Final Exam of ECE301, Prof. Wang's section

Monday 3:20–5:20pm, December 12, 2011, PHYS 114.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have two hours to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Neither calculators nor help sheets are allowed.

Name:

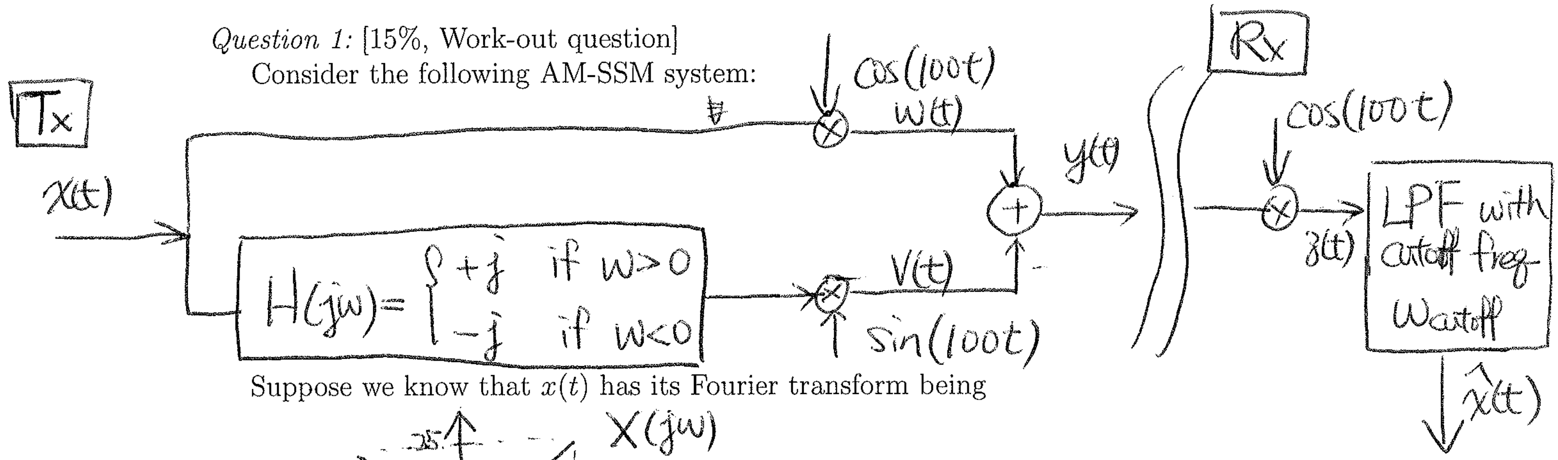
Student ID:

E-mail:

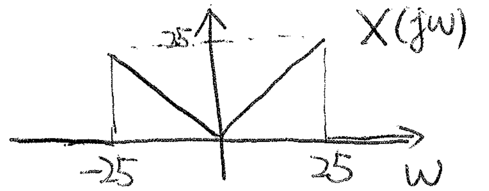
Signature:

Question 1: [15%, Work-out question]

Consider the following AM-SSM system:



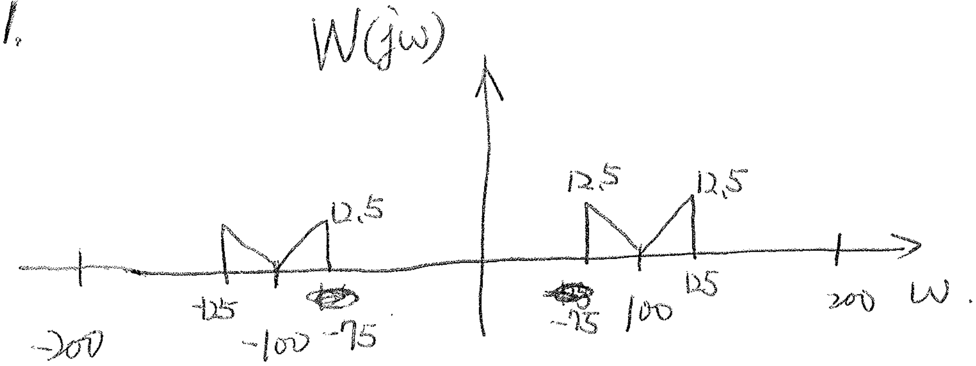
Suppose we know that $x(t)$ has its Fourier transform being



