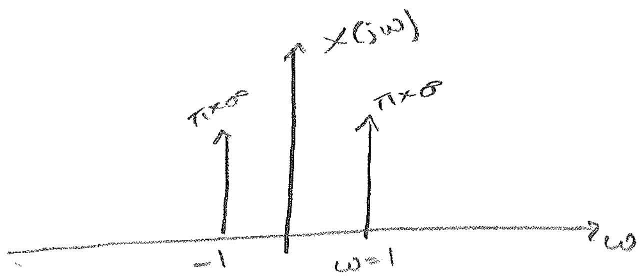
   Let $z(t)$ denote the output after passing $w(t)$ through this system. Find the expression of $z(t)$.

   If you do not know the answer of Q4.2, you can assume that $w(t) = \sum_{n=-\infty}^{\infty} 2^{-|n|} e^{jnt}$ and solve this question. You will still get 12 points if your answer is correct.

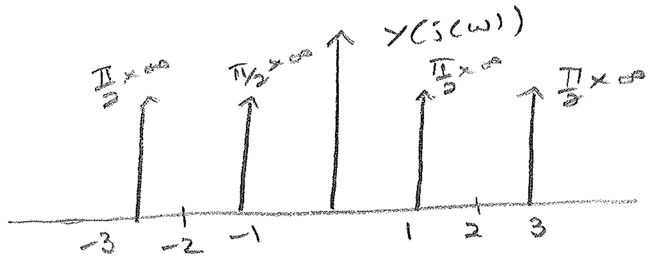
(1.) $\omega_c = 2\pi \times \frac{1}{\pi} = 2$ rad/sec.

$y(t) = x(t)\cos(2t)$

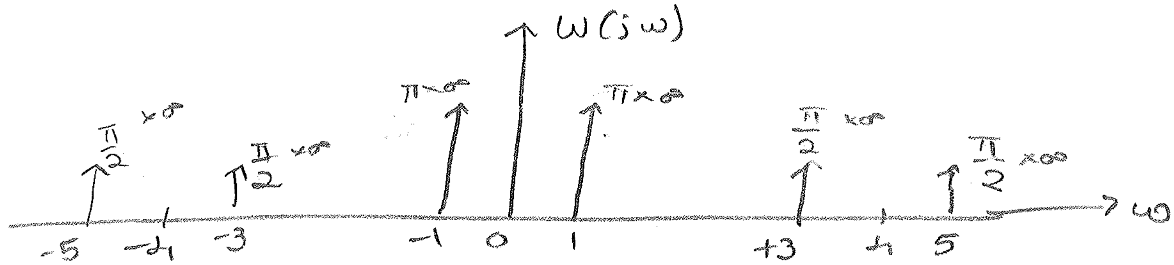
(2)



$$Y(j\omega) = \frac{X(j\omega) * (\pi\delta(\omega-2) + \pi\delta(\omega+2))}{2\pi}$$



$$W(j\omega) = \frac{1}{2\pi} Y(j\omega) * \pi\left(\delta(\omega-2) + \delta(\omega+2)\right) \times 2$$



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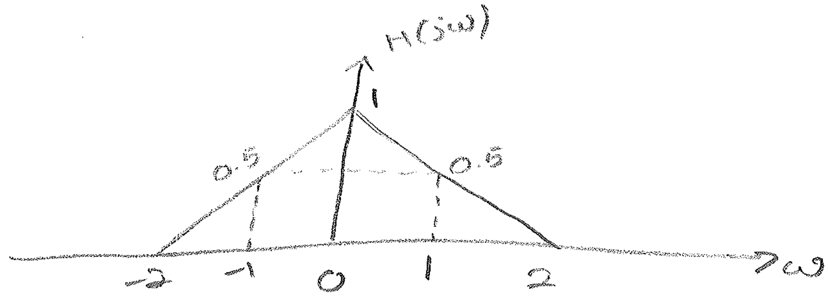

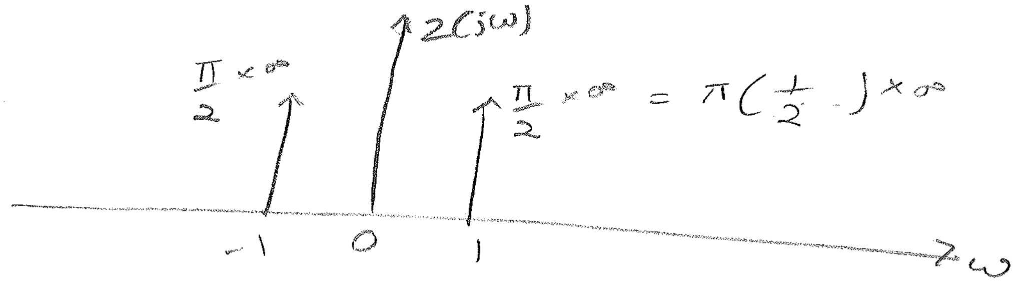

$$\Rightarrow z(t) = \frac{1}{2}\cos(t)$$

If $w(t) = \sum_{n=-\infty}^{\infty} 2^{-|n|} e^{jnt}$

$$z(t) = \frac{1}{2}\left[\frac{1}{2}\left(e^{jt} + e^{-jt}\right)\right] + 1$$

$$= \frac{\cos(t)}{2} + 1$$