Answer the following questions with carefully marked horizontal and vertical axes:

1. [3%] Plot $W(j\omega)$, the Fourier transform of $w(t)$ for the range of $-250 < \omega < 250$.
2. [3%] Plot $V(j\omega)$, the Fourier transform of $v(t)$ for the range of $-250 < \omega < 250$.
3. [3%] Plot $Y(j\omega)$, the Fourier transform of $y(t)$ for the range of $-250 < \omega < 250$.
4. [3%] Plot $Z(j\omega)$, the Fourier transform of $z(t)$ for the range of $-250 < \omega < 250$.
5. [3%] What is the range of the cutoff frequency ω_{cutoff} of the low-pass filter for which the demodulated signal $\hat{x}(t)$ is identical to that of $x(t)$?

1.



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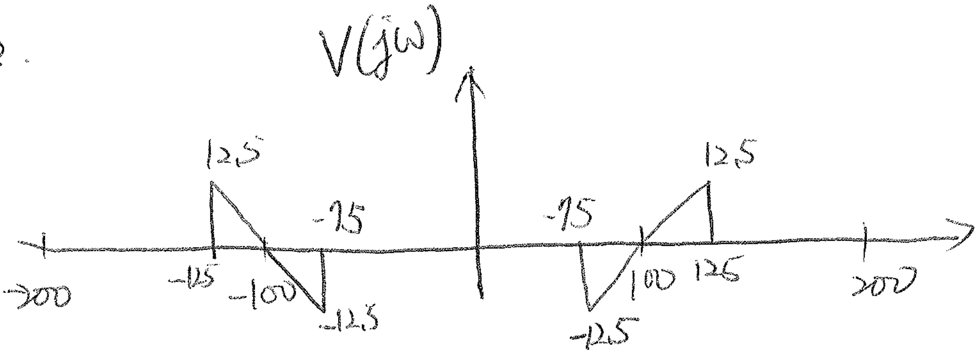

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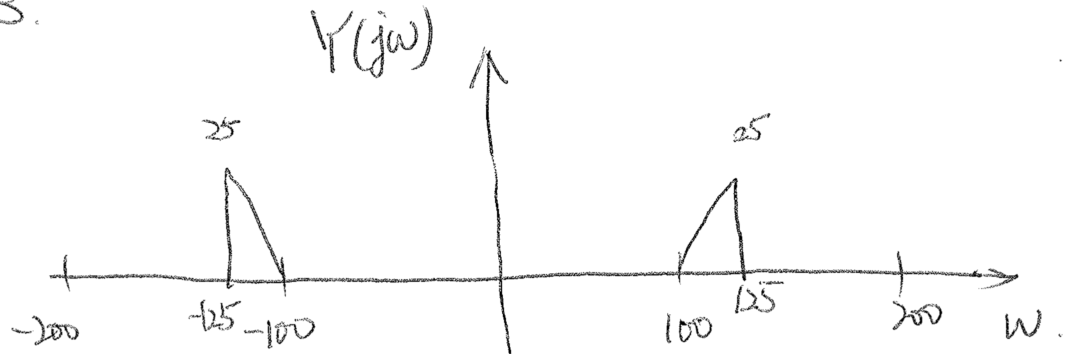

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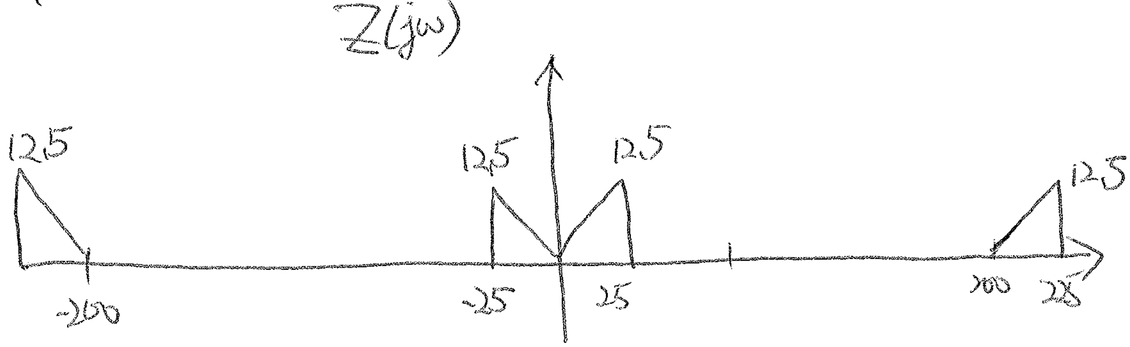


5. W_{cutoff} can be in the range of

+25 ~ 200

Question 2: [14%, Work-out question]

1. [1%] Let $x(t) = \cos(\pi t)$. What is the corresponding Nyquist frequency? Please use the unit Hertz for your answer.

2. [4%] If we sample $x(t)$ with sampling period $T = \frac{1}{2}$ seconds, we can convert the continuous-time signal $x(t)$ into a discrete-time signal $x[n]$. Plot $x[n]$ for the range of $-2 \leq n \leq 2$. Plot only. There is no need to write down the actual expression.

3. [4%] If we use the impulse-train sampling with sampling period $T = \frac{1}{2}$ seconds, we can convert $x(t)$ into another signal $x_p(t)$. Plot the Fourier transform $X_p(j\omega)$ of $x_p(t)$ for the range of $-5.5\pi \leq \omega \leq 5.5\pi$. Plot only. There is no need to write down the actual expression.

4. [5%] The discrete-time signal $x[n]$ can be stored in a computer as an array. Suppose the hard drive is corrupted and the signal values outside the range $|n| \leq 1$ are erased and set to zero. That is, the new signal $y[n]$ becomes

$$y[n] = \begin{cases} x[n] & \text{if } |n| \leq 1 \\ 0 & \text{otherwise} \end{cases}. \tag{1}$$

Suppose we pass $y[n]$ through the ideal reconstruction, also known as the band-limited interpolation. Plot the reconstructed signal $y(t)$ for the range of $-2 \leq t \leq 2$. Plot only. There is no need to write down the actual expression.

Hint: If you do not know $x[n]$ in the previous question, you can assume that $x[n] = \mathcal{U}[n-1]$. You will get 4 points if your answer is correct.

1. $w = \pi \left(\frac{rad}{sec}\right) = \frac{1}{2}$ (Hz)

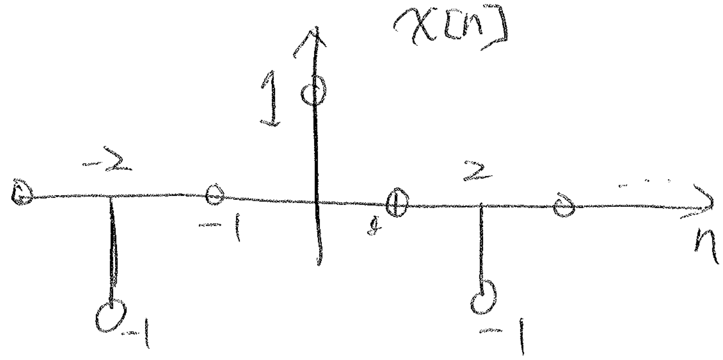
Nyquist Freq $= 2 \cdot w = 1$ (Hz)

2. $x[-3] = \cos(-1.5\pi) = 0$ $\qquad x[1] = \cos(0.5\pi) = 0$

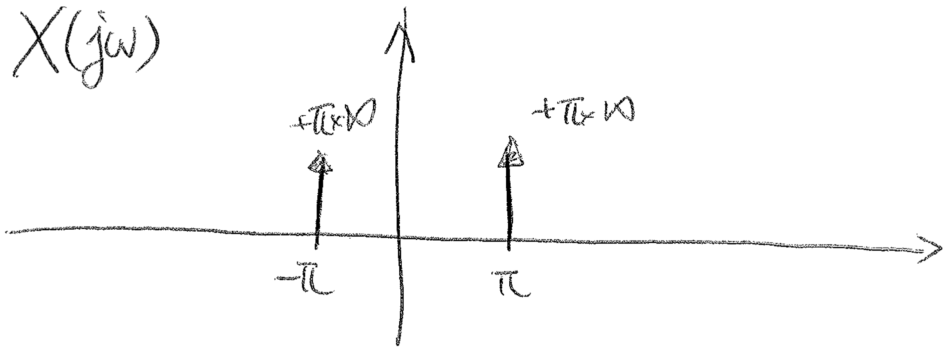
$x[-2] = \cos(-\pi) = -1$ $\qquad x[2] = \cos(\pi) = -1$

$x[-1] = \cos(-0.5\pi) = 0.$ $\qquad x[3] = \cos(1.5\pi) = 0.$

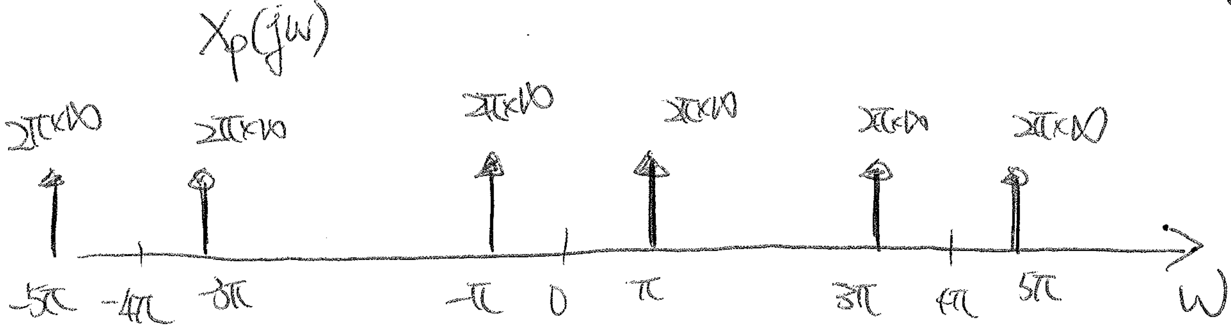
$x[0] = \cos(0) = 1$



3.



$$\omega_s = \frac{2\pi}{T} = 4\pi$$



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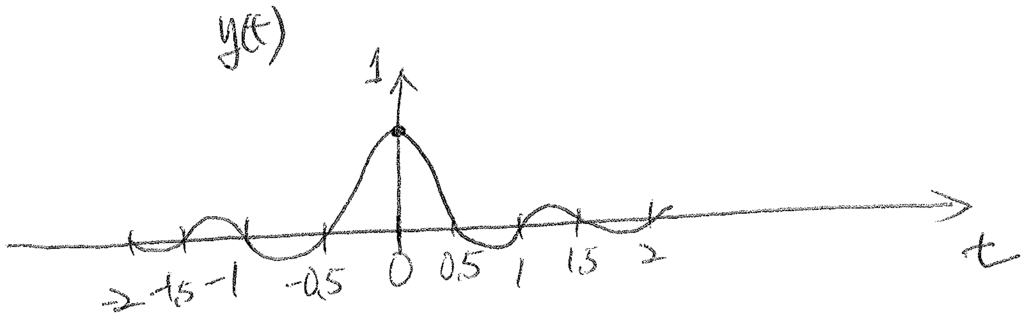
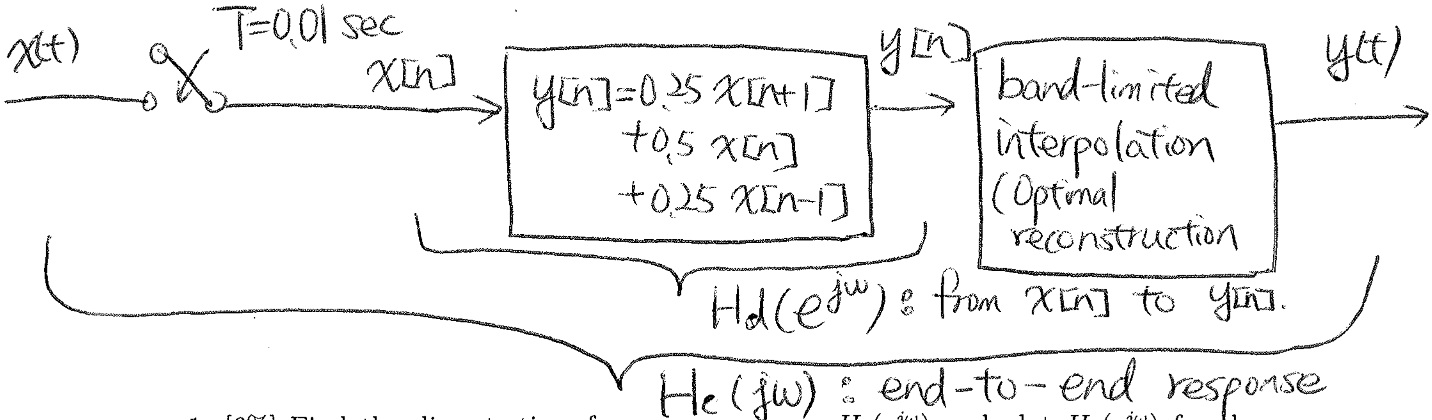


Question 3: [14%, Work-out question] Consider the following discrete-time processing system for continuous-time signals.



1. [6%] Find the discrete-time frequency response $H_d(e^{j\omega})$ and plot $H_d(e^{j\omega})$ for the range of $-2\pi \leq \omega \leq 2\pi$.

2. [8%] Find the continuous-time frequency response $H_c(j\omega)$ plot $H_c(j\omega)$ for the range of $-200\pi < \omega < 200\pi$.

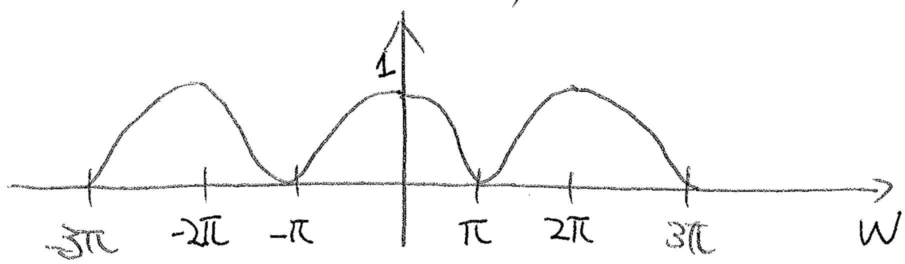
   Hint: If you do not know the answer to this question, you can answer the following questions instead: (i) If the input is $x(t) = 5$, what is the output? (ii) What type of filter is the overall system? A low-pass filter? A high-pass filter? Or a band-pass filter? Use a single sentence to justify your answer. If both your answers are correct, you will still get 4 points.

1.

$$Y(e^{j\omega}) = 0.25\, e^{+j\omega} X(e^{+j\omega}) + 0.5\, X(e^{+j\omega}) + 0.25\, e^{+j\omega\cdot(-1)} X(e^{+j\omega})$$

$$H(e^{+j\omega}) = 0.25\, e^{+j\omega} + 0.5 + 0.25\, e^{-j\omega}$$

$$= 0.5\,(1 + \cos(\omega))$$

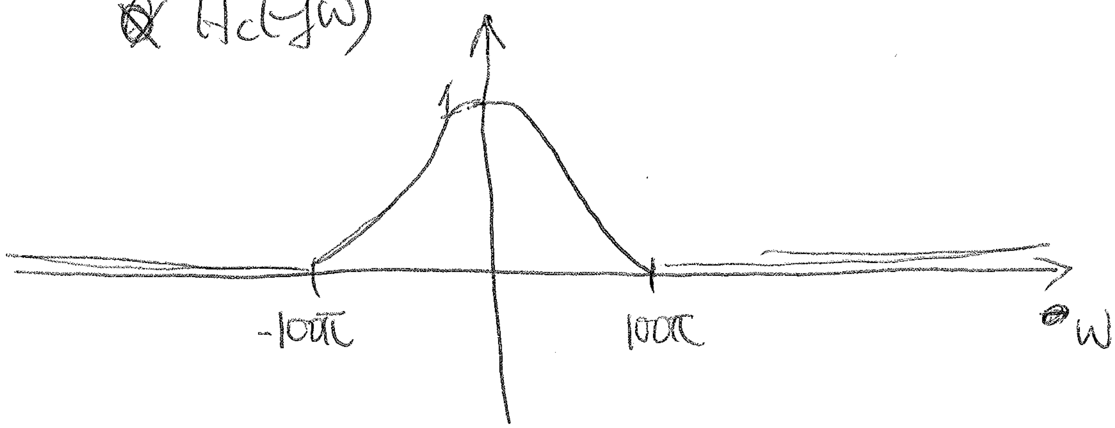


2.

$T = 0.01$ sec

$W_s = 200\pi$

$H_c(jw)$

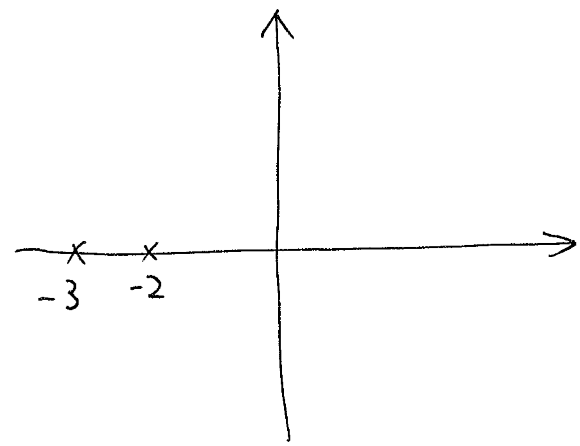


Question 4: [20%, Work-out question] Consider a discrete-time signal $x[n]$ with the expression of its Z-transform being $X(z) = \frac{1}{z+5+6z^{-1}}$. Suppose we also know that the discrete-time Fourier transform of $x[n]$ exists. Answer the following questions.

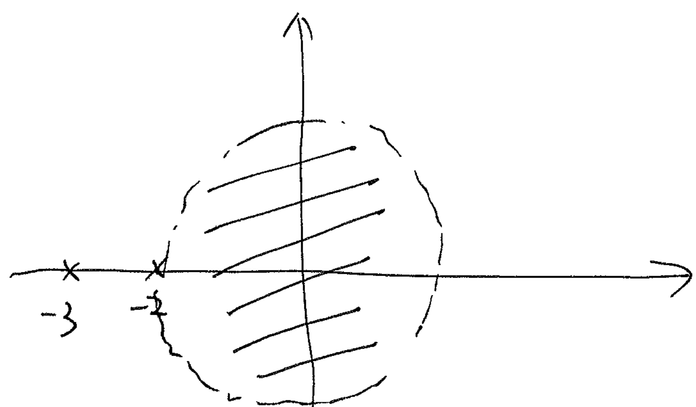
1. [4%] Find out all the poles of $X(z)$ and draw them in a z-plane.
2. [4%] What is the ROC of the Z-transform $X(z)$?
3. [4%] What is the value of $\sum_{n=-\infty}^{\infty} x[n]$. Hint: You may want to find the discrete-time Fourier transform $X(e^{j\omega})$ first.
4. [8%] Find the expression of $x[n]$.

1. $$X(z) = \frac{1}{z(1+2z^{-1})(1+3z^{-1})}$$

poles : $z = \cancel{0}, -2, -3.$



2. ROC contains the unit circle



3. $$\sum_{n=-\infty}^{\infty} x[n] = X(1) = \frac{1}{12}$$ ※

4. $$X(z) = z^{-1} \cdot \left(\frac{-2}{1+2z^{-1}} + \frac{3}{1+3z^{-1}} \right)$$

$x[n]$ = Time-shift to the right $\left((-2)\cdot(-1)\cdot(-2)^n u[-n-1] + 3\cdot(-1)\cdot(-3)^n u[-n-1] \right)$

$$= 2 \cdot (-2)^{n-1} u[-n]$$

$$- 3 \cdot (-3)^{n-1} u[-n]$$

$$= \left[(-3)^n - (-2)^n \right] u[-n]$$

Question 5: [10%, Work-out question] Consider the following differential equation:

$$\frac{d^2}{dt^2}y(t) + 2\frac{d}{dt}y(t) + y(t) = \frac{d}{dt}x(t) + x(t) \tag{2}$$

Find out the output $y(t)$ when the input is $x(t) = \cos(t) + \sin(\sqrt{3}t)$. Hint: If you do it right, there is no need to use partial fraction.

$$\left((j\omega)^2 + 2(j\omega) + 1\right) Y(j\omega) = (j\omega + 1) X(j\omega)$$

$$\Rightarrow H(j\omega) = \frac{j\omega + 1}{(j\omega)^2 + 2j\omega + 1}$$

$$H(j\cdot 1) = \frac{1+j}{2j+0}$$

$$|H(j\,1)| = \sqrt{2}/2 \qquad \measuredangle H(j\cdot 1) = \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4}.$$

$$H(j\cdot\sqrt{3}) = \frac{\sqrt{3}j + 1}{-3 + 2\sqrt{3}j + 1} = \frac{\sqrt{3}j + 1}{2\sqrt{3}j - 2}.$$

$$|H(j\sqrt{3})| = 2/4 = \frac{1}{2}. \qquad \measuredangle H(j\sqrt{3}) = \frac{\pi}{3} - \frac{2\pi}{3}$$

$$= -\frac{\pi}{3}.$$

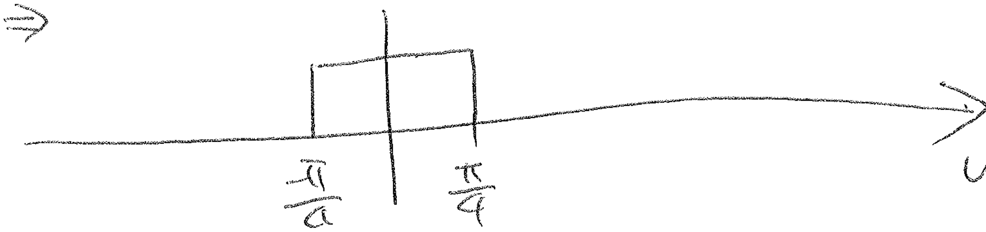
$$y(t) = \cos\left(t - \frac{\pi}{4}\right)\cdot\frac{\sqrt{2}}{2} + \sin\left(\sqrt{3}t - \frac{\pi}{3}\right)\cdot\frac{1}{2} \#$$

Question 6: [12%, Work-out question] Compute the continuous-time Fourier transform $H(j\omega)$ of the following signal. You can use a plot to represent your $H(j\omega)$. No need to write-down the actual expression:

$$h(t) = \frac{\sin(\frac{\pi t}{4})\sin(\frac{\pi t}{2})}{\pi^2 t} \tag{3}$$

$$h(t) = t \cdot \left(\frac{\sin(\frac{\pi t}{4})}{\pi t}\right)\left(\frac{\sin(\frac{\pi t}{2})}{\pi t}\right)$$

$\Rightarrow$



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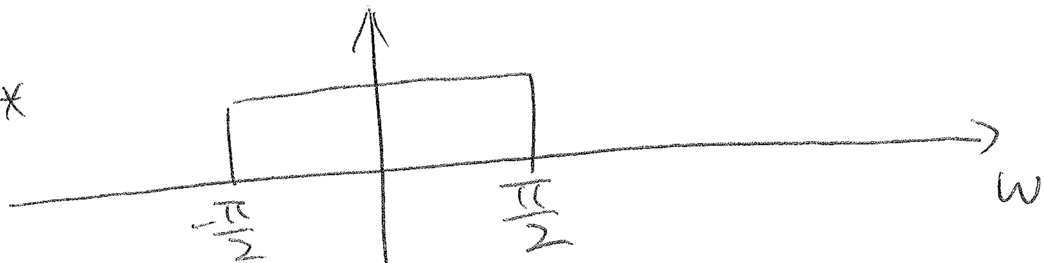

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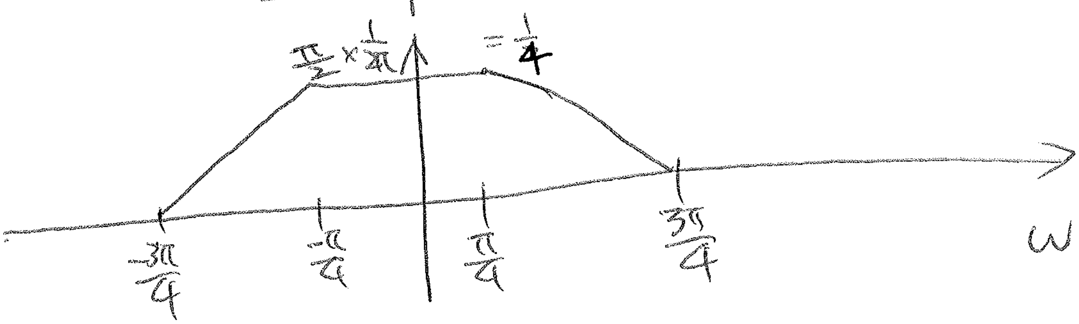
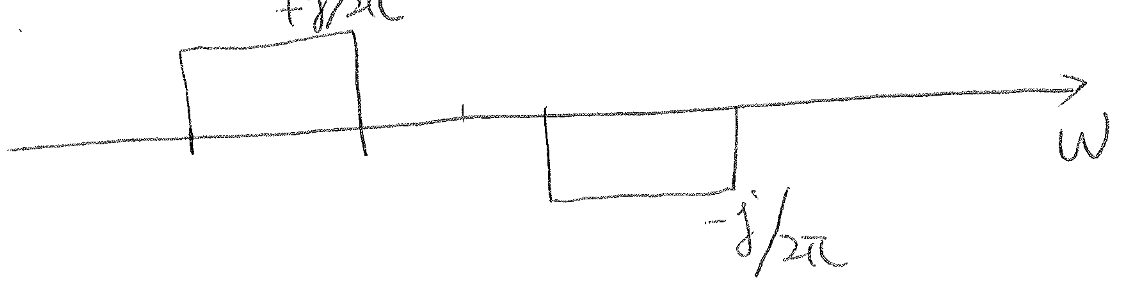


$\downarrow \; j\frac{d}{d\omega}$

$=$



Question 7: [15%, Multiple-choice question] Consider two signals $h_1(t) = \int_{s=-2t}^{2t} \cos(\pi s) ds$ and $h_2[n] = \max(\sin((n^2 + 0.5)\pi), 0)$

1. [1%] Is $h_1(t)$ periodic? Yes
2. [1%] Is $h_2[n]$ periodic? Yes
3. [1%] Is $h_1(t)$ even or odd or neither? odd
4. [1%] Is $h_2[n]$ even or odd or neither? even
5. [1%] Is $h_1(t)$ of finite energy? No
6. [1%] Is $h_2[n]$ of finite energy? No

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 causal? No
2. [1.25%] Is System 2 causal? No
3. [1.25%] Is System 1 stable? No
4. [1.25%] Is System 2 stable? No
5. [1.25%] Is System 1 invertible? No
6. [1.25%] Is System 2 invertible? No